ON THE IMPLICATIONS OF THE SYMMETRIC COMPONENT OF THE FREQUENCY SPLITTING REPORTED BY DUVALL, HARVEY AND POMERANTZ

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ABSTRACT. The component of the frequency splitting of solar five-minute oscillations observed by Duvall, Harvey and Pomerantz that is even in azimuthal degree measures latitudinal and depth variations in the structure of the sun. We indicate how the data hint that there is a shallow perturbation, possibly associated with a magnetic field, that is concentrated at low latitudes.

1. INTRODUCTION

This discussion is motivated by a recent paper by Duvall, Harvey and Pomerantz (1986) reporting degeneracy splitting of five-minute solar oscillations. The splitting is expressed in the form

$$s(L,m) \equiv \nu_{n,l,m} - \nu_{n,l,o} = L \sum_{i=o}^{5} a_i P_i\,(-m/L), \qquad (1)$$

where $\nu_{n,l,m}$ is the cyclic frequency of the mode of order n, degree ℓ and azimuthal order m, $L^2 = \ell(\ell+1)$ and P_i is the Legendre polynomial of degree i. (The dependence of s on n has been suppressed because Duvall et al. provide only data that have been averaged over n, and our definition of a_o, which is not directly relevant to this discussion, differs from theirs.) The odd terms in the sum measure rotation, and the even terms are produced by latitudinal variations in the solar structure and any perturbing force (such as a Lorentz force) that cannot distinguish between east and west. Here we are concerned only with the even terms.

2. DEPENDENCE ON m

The m^2-dependent component of s is plotted against m/L in Figure 1. Aside from the modes of lowest and highest degrees, the curves are flatter at low values of m/L and relatively steeper at high values

J. Christensen-Dalsgaard and S. Frandsen (eds.), Advances in Helio- and Asteroseismology, 175–180.

https://doi.org/10.1017/S0074180900158012 Published online by Cambridge University Press

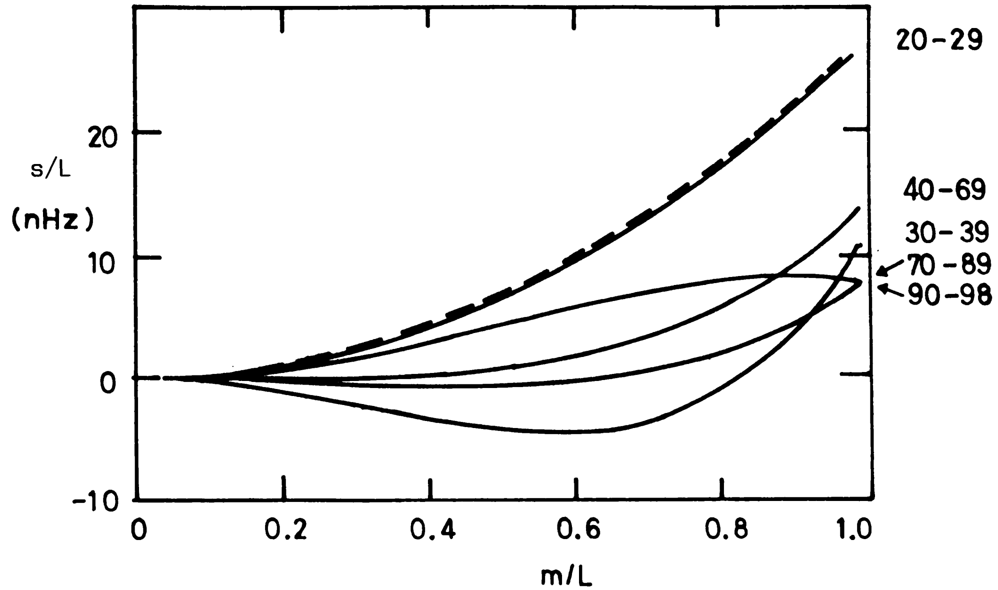

Figure 1. Symmetric component of the frequencysplitting factor s/L taken from the data of Duvall et al. (1986). The splittings s(L,m/L) are averages over all orders n of the modes observed with frequencies between 2.4 and 4 mHz and over the degrees ℓ in the ranges indicated in the figure. The dashed line is proportional to m^2/L^2, and is what is obtained from a perturbation proportional to $\sin^2\theta$.

