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IN MEMORIAM, J. DOUGLAS CARROLL
1939–2011

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John Douglas Carroll, widely regarded as a leading psychometrician and mathematical psychologist of the last four decades, and one of the founding fathers of classification as an interdisciplinary methodological field, passed away at the age of 72 on Tuesday, June 7, 2011, at Overlook Hospital in Summit, NJ. He is survived by his wife of 46 years, Sylvia B. Carroll, and two sons, Steven D. Carroll of Saratoga Springs, New York, and Gregory A. Carroll of Columbus, Ohio, as well as by his daughter-in-law, Maria Ryan Carroll, and two grandchildren, Jenny Morgan Ryan and Bradley Ryan Carroll.

Gifted people learn rapidly. Born on January 3, 1939, in Philadelphia, Pennsylvania, Doug Carroll finished his undergraduate studies with honors at the University of Florida in August 1958, with comajors in Mathematics and Psychology, at the age of nineteen. He was elected to Phi Beta Kappa, and won an Educational Testing Service Psychometric Fellowship to Princeton University, where he received his M.A. in 1960. He was immediately elected to the honorary Scientific Research Society Sigma Xi, and continued at Princeton with Ph.D. research on human learning and concept formation, which he completed in 1963. His major areas were Mathematical-Experimental Psychology and Psychometrics, his minor was in Mathematics, and the dissertation was put on record as Carroll (1963). The Psychometrics Program at Princeton was directed at the time by Harold Gullikson, an eminent psychometrician who was up to date in the (multidimensional) scaling field (Gulliksen & Messick 1960; see, also, the interesting perspective in Gulliksen, 1961). Carroll got his first academic job (1961–1963) as an instructor in Analysis of Variance and as a research assistant of Robert Abelson at the Psychology Department of Yale University, working on computer simulation of cognitive balancing, leading to his first journal publication (Abelson & Carroll, 1965).

In 1963, Doug Carroll took a research position as a Member of Technical Staff at Bell Laboratories in Murray Hill, NJ, a long-term relationship which lasted until 1990. It was there that he started enduring research collaborations with several people, in particular, Roger Shepard, Joe Kruskal, Jih Jie Chang, Mike Wish, and Sandy Pruzansky inside Bell Labs, and with Paul Green, Seymour Rosenberg, and Phipps Arabie outside the Labs. With Rosenberg, he did some work as a continuation of his dissertation (Carroll & Rosenberg, 1976), but the contacts with Shepard and Kruskal made him switch completely to the rapidly expanding research area of multidimensional scaling (MDS) and related techniques, of which he became the most prominent proponent for the rest of his career. Gradually, many more people came along, often invited by him personally, for a post-doc or a post-post-doc period at Bell Labs, which developed into the world-wide leading center of MDS and related techniques for the next 25 years.

My thanks are due to Sylvia Carroll for providing me with some crucial biographical details, and for stimulating me to write this obituary.

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While at Bell Labs, he also had various academic positions over the years. For instance, in 1965–1966, he taught a full-year course in mathematical statistics and statistical methods at both graduate and undergraduate levels, as Assistant Professor of Industrial Engineering and Operations Research at New York University; in 1975–1976, he taught a multivariate analysis course and seminar on multidimensional scaling as the Professor of Psychology at University of California at San Diego and Irvine, and in 1987–1989, a Ph.D. seminar on multidimensional scaling and related techniques applied to marketing research as Procter and Gamble Adjunct Professor of Marketing in the Wharton School of the University of Pennsylvania. In 1990, the Dean of the Business School of Rutgers University at Newark made him a very good offer to become Board of Governors Professor of Management and Psychology in the Marketing Department, which he accepted. Here, he started a second career in academia, teaching in the Graduate School of Management, and continuing with his research as usual. Soon after, his good friend and coauthor, Phipps Arabie, was able to join the same school.

It is hard to do justice to the creativity and stamina apparent in the 230+ Carroll papers and books, but I will give a summary of the highlights by grouping his contributions to the literature into seven clusters.

