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ARTICLE

Improved lower bound for the list chromatic number of graphs with no K_t minor*

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Abstract
Hadwiger's conjecture asserts that every graph without a K_t-minor is $(t-1)$-colourable. It is known that the exact version of Hadwiger's conjecture does not extend to list colouring, but it has been conjectured by Kawarabayashi and Mohar (2007) that there exists a constant c such that every graph with no K_t-minor has list chromatic number at most ct. More specifically, they also conjectured that this holds for $c = \frac{3}{2}$. Refuting the latter conjecture, we show that the maximum list chromatic number of graphs with no K_t-minor is at least $(2-o(1))t$, and hence $c \geq 2$ in the above conjecture is necessary. This improves the previous best lower bound by Barát, Joret and Wood (2011), who proved that $c \geq \frac{4}{3}$. Our lower-bound examples are obtained via the probabilistic method.

Keywords: list colouring; list chromatic number; choosability; graph minors; Hadwiger's conjecture

2020 MSC Codes: Primary: 05C15, 05C83

1. Introduction

Preliminaries. Given a number $t \in \mathbb{N}$, a *K_t-minor* is a graph G whose vertex set can be partitioned into t pairwise disjoint non-empty sets $Z_1, \ldots, Z_t$, such that for every $i \in [t]$, the induced subgraph $G[Z_i]$ is connected, and furthermore, for every two distinct $i, j \in [t]$, there exists at least one edge in G with endpoints in the sets Z_i and Z_j. We say that a graph *contains K_t as a minor* or that it *contains a K_t-minor* if it admits a subgraph which is a K_t-minor.

Given a graph G and a colour-set S, a *proper colouring* of G with colours from S is a mapping $c: V(G) \to S$ such that $c^{-1}(s)$ is an independent set, for every $s \in S$. Given a graph G, a *list assignment* for G is an assignment $L: V(G) \to 2^{\mathbb{N}}$ of finite sets $L(v)$ (called lists) to the vertices $v \in V(G)$. An *L-colouring* of G is defined as a proper colouring $c: V(G) \to \mathbb{N}$ of G in which every vertex is assigned a colour from its respective list, that is, $c(v) \in L(v)$ for every $v \in V(G)$. With this, we may define the chromatic number $\chi(G)$ of a graph G as the smallest integer $k \geq 1$ such that G admits an L-colouring, where $L(v) := [k]$ for every $v \in V(G)$.

In a similar way, the *list chromatic number* $\chi_\ell(G)$ of a graph G is defined as the smallest number $k \geq 1$ such that G admits an L-colouring for *every* assignment $L(\cdot)$ of colour lists to the vertices of G, provided that $|L(v)| \geq k$ for every $v \in V(G)$.

Clearly, $\chi(G) \leq \chi_\ell(G)$ for every graph G, but in general $\chi_\ell(G)$ is not bounded from above by a function in $\chi(G)$, as shown for example by complete bipartite graphs.

Hadwiger's conjecture, arguably one of the most important open problems in graph theory, states the following upper bound on the chromatic number of graphs with no K_t-minor:

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Conjecture 1. (Hadwiger [7], 1943) *For every $t \in \mathbb{N}$, if G is a graph not containing a K_t-minor, then $\chi(G) \le t-1$.*

Hadwiger's conjecture and its variants have received a lot of attention in the past, and a very good overview of partial results on this topic until about 2 years ago can be found in the survey article [17] by Seymour. In the following, let us briefly highlight the milestone results regarding Hadwiger's conjecture obtained so far.

As a result of Wagner [22], it was known that the case $t=5$ of Hadwiger's conjecture is equivalent to the statement of the famous *four colour conjecture*. After its confirmative resolution by Appel, Haken and Koch [1, 2] in 1977, Hadwiger's conjecture had been proved for all values $t \le 5$. Notably, in 1993, Robertson, Seymour and Thomas [16] managed to go one step further and to prove Hadwiger's conjecture also for the case $t=6$. As of today, all the cases $t \ge 7$ of Hadwiger's conjecture remain open problems.

