INFLUENCE OF NON-GRAVITATIONAL FORCES ON THE ORBITAL EVOLUTION OF THE SHORT-PERIOD COMETS

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ABSTRACT. This paper is concerned with the influence of non-gravitational forces on the orbital evolution of the short-period comets. This influence is variable for different comets and is especially noticeable for those comets which undergo approaches with Jupiter. When studying the dynamic evolution it is desirable to take into account the non-gravitational effects.

INTRODUCTION

An idea of the influence of the non-gravitational forces upon the cometary motion was first advanced by Encke in 1819. It had been revealed in the course of the 20th century that these forces acted upon some more comets, the number of comets whose motions experienced secular accelerations and decelerations being approximately equal. This indicated that the cause of the irregularities in cometary motion was connected with the comet's nucleus. However, consideration of the non-gravitational effects appeared to be possible without regard to the physical nature of these forces. This way of research was chosen by J.Encke, O.Backlund, M.Kamienski, A.Dubyago, G.Sitarski and other researchers of comets.

At present, there exists a fairly well developed icy nucleus model of a comet, proposed by F.Whipple and the method of taking into account the non-gravitational forces based on the above model, comprehensively presented by B. Marsden et al. in the series of their publications (Marsden, Marsden et al. 1968, 1969, 1970, 1971). There is no need to dwell upon this method at any length. It will suffice to mention that when improving cometary orbits from observations, one determines two non-gravitational parameters A1 and A2 along with the orbital elements. These two parameters define the amplitude and direction of the

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acceleration components caused by the non-gravitational forces and they are directed along the radius-vector (parameter A1) and along the transversal (parameter A2). For A1 > 0 the acceleration is oriented away from the sun whereas for A2 > 0 - it is in the direction of increasing true anomaly.

The new method has been applied by B.Marsden for a whole series of comets and, at present, the 4th edition of Marsden's Catalogue of Cometary Orbits (1982) includes the non-gravitational parameters A1 and A2 for 34 comets.

Thus, it may be viewed upon as a real fact that 34 short-period comets experience to a greater or lesser extent the effect of the non-gravitational forces during the time interval, limited, at least, by observations of these comets. There are no reasons to think that these forces (under definite conditions) did not act prior to the comet's discovery nor is there any reason to believe they will not act in the future.

The orbital evolution of some comets has been studied with consideration of both the influence of planetary perturbations and the non-gravitational effects.

Thus, Marsden and Sekanina (1974) investigated the motion of P/Encke not only over the time interval covered by observations (from 1786), but also over the interval spanning 200 yr preceding its discovery employing the two parameters of $A1= -0.0335 \cdot 10^{-8}$ and $A2= -0.0180 \cdot 10^{-8}$. The authors have established that after 100 years, there is a discrepancy between the predicted and computed times of perihelion passage of 6 weeks and after a period of 200 years, this discrepancy amounts to 5 months.

Yeomans (1977) has studied P/Halley's motion backward in time to 837 of our epoch and later on in collaboration with Kiang (Yeomans and Kiang, 1981) to 1404 B.C. They have substantiated an earlier established fact that the influence of non-gravitational forces affects the time of the perihelion passage by about four days per one revolution.

Kazimirchak-Polonskaya (1968), when studying the motion of P/Wolf with account of the non-gravitational effects, arrived at conclusion that the influence of these forces on P/Wolf's orbital evolution was negligible.

This fact, however, does not allow to draw a general conclusion concerning the negligibly small influence of these forces on the motion of all 34 comets for which the numerical values of the non-gravitational forces are known. For a brief review of non-gravitational forces affecting the motions of comet, see the recent work by Marsden (1984).

METHOD OF SOLUTION

The aim of the present paper is to carry out, as far as possible, the most careful and complete analysis on the influence of non-gravitational forces on the dynamic evolution of short-period comets over long time periods. The time span under consideration covers 200 yr (1800-2000). Orbital elements for all numerical integration were taken from the 4th edition of Marsden's Catalogue of Cometary Orbits (1982) as well as values of the non-gravitational parameters A1 and A2. Equations of cometary motion were integrated by Cowell's quadrature method with consideration of the fourth order differences. Perturbations by nine planets (Mercury-Pluto) were taken into account. We have employed the system of masses adopted by the IAU in 1964.

The non-gravitational parameters A1 and A2 chosen as the starting ones were considered for the whole interval of integration to be constant. That is,it has been assumed that the non-gravitational forces remained unchanged in time over the span of 200 yr.

Of the 80 comets, observed in two or more apparitions, the non-gravitational parameters are known only for 34. Among these are the comets which have constant A1 and A2, the comets which have changing values but retain the sign of A2 and, finally, those for which A2 has changed in quantity and in sign.