The Basic INDSCAL Model and CANDECOMP Algorithm Undoubtedly, the single most important innovative contribution was the Carroll and Chang (1970) Psychometrika paper, which introduced a least squares criterion for estimating all parameters of a Euclidean MDS model that accommodates systematic differences between individuals, and a convergent iterative algorithm for calculating actual estimates, called CANDECOMP. In the words of the President of the Psychometric Society, Professor Jos ten Berge, when he awarded the 2010 Psychometric Society Lifetime Achievement Award to professor Carroll, the importance of this paper was in creating a whole new area of three-way analysis:

> Whereas Test Theory and Item Response Theory were developed to take account of individual differences, this was much less the case for Multidimensional Scaling. The Carroll and Chang paper has changed the landscape of multidimensional scaling by modeling individual differences directly. This paper alone has been cited over 1600 times [meanwhile, over 2300 times, WH] in a wide variety of disciplines, not just in psychometrics, classification, and marketing, but even more so in mathematics, chemometrics, and electrical engineering. The reason for this is not the INDSCAL model itself. To fit the INDSCAL model, Carroll and Chang proposed the CANDECOMP algorithm, which, incidentally, has proven to be also useful as a least squares algorithm for fitting the late Richard Harshman's PARAFAC model. The so-called CANDECOMP/PARAFAC decomposition became immensely popular as a method of decomposing three-way arrays ever since; some 20 years ago, chemometricians realized that the three-way data they were analyzing in their laboratories called for this very decomposition.

The CANDECOMP algorithm was in fact a prime example *avant-la-lettre* of Alternating Least Squares (ALS), a term coined by De Leeuw, Young, and Takane (1976) that became the brand name of all of these types of algorithms. It was extended with the possibility to impose linear constraints on the parameters (CANDELINC) in Carroll, Pruzansky, and Kruskal (1980), allowing for prediction and interpolation based on known or measured attributes of the objects and individuals scaled. An influential application paper was Chang and Carroll (1980), in which they demonstrated that the INDSCAL model could detect color-blindness. But perhaps more importantly, they also made the methodological point that high-dimensional solutions are possible under the INDSCAL model, because there is not such a strong proliferation of the number of parameters in relation to the number of data elements. In one of the first bootstrap applications for MDS, Weinberg, Carroll, and Cohen (1984) showed how to obtain confidence regions for the INDSCAL parameters. The basic model was extended with additional parameters for main effects (called specificities) in Carroll and Winsberg (1995); one of several papers that had a long incubation period. The INDSCAL program had been described in Carroll and Chang (1969). Many more papers contained INDSCAL ideas or applications, as will become apparent in the following paragraphs.

MDS and Parametric Mapping An important contribution to ordinary MDS was a theoretical study of different ways to normalize the least squares loss functions used (Kruskal & Carroll, 1969). The same paper also discussed possible loss functions to solve the so-called degeneracy problem in multidimensional unfolding, in which two sets of points are fitted to proximities arranged in a rectangular table. Unfortunately, their proposals and experiments were inconclusive for settling the degeneracy problem. For specific types of designs and data collection, specific distributional assumptions can be made, leading to all the advantages of maximum likelihood estimation. One example is Takane and Carroll (1981) for the case in which ranking processes are modeled; another example is Winsberg and Carroll (1989), which paper assumed normally distributed errors and introduced spline transformations of the distances, in an MDS model that includes both common and specific dimensions.

With Roger Shepard, who was regularly working at Bell Labs at the time, the idea of local MDS was developed, in which relatively more weight is given to fitting the small dissimilarities with small distances, compared to the large ones (Shepard & Carroll, 1966). They called this technique parametric mapping (PARAMAP), and the software was described in Chang (1969). It was based on the idea of optimizing the "continuity" of the function mapping high-dimensional distances into low-dimensional distances. Local MDS often can more easily unfold and spread out severely nonlinear manifolds—such as points on a sphere or on a torus—than least squares MDS and has remained an active field of research until the present day. Triggered by the apparent success of less computationally intensive algorithms, Akkucuk and Carroll (2006) demonstrated that PARAMAP can still be a competitive option, especially in cases of severe nonlinearity.

Clustering Models and Methods In the clustering area, we may distinguish partition clustering (most typically with K-means), hierarchical clustering, which forms a nested sequence of partitions, and overlapping clustering, which forms a nonnested sequence of binary partitions.