than the parabolic dashed curve that one would expect from a perturbation proportional to $\sin^2 \theta$, such as is produced by centrifugal distortion due to an angular velocity that is independent of colatitutde θ. One would expect this to be a symptom of a perturbation $\Delta(r,\theta)$ from spherical symmetry that is confined to low latitudes, and which therefore influences the equatorially concentrated, nearly sectoral modes, with $m \simeq L$, the most strongly.

The two even m-dependent terms in the expansion (1) permit a two-term expansion of the perturbation Δ in powers of $\sin^2 \theta$:

$$\Delta \propto \sin^2 \theta - \alpha \sin^4 \theta \qquad (2)$$

where $\alpha = 35a_4/(40a_4 - 9a_2)$. For modes with $40 \lesssim \ell \lesssim 70$, for example, $\alpha \simeq 2$ and Δ is substantial only between latitudes ±40°. Its variation is not grossly different from the distribution of sunspots

3. DEPENDENCE ON L

The L dependence of s determines the variation of the perturbation with depth. The coefficients La_2, La_4 are tabulated by Gough and Thompson in these proceedings, and show little evidence of systematic

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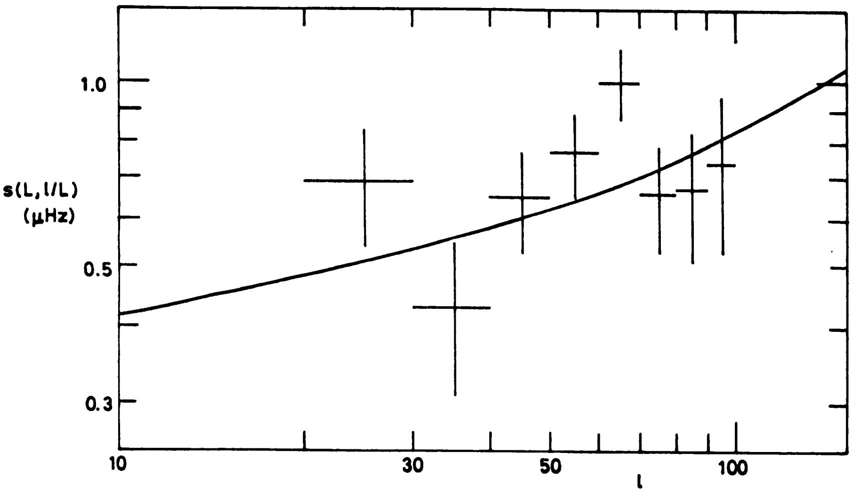

Figure 2. The crosses represent the even component of the frequency splitting s(L,ℓ/L) between sectoral and zonal modes (in μHz) evaluated from data reported by Duvall et al. (1986). Vertical bars estimate standard errors; horizontal bars the range of ℓ over which the data are averaged. The continuous line is the theoretical estimate of the same quantity obtained by assuming a latitudinally varying mixing length $l = l_0\,(1 + \varepsilon\Delta)$, where l_0 is the mixing length at the poles, $\Delta(\theta)$ is given by Equation (2) with $\alpha = 2$, and $\varepsilon = 0.06$.

variation. This can be seen also in Figure 2, where the splittings s(L, ℓ/L) between sectoral and zonal modes are plotted. There is perhaps a hint of a slight increase in s with ℓ, which would imply that the influence of Δ on the oscillations decreases with depth.