The evident difficulty of the exact version of Hadwiger's conjecture has inspired many researchers to study its asymptotic relaxation, known as the *Linear Hadwiger's conjecture*:

Conjecture 2. *There exists an absolute constant $c>0$ such that every graph G not containing a K_t-minor satisfies $\chi(G) \le ct$.*

While also the Linear Hadwiger's conjecture remains open, there has been a lot of progress. By classical results of Kostochka [10] and Thomason [19] from 1984, it was known that K_t-minor-free graphs are $O(t\sqrt{\log t})$-colourable. While already quite close to a linear bound, it has proven difficult to overcome this order of magnitude during more than 30 years of research. Finally, in 2019, Norine, Postle and Song [11] managed to break this barrier, by proving that the maximum chromatic number of K_t-minor-free graphs is in $O(t(\log t)^{1/4+o(1)})$. Very soon afterwards, several related results, extensions and improvements of this bound have been obtained, see [12–15]. The current state of the art-bound of $O(t \log \log t)$ has been obtained only couple of months ago by Delcourt and Postle [6].

Parallel to the development of Hadwiger's conjecture, which concerns the ordinary chromatic number, the list chromatic number of graphs with no K_t-minor has also received a considerable amount of interest. For example, Borowiecki [5] asked whether every graph with no K_t-minor has list chromatic number at most $t-1$ (which would strengthen Hadwiger's conjecture). While this is easily seen to be true for $t \le 4$, already for $t=5$ there exist examples of planar graphs (hence K_5-minor-free) with list chromatic number 5, as constructed first by Voigt [21]. Later, Thomassen [20] proved that 5 is also the correct upper bound for the list chromatic number of planar graphs, and using Wagner's result [22], this carries over to K_5-minor-free graphs, see [8, 18]. For every $t \ge 6$, the maximum list chromatic number of K_t-minor-free graphs remains unknown.

Since the exact version of Hadwiger's conjecture does not extend to list colouring, it is natural to study asymptotic versions. The current state of the art-upper bound on the list chromatic number of K_t-minor-free graphs is $O(t(\log \log t)^2)$, as was recently proved by Delcourt and Postle [6]. Compare also [12, 15] for the previous asymptotic upper bounds of magnitudes $O(t(\log t)^{1/4+o(1)})$ and $O(t(\log \log t)^6)$, respectively.

The following *List Hadwiger conjecture* was first stated by Kawarabayashi and Mohar [9] in 2007, compare also the entry [24] in the Open Problem Garden.

Conjecture 3. *There exists an absolute constant $c>0$ such that every K_t-minor-free graph G satisfies $\chi_\ell(G) \le ct$.*

At first, an even stronger conclusion, namely that every K_t-minor-free graph is t-list-colourable, was believed to be possible, compare for example [9, 23]. However, this stronger conjecture was disproved by the following result of Barát, Joret and Wood [3] from 2011, which shows that $c \ge \frac{4}{3}$ in Conjecture 3 is necessary.

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Theorem 4. *For every integer $t \geq 1$ there exists a graph with no K_{3t+2}-minor and list chromatic number greater than $4t$.*

Kawarabayashi and Mohar stated in [9] that they believe that Conjecture 3 holds true for $c = \frac{3}{2}$, and this statement also appears as Conjecture 8.4 in the survey article [17] by Seymour:

Conjecture 5. *Every graph G without a K_t-minor satisfies $\chi_\ell(G) \leq \frac{3}{2}t$.*

In this note, we disprove Conjecture 5 by showing that the maximum list chromatic number of K_t-minor-free graphs is at least $2t - o(t)$, and hence $c \geq 2$ in Conjecture 3 is necessary. The proof enhances an idea from [3] by using probabilistic arguments.

Theorem 6. *For every $\varepsilon \in (0, 1)$ there is $t_0 = t_0(\varepsilon)$ such that for every $t \geq t_0$ there exists a graph with no K_t-minor and list chromatic number at least $(2 - \varepsilon)t$.*

It would be interesting to see whether our lower-bound construction could be optimal up to the lower-order term, or whether further improvement of the lower bound is possible.

Problem 7. *Does every K_t-minor-free graph G satisfy $\chi_\ell(G) \leq 2t$?*

2. Proof of Theorem 6

In the following, for a natural number $n \in \mathbb{N}$ and a probability $p \in [0, 1]$, we denote by $G(n, n, p)$ the bipartite Erdős-Renyi graph, that is, a random bipartite graph G with bipartition $A \cup B$ such that $|A| = |B| = n$, in which every pair ab with $a \in A, b \in B$ is selected as an edge of G with probability p, independently from all other such pairs.