For overlapping clustering, Arabie and Carroll (1980) introduced a rigorous method for fitting the Shepard-Arabie ADCLUS (for ADditive CLUStering) model, called MAPCLUS. The technique was introduced to marketing researchers in Arabie, Carroll, DeSarbo, and Wind (1981), and extended to the INDSCAL situation in Carroll and Arabie (1983) under the name INDCLUS. Further algorithmic improvements for both ADCLUS and INDCLUS were proposed in Chaturvedi and Carroll (1994), leading to SINDCLUS, where the S indicated "Separability-based, and Speedy," which referred to a smart and powerful ALS strategy. To increase comprehension of MAPCLUS solutions, Carroll and Corter (1995) introduced a new graph-theoretic method to construct multiple tree or extended tree representations of additive clusters, called CLUSTREES.

How to select the variables in partition clustering is a major open question. The variable importance problem was approached in an original manner by introducing unknown, "INDSCAL-like," weights for each variable in a weighted squared distance between all pairs of objects, which is fitted to an unknown, binary, block-structured matrix of cluster distances (DeSarbo, Carroll, Clark, & Green, 1984), in a method called SYNCLUS (for SYNthesized CLUStering). In continuation of this approach, it was also applied to hierarchical clustering in De Soete, DeSarbo, and Carroll (1985), where it should be noted that here, too, the hierarchical clustering and the variable weighting were done simultaneously in a least squares sense, and not through one of the more traditional hierarchical clustering procedures—an important step ahead. Another way to approach weighting of variables is to do the (nonhierarchical) clustering in a low-dimensional subspace rather than in the full space of variables and again not in sequence but simultaneously; this was achieved in De Soete and Carroll (1994).

When the variables are categorical without an ordering of the categories (data are on a so-called nominal scale), it is not appropriate to use straight-forward least squares loss functions and to represent the clusters as multivariate means. To resolve this difficulty, Chaturvedi, Green, and Carroll (2001) proposed *K-modes* clustering, in which clusters are represented by category labels and the loss function is a counting measure.

Analysis of Preference (or Other Dominance) Data Preference data consist either of ratings or rankings of m stimuli or objects by n individuals or judges—in the terminology of Carroll and Arabie (1980) the *two-way two-mode* case—or they consist of one or more m by m tables of paired comparison judgments (or choices) by n judges. Here, the interest is in understanding the differences between the judges in their appreciation of the objects, and this was certainly one of Doug Carroll's favorite areas of research. He wrote two programmatic papers, in which he laid down several current and novel models for analyzing such data (Carroll, 1972, 1980).

If the objects have a specified design coded in design variables, the traditional way to account for the differences in preference was by regression and analysis of variance methods. For a categorical outcome variable, Carroll (1969) had suggested some good ideas. Due to the relentless efforts of Paul Green, decomposing preferences in terms of design factors had become known in marketing research as conjoint analysis, and in Green, Carroll, and Goldberg (1981) the authors introduced the POSSE system (Product Optimization and Selected Segment Evaluation) as a general empirical procedure for optimizing product/service designs in marketing research, based upon response surface methods. Next, DeSarbo, Carroll, Lehman, and O'Shaughnessy (1982) developed an extension called three-way multivariate conjoint analysis to account for the effects of both multiple dependent variables and individual differences in the model.

One step further was to use specific models for the interaction between objects and judges that have a good rationale for interpretation. As explained in Carroll (1972), there were two extant multidimensional models for this purpose: the vector model and the ideal point (or unfolding) model. Given a set of attributes of the objects (or coordinates obtained with some separate MDS analysis), these models could be fitted by the program PREFMAP (Chang & Carroll, 1972), and this was called an external analysis of preference. The case in which no attributes or coordinates of the objects were available called for an internal analysis. For the vector model, this could be done with MDPREF (Chang & Carroll, 1968); for the unfolding model, one could use a special option in MDSCAL (Kruskal, 1968; Kruskal & Carroll, 1969). If the preference data were collected under different conditions or scenarios, the natural way to proceed was to consider the INDSCAL model for the case of rectangular data (DeSarbo & Carroll, 1985). For paired comparison data, a new idea was to model the individual differences in a multidimensional fashion by assuming a random distribution of vectors (De Soete & Carroll, 1983, or a random distribution of ideal points (De Soete, Carroll, & DeSarbo, 1986).

A final idea to analyze rectangular proximity data should be mentioned. The vector and unfolding model are Euclidean representations of two sets of vectors or two sets of points in a joint space. De Soete, DeSarbo, Furnas, and Carroll (1984) showed that the joint space could also be discrete, with an ultrametric or an additive distance metric. The next section gives several more examples of this idea.

