LETTER TO THE EDITOR

Dear Editor,

Lindley and Singpurwalla (1986) have recently considered a reliability system of two parallel exponential components. The system operates in an environment which does not change over time but may be different from the test environment. Suppose the environment factor η is a random variable having the gamma density

$$b^{a+1}\eta^{a}\exp(-\eta b)/a!; \tag{1}$$

then the authors show that the joint life density of T_1 and T_2, the times to failure of the two components at t_1 and t_2 respectively, is

$$p(t_1, t_2 \mid \lambda_1, \lambda_2, a, b) = \frac{\lambda_1\lambda_2(a+1)(a+2)b^{a+1}}{(\lambda_1 t_1 + \lambda_2 t_2 + b)^{a+3}} \tag{2}$$

where $1/\lambda_i$ $(i = 1, 2)$ is the mean lifetime of the ith exponential component before the components were assembled as a system.

The model suggested by the authors is interesting and useful. However, I wish to add three points which might be helpful to the readers.

(i) The authors claimed that they were unable to show that the joint density (2) can be transformed to any one of the bivariate Pareto densities considered by Mardia (1962). However, it is easy to see that the density (2.1) of Mardia (1962) would reduce to (2) if we let $\lambda_1\lambda_2 = b$.

(ii) The bivariate distribution (2) when $\lambda_1\lambda_2 = b$ has also been derived by Hutchinson (1979) but from a different context. He has discussed interesting applications of this distribution to the following problems: (a) Observers agreement studies on diagnosis of multiple sclerosis; (b) Classification of various degrees of injuries viz fatal, serious, slight and non-injury in road accidents; (c) Multifactored model of some schizophrenia or pyloric stenosis, and (d) Replicated paired comparison tests of brand products.

(iii) Equation (2) may also be derived through a trivariate reduction technique, Suppose that Y_i is the lifetime of the ith exponential component with parameter λ_i $(i = 1, 2)$ before the components are assembled as a system and η is the common environmental factor having a density given in (1). It is reasonable to assume that Y_1, Y_2 and η are mutually independent. Let

$$T_1 = Y_1/\eta, \quad T_2 = Y_2/\eta,$$

that is, each component life is weighted by an environmental factor; then T_1 and T_2 have a joint distribution given by (2). The technique used in this derivation is in essence the same technique as that used by Lindley and Singpurwalla (1986) except that our proof reveals more clearly the structure of the model.

References

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