CONCEPTS OF REFERENCE FRAMES FOR A DEFORMABLE EARTH

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ABSTRACT

In order to describe geometrical facts in geodesy, geodynamics, or astronomy, one must have suitable reference frames. This paper deals with some basic concepts for the definition of both terrestrial and celestial systems which make it possible to describe position and motion of mass elements of the earth or stars, respectively. It is suggested to use such frames with respect to which coordinate changes of points on the earth's surface or of stars, respectively, become minimal. The rotation vector should be defined by applying the rotation operator on the velocity field.

1. GENERAL REQUIREMENTS OF REFERENCE FRAMES, CLASSIFICATION IN LOCAL, GLOBAL, AND UNIVERSAL FRAMES

The use of the expression "motion" requires a statement as to what is regarded as fixed. Such a definition is necessary to identify points or directions at the present epoch with respect to points or directions at an earlier epoch. There are two extreme choices to define what is fixed: 1) Local: The mass on which one is standing or on which one has a geodetic point and the observational instrument is regarded as fixed. Then all other mass elements of the deformable earth and all other celestial bodies will move with respect to this fixed point. 2) Universal: The axes of an inertial system can be considered as universally fixed. However, a strict practical realization of an inertial system is hardly possible. Later on, a certain approximation will be introduced.

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Neither of these two choices is satisfying the needs of geodesy or geodynamics. For the deformable earth, one needs something that is earth-fixed in some "average" sense over the whole globe. Let us call this "globally fixed". After one has defined what is to be regarded as fixed, one can establish a reference frame, by defining for one epoch the origin and the direction of the axes. This is in principle quite arbitrary; but once one has done this, the frame will be defined for all subsequent epochs. The special frame one may choose will, however, be less important in this contribution, since all frames which do not move with respect to each other are equivalent.

2. PRINCIPLES OF DEFINING WHAT IS FIXED, IN GEODESY AND ASTRONOMY

2.1 Global Concepts for the Earth

If we exclude such principles which privilege a single point or few points of the earth surface, there remain essentially two concepts: a dynamical one and a geometrical one. For a dynamical definition, one can take the mass centre and the principal axes of inertia. Any system with respect to which these elements do not move is globally fixed in the dynamical sense. The practical disadvantages of this definition are well known, namely that the mass centre is not directly accessible, that the equatorial axes are ill defined, and that both have motions with respect to the earth surface that may be larger than the relative motions of different parts of the surface.

These circumstances favour the geometrical concept: Every geodesist would be happy if the crust of the earth were rigid because coordinates of terrestrial points would not change in this case. Therefore, it is a good choice to fix one's reference frame in such a way that the square sum of coordinate changes or, equivalently, the square sum of the displacements of all points of the earth surface becomes minimal from one epoch to the immediately following. This means that the square sum of all velocities $\underset{\sim}{v}$ is minimal.

$$\iint_{\Sigma} |\underset{\sim}{v}|^2 \, d\sigma = \min \tag{1a}$$

where Σ = surface of the earth and $d\sigma$ = element of Σ. Taking into consideration that along the edges of crustal plates mass flows out or sinks into the interior, one could extend the integration area:

$$\iint_{W} |\underset{\sim}{v}|^2 \, dw = \min \tag{1b}$$

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where W = earth crust and a certain domain below the crust, dw = element of W. This is, of course, in practice not strictly realizable, because observations would have to continuous both in space and in time. This leads to the approximation:

$$\sum_i p_i \; |\underset{\sim}{x}_i(t_j) - \underset{\sim}{x}_i(t_{j-1})|^2 = \min \tag{1c}$$

where $\underset{\sim}{x}$ is the position vector, i a discrete point, j the discrete epoch, p_i a positive weight factor.

Thus, if at an initial epoch t_0 a reference frame has been defined and coordinates $\underline{x}(t_0)$ been computed, the frame can be updated from one epoch to the next. Therefore, it is necessary to determine for each epoch t_j preliminary coordinates $\underline{y}(t_j)$ in an arbitrary free frame from geodetic observations at epoch t_j. Then the transformation parameters $\theta_1,\theta_2,\theta_3$ (for rotation) and s_1,s_2,s_3 (for translation) from the free frame are determined from the adjustment condition

$$\sum_i p_i \; \| \underline{R}(\theta_1,\theta_2,\theta_3)\; \underline{y}_i(t_j) + \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} - \underline{x}_i(t_{j-1}) \|^2 = \min \tag{2}$$

using Cartesian coordinates.

