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Corrigendum: Varieties with ample cotangent bundle

(Compositio Math. 141 (2005), 1445–1459)

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O. Benoist pointed out several errors and inaccuracies in the article [Deb05].

(1) Proposition 5 is wrong as stated and should read as follows.

PROPOSITION 5. *The normal bundle of a smooth nondegenerate subvariety of an abelian variety is nef and big.*

The converse is obviously wrong (consider any smooth curve C in a simple abelian variety A of dimension at least 2; if B is any nonzero abelian variety, the normal bundle of C in $A \times B$ is nef and big, although C is degenerate). The error occurs in the last paragraph of the proof: the tangent spaces to X along a general fiber of $\pi|_X$ are indeed contained in a fixed hyperplane, but this hyperplane is not general, so we cannot conclude that the fiber is finite.

As a result, the second implication of Proposition 6 is unproven. That proposition should read as follows.

PROPOSITION 6. *Let X be a smooth subvariety of an abelian variety A . If Ω_X is ample, $N_{X/A}$ is nef and big.*

(2) Lemma 12 is wrong as stated: if Y is an integral surface with isolated singularities in \mathbf{P}^n which is not Cohen–Macaulay at some point p , the local ring $\mathcal{O}_{Y,p}$ has depth 1. If F is any hypersurface in \mathbf{P}^n which passes through p but does not contain Y , the local ring $\mathcal{O}_{Y \cap F, p}$ has depth 0, hence is not reduced: the intersection $Y \cap F$ has an embedded point at p . Therefore, the variety $\mathcal{V}_e(Y)$ has codimension 1 in $\mathcal{V}_{e,n}$ for any $e \geq 1$.

O. Benoist proved in [Ben11, Théorème 1.5] that this is essentially the only case when things go wrong, and that Lemma 12 holds when Y is for example normal. To correct the proofs of Theorems 7–9 of [Deb05], which all rely on this lemma, we need a slightly modified version of Benoist’s theorem which follows easily from [Ben11, Théorème 1.4].

LEMMA 12 (O. Benoist). *Let Y be an integral subscheme of \mathbf{P}^n of dimension at least 2, let $\nu: \widehat{Y} \rightarrow Y$ be its normalization, and let $\mathcal{V}_{e,n}$ be the projective space of hypersurfaces of degree e in \mathbf{P}^n . The codimension of the complement $\mathcal{V}_e(Y)$ of*

$$\{F \in \mathcal{V}_{e,n} \mid \nu^*F \text{ is integral of codimension 1 in } \widehat{Y}\}$$

in $\mathcal{V}_{e,n}$ is at least $e - 1$.

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The proofs of Theorems 7–9 of [Deb05] can then be easily adapted. For example, in the proof of Theorem 7, just replace diagram (4) with the following.

$$\begin{array}{ccc} H^0(H, L_c^e|_H) & \longrightarrow & H^1(A, \mathcal{O}_A) \\ \downarrow & & \downarrow \rho \\ H^0(\nu^*H, \nu^*L_c^e) & \longrightarrow & H^1(\widehat{Y}, \mathcal{O}_{\widehat{Y}}) \end{array}$$

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- Ben11 O. Benoist, *Le théorème de Bertini en famille*, Bull. Soc. Math. Fr. **139** (2011), 555–569.
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