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ANALYSIS OF ELECTROMAGNETICALLY DRIVEN FLOW IN AN ANNULAR LAYER OF CONDUCTING FLUID

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Rotating flows and the formation of swirling patterns, know as vortices, can be simulated by a number of simple mechanical methods such as physical stirring. However, mechanical methods for inducing flow may fail in some industrial and laboratory settings due to extreme congestion or aggressive thermal or chemical environments. In such conditions applying electromagnetic forcing to a conducting fluid such as electrolytes (solutions of salts in water) offers a practical alternative.

We consider a set-up prototypical to a realistic electromagnetic stirrer, which consists of a shallow conducting fluid layer contained between two coaxial vertical cylinders acting as electrodes and placed over a magnet. When electric current flows between the electrodes, a Lorentz force is created due to the interaction between the vertical magnetic field and the radial electric current, which drives the fluid circumferentially.

Experimental studies reported in [3] demonstrated that when the electric current (and therefore the azimuthal flow speed) is increased, quite unexpectedly, a robust anticyclonic vortex system appears near the outer cylinder. The goal of the present study is to determine physical reasons for the appearance of and the mechanisms maintaining such vortices using linear and weakly nonlinear hydrodynamical stability analyses over a range of a fluid depths, electric current strengths and magnetic field magnitudes.

We show computationally that for small values of electric current a steady axisymmetric basic flow exists consisting of a single toroidal structure, referred to as Type 1 basic flow. As the electric current is increased a second counter-rotating toroidal structure suddenly appears, and we refer to such a flow as Type 2. Subsequently,

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vortices develop on the background of such a two-torus basic flow near the corner of the flow domain formed by the outer cylinder and the free surface. As the electric current is increased further these two steady basic flow types become topologically indistinguishable and eventually disappear, suggesting the existence of a saddle-node bifurcation. However, the experimental observations demonstrate that the anticyclonic vortices continue to exist.

A comprehensive linear analysis of the two steady axisymmetric basic flows is performed in a three-parameter space including the Reynolds and Hartmann numbers and the layer aspect ratio characterising electric current strength, magnetic field magnitude and electrolyte depth, respectively. The real part of the linear amplification rate is an indicator of stability and the azimuthal wavenumber determines the number of vortices.

In the scenario of varying Reynolds number at fixed Hartmann number and aspect ratio, the Type 1 basic solution is linearly stable for all wavenumbers. The Type 2 basic flow is unstable to several modes at its onset. As the Reynolds number is increased the number of vortices and their growth rates change. In particular, at larger values of the Reynolds number the Type 2 flow stabilises with respect to modes with larger azimuthal wavenumbers. As the Reynolds number approaches a certain value, most perturbations attain a negative growth rate, except for the azimuthally invariant mode. Varying electrolyte depth at fixed Reynolds and Hartmann numbers leads to a profound and nontrivial conclusion that even though the electric current, the interaction of which with a magnetic field creates the driving Lorentz force, and the electrolyte depth are two completely independent parameters, the two physically different influences they characterise lead to a physically similar flow evolution. This emphasises the robustness of the formation of the observed vortices. It was also found that as the electrolyte layer becomes deeper, smaller Reynolds numbers (slower azimuthal flow) are required for the existence of the Type 2 basic flow, and that the layer only supports instabilities producing fewer vortices. It was demonstrated that any results and conclusions for the variable magnetic field strength can be inferred from those obtained by varying the electric current.

Subsequently, we derive the linearised perturbation energy balance equation to pinpoint the mechanism responsible for the development of the observed vortices. It was shown that even though the background flow is driven electromagnetically, the appearance of the vortices is due to purely hydrodynamic reasons. It is concluded that the Rayleigh centrifugal instability mechanism predominantly drives the formation of the observed vortices.

Finally, a weakly nonlinear amplitude expansion is performed to find an approximate component of the mean flow solution beyond the critical point. The rigorous procedure developed in [4] was used, which enabled us to derive the equation governing the nonlinear evolution of a disturbance amplitude at a finite distance from the critical point that was shown to follow a canonical fold catastrophe scenario. The resulting analysis shows that past the fold point, the primary meridional circulation and circumferential jet slowed, which allows for the reappearance of the secondary toroidal structure that had become suppressed prior to the fold point. As a consequence, a jet structure with growing amplitude opposing the mean azimuthal flow begins to develop. Note that we cannot calculate a basic flow beyond a fold point. Once the amplitude becomes finite, the nonlinear interactions of the perturbation mode further enhance the corner jet. This results in more energetic collisions of the basic flow and perturbations near the free surface and, subsequently, past the fold point the vortices still arise on the two-torus background, even though it originates from a different branch of the fold.

Some of this research has been published in [1, 2].

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