Let the solution, for which the minimum is achieved, be $\theta_k = \bar{\theta}_k$, $s_l = \bar{s}_l$, (k,l = 1,2,3); then

$$\underline{x}_i(t_j) = \underline{R}(\bar{\theta}_1,\bar{\theta}_2,\bar{\theta}_3)\; \underline{y}_i(t_j) + \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}$$

will be the corrdinates in the updated globally fixed frame at epoch t_j. It should be noted that this frame at epoch t_j and that at epoch t_{j-1} are by definition identical. Only the coordinates have changed a little due to deformation. The establishment of the frame at the initial epoch t_0 should be done in such a way that it coincides then with a traditional frame, as, for example, the one defined by the mass centre, the CIO, and the meridian of Greenwich. We shall call such a frame an earth-fixed one.

2.2 Generalization and Specialization of the Minimum Principle

The same problem of deformation as on the earth arises "in the sky"; for the relative positions of the stars, too, change in time (2- or 3-dimensional proper motion). Therefore, eq. (2) can be applied for defining a "fixed" celestial frame (as an approximation to an inertial frame) in the same way as for an earth-fixed frame. The index i denotes then the star (including quasars) instead of the point. The fact that the proper motion

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of quasars is much smaller (at least the directional component) than the proper motion of visible stars can be taken into account by assigning a lower or zero weight to visible stars. In astrometry one is rather interested only in the direction than in the absolute position of a star. Therefore, it makes sense not to minimize the square norm of the absolute displacement vector ($\Delta x^2 + \Delta y^2 + \Delta z^2$, in Cartesian coordinates), but the projection of the displacement vector on the unit sphere ($\cos^2\delta\ \Delta\alpha^2 + \Delta\delta^2$, in spherical coordinates). Further, one cannot determine rotation and translation parameters, but rotation parameters only.

Also in the terrestrial case, we can split the absolute displacement vector in its radial and its directional component, if we use spherical coordinates r,λ,ϕ. As the tides cause mainly short period radial deformations, one can reduce eq. (2) on the radial displacement for a short period updating. On the other hand, plate tectonics cause mainly secular directional deformations (as seen from the origin near the geocentre). After reducing the observations for short period tidal deformations, one can, therefore, define a long-period earth-fixed system by specifying eq. (2) on the directional displacement.

3. THE ROTATION VECTOR

The rotation vector, which usually plays an important role for defining terrestrial or celestial reference frames, has not yet been mentioned. It is, however, necessary for the definition of a time system which is consistent with the natural run of day. There are two ways for defining the rotation of a body. Rotation in the first sense is, loosely speaking, when the direction of the position vector from the mass centre to some mass element is changing. Rotation in the second sense is a change of orientation of a mass particle. It can be realized without any reference to the centre of mass and is defined by $\underset{\sim}{\omega} = 1/2 \text{ rot } \underset{\sim}{v}$, where $\underset{\sim}{v}$ is the velocity and $\underset{\sim}{\omega}$ the rotation vector, both referring to the same "fixation". A suitable separation of deformation from rotation, according to the principle of polar decomposition, is already included in this definition. As it is independent of the mass centre it is better fitting the earth-fixed frame introduced above in eq. (2,3) than rotation of the first kind and should, therefore, be preferred. In the special case of a rigid earth, both definitions, of course, coincide.

In the general case of a deformable earth, however, the rotation vector changes not only in time, but also from point to point. This is quite clear because the relative rotation of plates with respect to the earth-fixed frame is superposed on the rotation of this frame with respect to a universally fixed frame; the resulting rotation will, therefore, be different for each plate. For defining a global rotation vector there are two possibilities:

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one can either take the rotation of the earth-fixed frame or the average of all local rotation vectors. These two choices will in general not coincide.

REFERENCES

GRAFAREND, E. (1977): Space-Time Differential Geodesy: The Changing World of Geodetic Science (ed. U. Uotila), Vol. I, Rep. 250, Dep. Geod. Sci., The Ohio State Univ., Columbus.

GRAFAREND, E.; MUELLER, I.; PAPO, H ; RICHTER, B. (1979): Investigations on the Hierarchy of Reference Frames in Geodesy and Geodynamics, Rep. 289, Dep. Geod. Sci., The Ohio State Univ., Columbus.

HEITZ, S. (1975): Bezugssysteme geodätischer Erdmodelle, Zeitschrift für Vermessungswesen loo, pp. 251-260.

HEITZ, S. (1980): Mechanik fester Körper, Dümmler Verlag, Bonn.

MORITZ, H. (1979): Concepts in Geodetic Reference Frames, Rep. 294, Dep. Geod. Sci., The Ohio State Univ., Columbus.

TRUESDELL, C. (1954): The Kinematics of Vorticity, Science Series No. 19, Indiana University Publications, Bloomington.

https://doi.org/10.1017/S0252921100081355 Published online by Cambridge University Press