Lemma 8. *Let $\varepsilon \in (0, 1)$ be fixed, let $f = f(\varepsilon) \in \mathbb{N}$ and $\delta = \delta(\varepsilon) \in (0, 1)$ be constants chosen such that $f\delta < 1$. Let $p = p(n) := n^{-\delta}$. Then w.h.p. as $n \to \infty$, the random graph $G = G(n, n, p(n))$ with bipartition $A \cup B$ satisfies both of the following properties:*

- *For every subset $X \subseteq A$ such that $|X| \geq \varepsilon n$ and every collection of pairwise disjoint non-empty subsets $Y_1, \ldots, Y_k \subseteq B$ such that $k \geq \varepsilon n$ and $\max\{|Y_1|, \ldots, |Y_k|\} \leq f$, there exists a vertex $x \in X$ and some $j \in [k]$ such that G contains all the edges $xy, y \in Y_j$. The same statement holds symmetrically for the case when $X \subseteq B$ and $Y_1, \ldots, Y_k \subseteq A$.*
- *G has maximum degree at most εn.*

Proof.

- Let E_n denote the probability event that G does not satisfy the property claimed in the first item. We need to show that $\mathbb{P}(E_n) \to 0$ as $n \to \infty$. So consider a fixed subset $X \subseteq A$ (or symmetrically, $X \subseteq B$) such that $|X| \geq \varepsilon n$, and a fixed collection $Y_1, \ldots, Y_k$ of disjoint non-empty subsets of B (or symmetrically, A), where $k \geq \varepsilon n$ and $\max\{|Y_1|, \ldots, |Y_k|\} \leq f$. Let $E(X, Y_1, \ldots, Y_k)$ be the probability event 'there exists no pair $(x, j) \in X \times [k]$ such that x is fully connected to the vertices in Y_j'. Fixing a vertex $x \in X$ and an index $j \in [k]$, clearly the probability of the event that 'x is not fully connected to Y_j' equals $1 - p^{|Y_j|} \leq 1 - p^f$. Since these events are independent for different choices of (x, j), we conclude that

$$\mathbb{P}(E(X, Y_1, \ldots, Y_k)) \leq \left(1 - p^f\right)^{|X| \cdot k} \leq \left(1 - p^f\right)^{\varepsilon^2 n^2} \leq \exp(-p^f \varepsilon^2 n^2) = \exp\left(-\varepsilon^2 n^{2 - f\delta}\right).$$

With a rough estimate, there are at most

$$2 \cdot 2^n \cdot (n+1)^n = \exp(\ln(2)(n+1) + n\ln(n+1))$$

ways to select the sets $X, Y_1, \ldots, Y_k$. Hence, applying a union bound we find that

$$\mathbb{P}(E_n) \leq \exp(\ln(2)(n+1) + n\ln(n+1) - \varepsilon^2 n^{2 - f\delta}).$$

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The right hand side of the above inequality tends to 0 as $n \to \infty$, since $f\delta < 1$ and hence $\varepsilon^2 n^{2-f\delta} = \Omega(n^{2-f\delta})$ grows faster than $\ln(2)(n+1) + n\ln(n+1) = O(n \ln n)$. This proves that G satisfies the properties claimed by the first item w.h.p., as required.

- To show that also the property claimed by the second item holds true w.h.p., consider the probability that a fixed vertex $x \in A \cup B$ has more than εn neighbours in G. Note that the degree of x in $G(n, n, p)$ is distributed like a binomial random variable $B(n, p)$, and hence its expectancy is $np = n^{1-\delta}$, which is smaller than $\frac{\varepsilon n}{2}$ for n sufficiently large in terms of ε and δ. Applying Chernoff's bound we find for every sufficiently large n:

$$\mathbb{P}(d_G(x) > \varepsilon n) \le \mathbb{P}(B(n, p) > 2np) \le \exp\left(-\frac{1}{3}np\right) = \exp\left(-\frac{1}{3}n^{1-\delta}\right).$$

Since this bound applies to every choice of $x \in A \cup B$, applying a union bound we find that the probability that G has maximum degree more than εn is at most

$$2n \exp\left(-\frac{1}{3}n^{1-\delta}\right) = \exp\left(\ln(2n) - \frac{1}{3}n^{1-\delta}\right)$$

which tends to 0 as $n \to \infty$, as desired (here we used that $\delta < 1$ and hence $n^{1-\delta}$ grows faster than $\ln(2n)$). □

The next lemma uses Lemma 8 to obtain a useful deterministic statement about the existence of graphs with certain properties, which are then handy when constructing the lower-bound examples for Theorem 6.