We can estimate the frequency splitting from the asymptotic formula for p-mode eigenfrequencies ω:

$$\int_{r_1}^{r_2}\left[\frac{\omega^2-\omega_c^2}{c^2} - \frac{L^2}{r^2}\right]^{\frac{1}{2}} dr \sim \pi(n + \tilde{\alpha}), \qquad (3)$$

(cf Gough, 1986) which is derived from a dispersion relation

$$\omega^2 = \omega_c^2 + c^2k^2 \qquad (4)$$

for waves with a local wave number k, where r is radius, ω_c is Lamb's critical acoustic frequency, c is sound speed, $\tilde{\alpha}(\omega)$ is a frequency-dependent phase factor that depends on conditions near the upper turning point r_2, and the integral is actually an appropriate average along ray paths in a plane through the centre of the sun whose normal subtends an angle $\sin^{-1}(m/L)$ with the axis of symmetry. The perturbation Δ modifies the dispersion relation (4), which we might formally represent as variations $\delta\omega_c$, δc to ω_c and c. If those variations are small, and if furthermore they are not confined solely to a very narrow region near r_2 where ω_c is important, we can neglect ω_c and estimate the perturbed frequency $\delta\omega = 2\pi\delta\nu \propto s$ by

$$\frac{\delta\omega}{\omega} = \frac{\int F c^{-1} \delta c \, dr}{\int F \, dr} , \tag{5}$$

where

$$F = c^{-1}(1 - c^2 L^2 / r^2 \omega^2)^{-\frac{1}{2}} \tag{6}$$

and the integrals are evaluated between $r_1 = c(r_1)L/\omega$ and $r_2 \simeq R$, where R is the radius of the sun.

Suppose now that δc is nonzero only in a shallow subphotospheric layer, extending well beneath $r = r_2$ to a depth $d \ll R - r_1$. Then the numerator of the right-hand side of (5) depends simply on conditions in the superficial layer, whereas the denominator scales with the depth $R - r_1$ of the region within which the mode is trapped, which for high-degree modes is proportional to L^{-1}. We illustrate this by considering a constant relative perturbation $\Delta = \delta c/c$ in $R - r < d$ to an adiabatically stratified complete plane-parallel polytrope of index μ. In that case the integrals in (5) can be evaluated analytically. Provided $c^2 L^2/R^2\omega^2 \ll 1$ for $R - r < d$, the result is approximately

$$\delta\nu \simeq \frac{\Delta}{\pi^2} \frac{c(R-d)}{R} L = \frac{2}{\pi} \Delta \left(\frac{d}{\mu R}\right)^{\frac{1}{2}} \sigma L, \tag{7}$$

where $\sigma = (GM/R^3)^{\frac{1}{2}}/2\pi \simeq 0.1$ mHz, M being the solar mass and G the gravitational constant. This increases with L more rapidly than the data in Figure 2.

A similar result holds for a perturbation $\Delta = \delta c/c$ that varies beneath $r = r_2$ as $(1 - r/R)^{-a}$ with $a > \frac{1}{2}$. In that case

$$\delta\nu \simeq \frac{\Delta(r_2)}{(2a-1)\pi^2} \frac{c(r_2)}{r_2} L \tag{8}$$

for $[c(r_2)L/\omega r_2]^2 \simeq (10^{-3}L)^2 \ll 1$, which again is proportional to L. If, however, $a < \frac{1}{2}$, then

$$\delta\nu \simeq \frac{\Delta(r_2)}{4\pi^2} \left[\frac{2\omega r_2}{c(r_2)}\right]^{1-2a} \frac{[\Gamma(\frac{1}{2}-a)]^2}{\Gamma(1-2a)} \frac{c(r_2)}{r_2} L^{2a}, \tag{9}$$

where Γ is the gamma function. In that case $\delta\nu$ varies with L more slowly.