Discrete and Hybrid Scaling Methods In his Presidential Address to the Psychometric Society, Carroll (1976) had given a wide-ranging vision of how to integrate discrete models, like hierarchical clustering and additive tree structures, into a more comprehensive MDS framework that might still also include continuous dimensions. More examples of this "hybrid scaling" paradigm were given in Carroll and Pruzansky (1980). For the issue of how to distinguish between additive tree models and spatial representations, it turned out that it is useful to look at the skewness of the distribution of distance and the proportion of elongated triangles (Pruzansky, Tversky & Carroll, 1982).

An individual differences version of (multiple) hierarchical and additive tree fitting (called INDTREES) was proposed in Carroll, Clark, and DeSarbo (1984). Another three-way model (called CLUSCALE) combining overlapping clusters (INDCLUS) and continuous dimensions (INDSCAL) was proposed by Chaturvedi and Carroll (2006), in which they used the Green, Carroll, and DeSarbo (1978) idea for partitioning Variance-Accounted-For to figure out the contributions of the discrete and continuous parts of the model, respectively.

For cases in which one has certain a priori expectations about the discrete structure, it is useful being able to put constraints on the topological structure of the tree (De Soete, Carroll, & DeSarbo, 1987). Discrete structures more general than trees were considered by Klauer and Carroll (1989), who developed a rigorous least squares procedure for fitting general graphs with minimal path-length metric to proximity data. In this procedure, the user has to specify the number of links in the graph, and if this number is set equal to the number of objects minus one the procedure provides a tree where each node corresponds to an object. An extension to directed networks for asymmetric proximity data was given by Klauer and Carroll (1991).

Correspondence Analysis and Multiple Correspondence Analysis There are three papers in this area. In Carroll, Green, and Schaffer (1986) a rescaling was proposed of the row and column coordinates of a simple correspondence analysis solution. This rescaling, which had the objective to make inter-point distance comparisons between row and column points more meaningful, was further clarified in Carroll, Green, and Schaffer (1987), but still caused quite some controversy. On a different aspect, for the case of multiple correspondence analysis (equivalent to Guttman's principal components of scale analysis), Carroll and Green (1988) argued that the usual principal axes orientation of a solution may not always be desirable, and proposed an INDSCAL-type procedure when multiple groups of individuals are available, to arrive at a better orientation of the axes.

Reviews, Books and Some Special Papers During the course of his career, Doug Carroll received numerous invitations to write surveys of the literature, encyclopedia entries, and book chapters, and he often came up with extensive articles. Only a few of these can be mentioned here. An influential methodological review of three-way MDS was given in Carroll and Wish (1974), and a review of applications of the INDSCAL model in Wish and Carroll (1974). A textbook about three-way scaling appeared as Arabie, Carroll, and DeSarbo (1987). The whole area of MDS was covered in an *Annual Review of Psychology* paper by Carroll and Arabie (1980), and a Handbook of Statistics chapter by Wish and Carroll (1982).

The *Annual Review* paper contained a new taxonomy of data types and multivariate analysis methods that became the standard in the field. It was based on the number of ways and modes of data matrices, scale type and conditionality of the data, completeness of the data, and the number and nature of replications, and included a comprehensive scheme of models for analysis. It is worth noticing that Carroll (1966) had already phrased several criticisms of an earlier taxonomy of data and methods by Clyde Coombs in his 1964 book *A Theory of Data*. In his review of this book, he remarked that Coombs' taxonomy did not include important types of data in psychology, and that:

> Even in the measurement area it represents only one of a number of ways of dissecting the universe, and makes no room for many potential, though as yet underdeveloped, classes of data. The attempt at organizing data on the basis of abstract

> properties is a laudable one, but the present system appears, to this reviewer, a bit narrow in scope.

At the end of his review, Carroll expressed as his hope:

> that Coombs and his colleagues will relax their insistence on purely non-metric solutions, at least those of the limited class portrayed in this book. Recent developments, exemplified by the work of R.N. Shepard and J.B. Kruskal at Bell Laboratories, indicate that tightly constrained metric solutions can be obtained from non-metric data with a minimum of the restrictive assumptions decried by Coombs.

