CORRESPONDENCE.

SOLUTION OF PROBLEM PROPOSED BY PROFESSOR DE MORGAN.

To the Editor of the Assurance Magazine.

Sir,—In the October number of the Assurance Magazine, Professor De Morgan, referring to Surrender Values of the form $1-\frac{1+a_y}{1+a_x}$ or $\frac{a_x-a_y}{1+a_x}$, says:—

"Now a_x-a_y is the value to (x) of a counter-survivorship—as we "may call it—of the following kind. The executors of the first who dies "pay an annuity of £1 to the survivor; and $(a_x-a_y)\div(1+a_x)$ is the "whole life premium which (x) should pay to be put in this position. "How, from the nature of this contract, does it follow that one payment of "this premium, over and above the annual premium which (x) should pay, "admits (y) to a policy of £1 at the premium for the age (x)?"

The following appears to me to be the reason.

Since a_x-a_y is the value of a counter-survivorship annuity of £1, $\frac{a_x-a_y}{1+a_x}$ will equal the value of a similar annuity of $\frac{1}{1+a_x}$.

It is required to prove that if there be paid to an Office a single premium of $\frac{a_x-a_y}{1+a_x}$, and an annual payment during y's life of $\frac{1}{1+a_x}-d$, (the premium which x would require to pay for an assurance of £1 or a perpetuity-due of d on his death), this will entitle y to an assurance of £1 or its equivalent, a perpetuity-due of d on his death.

If $\frac{a_x-a_y}{1+a_x}$ be paid, x will *receive* from the Office, should y die first, an annuity of $\frac{1}{1+a_x}$; and, should x die first, y will *pay* the Office an annuity of $\frac{1}{1+a_x}$; if $\frac{1}{1+a_x}-d$ be paid annually during the life of x, the Office will pay £1, or a perpetuity-due of d, on death of x.

Let us suppose that y dies first, x will receive, under the first contract, an annuity of $\frac{1}{1+a_x}$; but, as the annual premium $\frac{1}{1+a_x}-d$ does not cease till the death of x, x will require to pay this premium out of his annuity of $\frac{1}{1+a_x}$, thus reducing his annuity to d, since $\frac{1}{1+a_x}-\left(\frac{1}{1+a_x}-d\right)=d$. On the death of x his heirs will become entitled to the perpetuity-due of d which will then become payable under second contract. In effect, the single payment of $\frac{a_x-a_y}{1+a_x}$ and the payment of $\frac{1}{1+a_x}-d$ annually during the life of y, will provide an assurance of £1, or its equivalent, a perpetuity-due of d, on death of y should he die before x.

Next, let us suppose that x dies first, y will *pay* to the Office under the first contract an annuity of $\frac{1}{1+a_x}$, and will *receive* from the Office the perpetuity-due of d which will become payable under second contract through death of x. This reduces y's payment to the Office to $\frac{1}{1+a_x}-d$; and this payment will cease on the death of y, when his heirs will become entitled to the perpetuity-due of d. In effect, the single payment of $\frac{a_x-a_y}{1+a_x}$ and the payment of $\frac{1}{1+a_x}-d$ annually during the life of y, will provide an assurance of £1, or its equivalent, a perpetuity-due of d, on death of y should he survive x.

It will therefore be seen from the above that, whether y dies first or second, the single payment of $\frac{a_x-a_y}{1+a_x}$ and an annual payment during the life of y of the premium which a person aged x would require to pay for an assurance of £1, viz. $\frac{1}{1+a_x}-d$, will entitle y to a policy for £1, or its equivalent, a perpetuity-due of d, on death of y.

I remain,
Your obedient servant,
THOMAS MARR.

Glasgow, 24th October, 1867.