A logarithmic regression of the values of the splittings between sectoral and zonal modes deduced from the observations of Duvall, Harvey and Pomerantz (1986) plotted in Figure 2 yields $s(L, \ell/L) \simeq 0.30 L^{0.2}$ μHz. This can be identified with equation (9), yielding a=0.1. Moreover, since the upper turning point r_2 is about $10^{-4}R$ beneath the photosphere where $c \simeq 10$ km s^{-1}, and $\omega/2\pi \simeq 3$ mHz, it follows that $\Delta(r_2) \simeq 4 \times 10^{-4}$ if $\alpha = 2$.

4. DIRECT MAGNETIC PERTURBATIONS

The evidence from the m/L dependence of s that Δ is somewhat concentrated near the equator suggests that magnetic perturbations might be important. The direct effect of the Lorentz force on wave propagation can be represented by a direction-dependent sound-speed perturbation to the dispersion relation (4). For example, we deduce from the work of Bogdan and Zweibel (1985) that for waves propagating perpendicular to a fibril field in a gas with adiabatic exponent γ,

$$\frac{\delta c}{c} = \Delta \simeq \frac{1}{2} \frac{\beta}{2-\beta} \frac{4+(\gamma-2)\beta}{\gamma+(\gamma-2)\beta} f, \tag{10}$$

where f is the fraction of volume occupied by parallel flux tubes in temperature equilibrium with their surroundings and β is the ratio of gas pressure to magnetic pressure in the flux tubes. $\Delta \simeq \beta f/\gamma$ when $\beta \ll 1$. Waves travelling nearly parallel to the field are influenced much less. There are no reliable predictions of how Δ should vary with depth; that we must infer from the L dependence of the observed frequency splitting.

It should be appreciated that the formula (10) was deduced for an unstratified medium, in which ω_c is zero. However, if the value a = 0.1 is to be believed, the influence of Δ on the waves is greatest near the upper turning point. So one might have suspected that the neglected perturbation to $\delta\omega_c$ might be important. It is evident, however, that its influence will be confined to a shallow region in the vicinity of r_2, and therefore its contribution to the frequency perturbation must be proportioned to L. The data suggest that any such contribution is small.

Since the dominant influence of Δ is near the top of the acoustic cavity $r_1 < r < r_2$ where the waves travel almost vertically (except, of course, very close to $r = r_2$ where the waves travel horizontally), the fibrils must have a strong horizontal component. Moreover, the magnitude of Δ is only weakly dependent on the orientation of the plane of propagation of the wave, and hence on m/L. The m dependence would therefore be expected to come predominantly from the latitudinal dependence of Δ. If we assume the fibrils are horizontal in the region where Δ is most important, we thus obtain a pole-equator difference in βf of about 7×10^{-4} at $r = r_2$, which is not substantially greater than the estimate inferred from the observations of Tarbell _et al_. (1979).

5. EFFECT OF VARIATION OF THE THERMAL STRATIFICATION

Another effect the magnetic field might have is to suppress convection preferentially at low latitudes. The effect on the stratification would be strongest in the superdiabatic boundary layer at the top of

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the convection zone, but the influence of that effect would extend to greater depths. Therefore we would anticipate values of $\delta\nu$ that rise with L more slowly than linearly.

To estimate the effect we computed model solar envelopes to represent the polar and equatorial regions, with an equatorial mixing length differing from the polar value by a factor $1 - \varepsilon$. The luminosity, effective temperature and surface gravity at the equator were adjusted so that in the lower regions of the convection zone, where the stratification is adiabatic, the polar and equatorial models were indistinguishable. We then compared the adiabatic eigenfrequencies of the two models.

The frequency splitting $s(L, \ell/L)$ computed with $\alpha=2$ is plotted in Figure 2. It is positive, as are the observations, and the variation with L, $s \propto L^{0.3}$, is similar to that observed. However, to obtain agreement with the magnitude of the splitting (which is linear in ε when ε is small) requires $\varepsilon = 0.1$, which yields a radiative flux from the photosphere some 15% smaller at the equator than at the poles. This value is unacceptably large, and we deduce that the observed frequency splittings cannot be explained solely by a latitudinal variation in the mixing length.

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