The Carroll–Arabie taxonomy certainly was of much wider scope and covered a much richer class of much stronger models than all earlier taxonomies.

Two further influential review papers that surveyed developments in conjoint analysis and multidimensional scaling for marketing research were Carroll and Green (1995, 1997). The two of them had already been collaborating—apart from the journal articles already mentioned—in the two popular textbooks Green (1976), about matrix algebra, of which a revised edition came out as Carroll, Green, and Chaturvedi (1997), and Green (1978), about multivariate analysis, of which later a new version came out as Lattin, Carroll, and Green (2003). Very recently, a review of two-way MDS with Carroll's Ph.D. student Stephen France reached out to computing researchers in the engineering community (France & Carroll, 2011).

A final mention deserve some less known application papers on a topic that was of special interest to Doug: the measurement and dimensions of clinical pain (Clark, Carroll, Yang, & Janal, 1986; Clark, Ferrer-Brechner, Janal, Carroll, & Yang, 1989; Clark, Janal, Hoben, & Carroll, 2001). This topic was of special interest to him because of his personal experience with pain. On February 18, 1979, he was skiing with his wife Sylvia on a steep, extremely icy slope called the Flying Mile at Mt. Tremblanc, Canada, near Montreal. An unfortunate turn made him slip into free fall. That severe ski accident made him paraplegic and wheelchair dependent for the rest of his life. Despite his dreadful health condition, he remained active and determined in all his intellectual pursuits throughout his life, until the very end: at the moment of his passing, he had just been launching a new project in high-dimensional data (France, Carroll, & Xiong, 2012). The work on pain led to yet another remarkable methodological innovation using INDSCAL: a multidimensional-psychophysical scaling procedure for generating verbal rating scales with quantifications at the ratio level of measurement precision (Clark, Carroll, & Janal, 2010).

My overview of scientific contributions to the literature does not suffice to describe Doug Carroll's presence in the scientific community. Apart from the Lifetime Achievement Award of the Psychometric Society already mentioned, he also received the Distinguished Scientific Contribution Award of the American Psychological Association and the William James Fellow Award of the Association for Psychological Science, both in 1989. He was invited to be Fellow at the Center for Advanced Studies in the Behavioral Sciences, Stanford, California, 1984–1985, and received Best Article Awards from the Journal of Marketing in 1981 and from the Journal of the Academy of Marketing Science in 1989. His professional network was enormous, and he took on a more than average share of service to the following seven professional societies:

- *Psychometric Society* (Board of Trustees 1971–1977, 1981–1987, 1993–1996; Editorial Council 1975–1981, President-Elect, President and Past President, 1974–1977; Associate Editor of Psychometrika 1973–2003)
- *Classification Society of North America* (Governing Council 1974–1977, President 1980–1983; Board of Directors 1984–1989, 1994–1997, 1999–2001; Member Editorial Board of Journal of Classification 1983–2011)
- *Society of Multivariate Experimental Psychology* (Editorial Advisory Board, 1978–1982, of which Chair 1980–1981; President 1982–1983)

- *American Psychological Association* (Fellow, 1977; Chair of Membership and Fellowship Committee, Division 5 (Evaluation, Measurement and Statistics) 1978–1981; President-Elect, President and Past President, Division 5, 1990–1993)
- *American Statistical Association* (Fellow, 1979; Chair-Elect, Chair and Past Chair, Section on Statistics in Marketing, 1992–1995; Member ASA Committee on Statistics and Disability, 1993–1995)
- *American Marketing Association* (Member Editorial Board of Journal of Marketing Research)
- *International Federation of Classification Societies* (Member of Founding Council 1985–1989; Member of IFCS Council, 1991–1994; Vice-President, 1995, 1998; President, 1996–1997)

Doug did all his work and duties with a dedication and joie de vivre that was truly inspirational. A crucial factor for being able to continue so actively with his favorite pursuits when he became disabled was the great and unconditional support that he received from the love of his life, Sylvia. He could always rely on her, at home, but also traveling around the world in often extraordinary and difficult circumstances. Many of us have experienced that Doug was an individual of integrity prepared to help others without expectation of return (although he did enjoy loyalty). He had a strong drive to do the right thing. He was not only a Justice of the Peace in his civilian life, but also a peacemaker in his professional life. That is why we do not only lose a good colleague, but also a good friend. He will be dearly missed and never forgotten.

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