Lemma 9. *For every $\varepsilon \in (0, 1)$, there is $n_0 = n_0(\varepsilon)$ such that for every $n \ge n_0$, there exists a graph H whose vertex set $V(H) = A \cup B$ is partitioned into two disjoint sets A and B of size n, and such that the following properties hold:*

- *Both A and B are cliques of H,*
- *every vertex in H has at most εn non-neighbours in H, and*
- *for every $t \in \mathbb{N}$ such that $t \ge (1 + 2\varepsilon)n$, H does not contain K_t as a minor.*

Proof. Let $f := \lceil \frac{1}{\varepsilon} \rceil \in \mathbb{N}$ and $\delta := \frac{\varepsilon}{2}$. Then $f\delta < 1$, and hence we may apply Lemma 8. It follows directly that there exists $n_0 = n_0(\varepsilon) \in \mathbb{N}$ such that for every $n \ge n_0$ there exists a bipartite graph G, whose bipartition classes A and B are both of size n, and such that

- For every subset $X \subseteq A$ such that $|X| \ge \varepsilon n$ and every collection of pairwise disjoint non-empty subsets $Y_1, \ldots, Y_k \subseteq B$ such that $k \ge \varepsilon n$ and $\max\{|Y_1|, \ldots, |Y_k|\} \le f$, there exists a vertex $x \in X$ and some $j \in [k]$ such that G contains all the edges $xy, y \in Y_j$. The same statement holds symmetrically for the case when $X \subseteq B$ and $Y_1, \ldots, Y_k \subseteq A$.
- G has maximum degree at most εn.

We now define H as the complement of G (also with vertex set $A \cup B$). Since G is bipartite, clearly A and B form cliques in H, verifying the first item. The second item follows directly from the fact that $\Delta(G) \le \varepsilon n$.

It hence remains to verify the last item. Towards a contradiction, suppose that there exists a number $t \in \mathbb{N}$, $t \ge (1 + 2\varepsilon)n$, such that H contains K_t as a minor. This implies that there exists a collection $\mathcal{Z}$ of non-empty and pairwise disjoint subsets of $V(H)$ such that $|\mathcal{Z}| = t$ and such that for every two distinct $Z, Z' \in \mathcal{Z}$, there exists at least one edge in H connecting a vertex in Z to a vertex in Z'. Let us denote $\mathcal{Z}_s := \{Z \in \mathcal{Z} | |Z| = s\}$, and $z_s := |\mathcal{Z}_s|$, for every $s \ge 1$. We clearly have

$$2n \ge \sum_{s \ge 1} s z_s \ge 2(t - z_1) + z_1 = 2t - z_1 \ge 2n + 4\varepsilon n - z_1.$$

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Rearranging yields that $z_1 \geq 4\varepsilon n$. From this, we may conclude that either A or B contains at least $2\varepsilon n$ singletons from $\mathcal{Z}$. By symmetry (possibly by renaming A and B), we may assume w.l.o.g. that B contains at least $2\varepsilon n$ singletons from $\mathcal{Z}$, and denote the set of these singletons by X. Let us now define $\mathcal{Z}_A := \{Z \in \mathcal{Z} | Z \subseteq A\}$ and $\mathcal{Z}_B := \{Z \in \mathcal{Z} | Z \cap B \neq \emptyset\}$. Since the sets in $\mathcal{Z}$ are pairwise disjoint, we can see that $|\mathcal{Z}_B| \leq |B| = n$, and therefore $|\mathcal{Z}_A| = t - |\mathcal{Z}_B| \geq t - n \geq 2\varepsilon n$. Since $|A| = n$, the latter implies that there are at least εn distinct sets in $\mathcal{Z}_A$ which have size at most $\frac{1}{\varepsilon}$. Let $Y_1, \ldots, Y_k$ with $k \geq \varepsilon n$ be an enumeration of the sets in $\mathcal{Z}_A$ of size at most $\frac{1}{\varepsilon} \leq f$. By the above, there exists $j \in [k]$ and a vertex $x \in X$ such that $xy \in E(G)$ for every $y \in Y_j$. Since H is the complement of G, this means that $\{x\}$ and Y_j are distinct sets in $\mathcal{Z}$, which do not have any connecting edge. This is a contradiction to our initial assumptions, and hence we have shown that the third item claimed in the lemma is also satisfied. This concludes the proof. □

We are now ready for the proof of Theorem 6. The only missing ingredient is the following well-known 'pasting-lemma', compare Lemma 3 in [3].

Lemma 10. *Let G_1 and G_2 be -minor-free graphs, and let $V(G_1) \cap V(G_2) = C$. If C forms a clique in both G_1 and G_2, then the graph $G_1 \cup G_2$ is also K_t-minor-free.*

Proof of Theorem 6. Let a fixed $\varepsilon \in (0,1)$ be given. Pick some $\varepsilon' \in (0,1)$ such that $\frac{2-\varepsilon'}{1+2\varepsilon'} \geq 2 - \frac{\varepsilon}{2}$. Let $n_0 = n_0(\varepsilon') \in \mathbb{N}$ be as in Lemma 9, and define $t_0 := \max\{\lceil (1+2\varepsilon')n_0 \rceil, \lceil \frac{4}{\varepsilon} \rceil\}$. Now, let $t \geq t_0$ be any given integer. Define $n := \left\lfloor \frac{t}{1+2\varepsilon'} \right\rfloor \geq n_0$. Applying Lemma 9, we find that there exists a graph H whose vertex set is partitioned into two non-empty sets A and B of size n, such that both A and B form cliques in H, every vertex in H has at most $\varepsilon' n$ non-neighbours, and H is K_t-minor-free (since $t \geq (1+2\varepsilon')n$, by definition of n).

For every possible assignment $c \in [2n-1]^A$ of colours from $[2n-1]$ to vertices in A, denote by $H(c)$ an isomorphic copy of H, such that the vertex set of $H(c)$ decomposes into the sets A and $B(c)$ of size n. More precisely, the distinct copies $H(c)$, $c \in [2n-1]^A$ of H share the same set A but have pairwise disjoint sets $B(c)$. Since A forms a clique of size n in the K_t-minor-free graph $H(c)$ for every colouring $c: A \to [2n-1]$, it follows by repeated application of Lemma 10 that the graph $\mathbf{G}$ with vertex set $A \cup \bigcup_{c \in [2n-1]^A} B(c)$, obtained as the union of the graphs $H(c)$, $c \in [2n-1]^A$, is K_t-minor-free.

Now, consider an assignment $L: V(\mathbf{G}) \to 2^{\mathbb{N}}$ of colour lists to the vertices of $\mathbf{G}$ as follows: For every vertex $a \in A$, we define $L(a) := [2n-1]$, and for every vertex $b \in B(c)$ for some colouring $c \in [2n-1]^A$ of A, we define $L(b) := [2n-1] \setminus \{c(a) | a \in A, ab \notin E(H(c))\}$. Note that since every vertex in $B(c)$ has at most $\varepsilon' n$ non-neighbours in $H(c)$, we have $|L(v)| \geq 2n - 1 - \varepsilon' n$ for every vertex $v \in V(\mathbf{G})$.

We now claim that $\mathbf{G}$ does not admit a proper colouring with colours chosen from the lists $L(v)$, $v \in V(\mathbf{G})$, which will then prove that $\chi_\ell(\mathbf{G}) \geq 2n - \varepsilon' n$. Indeed, suppose towards a contradiction there exists a proper colouring $c_{\mathbf{G}}: V(\mathbf{G}) \to \mathbb{N}$ of $\mathbf{G}$ such that $c_{\mathbf{G}}(v) \in L(v)$ for every $v \in V(\mathbf{G})$. Let c be the restriction of $c_{\mathbf{G}}$ to A, and consider the proper colouring of $H(c)$ obtained by restricting $c_{\mathbf{G}}$. Since $H(c)$ has order $2n$ and $c_{\mathbf{G}}(v) \in [2n-1]$ for every $v \in V(H(c))$, there must exist two (necessarily non-adjacent) vertices in $H(c)$ which have the same colour with respect to $c_{\mathbf{G}}$. Concretely, there exist $a \in A$, $b \in B(c)$ such that $ab \notin E(H(c))$ and $c_{\mathbf{G}}(a) = c_{\mathbf{G}}(b)$. This however yields a contradiction, since $c_{\mathbf{G}}(b) \in L(b)$ and by definition $c(a) = c_{\mathbf{G}}(a)$ is not included in the list of b.

We conclude that indeed, $\mathbf{G}$ is a K_t-minor-free graph which satisfies

$$\chi_\ell(\mathbf{G}) \geq (2-\varepsilon')n = (2-\varepsilon')\left\lfloor \frac{t}{1+2\varepsilon'} \right\rfloor > (2-\varepsilon')\left(\frac{t}{1+2\varepsilon'} - 1\right)$$
$$\geq \left(2 - \frac{\varepsilon}{2}\right)t - (2-\varepsilon') \geq (2-\varepsilon)t,$$

where for the last inequality we used $t \geq t_0 \geq \frac{4}{\varepsilon}$. □

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