Therefore, for convenience of the study 34 comets have been divided into three groups in accordance with the above mentioned peculiarities; these groups are presented in Table I. To the first group (with constant A1 and A2) belong 13 comets; to the second group (A1 and A2 change in absolute value) - 6 (figures indicate how often the parameters changed) and into the third group were included 15 comets (numerator indicates the total number of the parameter changes and denominator - the number of variations with the change of a sign).

Before passing to a presentation of the results, we should like to point out the difficulties involved in solving the problem of the dynamic evolution of cometary orbits with full account of both the planetary perturbations and the non-gravitational effects. If a comet was observed in a sufficiently great number of apparitions, then, as a rule, all observations in these apparitions cannot be linked with a required accuracy by one system of elements and one is forced to link the apparitions by groups.

But in such cases it is up to the researcher which orbit should be chosen as a starting one, if he is not willing to proceed by utilizing the first, with regard to time, apparition for backward integration and the last one - for forward integration in time. The preferable technique would be a study based on one system of elements.

TABLE I

Comets experiencing non-gravitational effects

A1, A2: Constants	Variations without a change of sign	Variations with a change of sign
1. Arend (1951 X)	1. Biela (1772)-2 1/	1. Brooks 2 (1889 V)-4/2 2/
2. Daniel (1909 IV)	2. Borelly (1905 II)-2	2. Brorsen (1846 III)-2/1
3. Jackson--Neujmin (1936 IV)	3. Halley (-239)-2	3. Comas-Sola (1927 III)-2/1
4. Johnson (1949 II)	4. Honda-Mrkos--Pajdusakova (1948 XII)-2	4. D'Arrest (1851 II)-7/6
5. Olbers (1815)	5. Schaumasse (1911 VII)-2	5. Encke (1786 I)-18/10
6. Perrine--Mrkos (1896 VII)	6. Schwassmann-Wachmann 2 (1929 I)-3	6. Faye (1843 III)-4/1
7. Reinmuth 1 (1928 I)		7. Finlay (1866 VII)-4/1
8. Tempel- -Swift (1869 III)		8. Forbes (1929 II)-2/1
9. Tempel--Tuttle (1366)		9. Giacobini-Zinner (1900 III)-4/1
10. Whipple (1933 IV)		10. Grigg-Skjellerup (1902 II)-3/1
11. Wirtanen (1947 XIII)		11. Kopff (1906 IV)-4/1
12. Wolf (1884 III)		12. Pons-Winnecke (1819 III)-5/4
13. Wolf--Harrington (1924 IV)		13. Tempel 2 (1873 II)-4/2
		14. Tuttle (1790 II)-3/2
		15. Tuttle-Giacobini-Kresak (1858 III)-2/1

1/ Figure indicates how many times the non-gravitational parameters change; 2/ Numerator has the same meaning as in 1/; denominator denotes the number of A1 and A2 variations that include a sign change for A2.

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Inthe presence of several orbits and a pair of non-gravitational parameters the problem is reduced to the choice of the starting system of elements but for the second and the third groups from table I this choice is complicated by selection of the non-gravitational parameters. For the solution of the first problem we have chosen the way of Yeomans and Kiang (1981).

As a criterion we have taken the differences (ΔT) between the perihelion passage moments in all observed apparitions of the comet derived by numerical integration (Tc) and those values of T, given by B.Marsden (1982). Needless to say, this was a labour-consuming work, entailing repeated numerical integrations within the range of the observed apparitions which for some comets (for instance, P/Pons-Winnecke 1819 III = 1976 XIV) amounted to more than 150 yr.

We had no clear-cut criterion for evaluating ΔT values derived in the computing process, as good or bad. However, on the whole, we were adhering to two principles: firstly, not to employ any sharp deviations of ΔT; secondly, all the ΔT values for a given comet should be minimized in absolute values.

Sometimes, it may be reached by a selection of the system of elements.[1] But more often we also used the selection of non-gravitational effects.

Table II presents for some comets those values of A1 and A2 which are given in B.Marsden's Catalogue of Cometary Orbits and which have been selected for studying the dynamic evolution of cometary orbits. It is apparent from this Table, that sometimes the most optimal values turned out to be one of the A1 and A2 pairs, given by B.Marsden in his Catalogue, sometimes A1 taken from one group and A2 from the other one, and, finally, for some of the comets we had to select A1 and A2 artificially so that the ΔT values would be minimized.

DISCUSSION AND CONCLUSIONS

Table III presents a partial summary of our preliminary study of the non-gravitational parameters for the comets indicated in column 2. In column 4 of the Table are given the maximum values of the ΔT discrepancies from all the observed apparitions derived with the highest possible (in

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1) For instance, for P/Pons-Winnecke $\Delta T = 5^{d}.7$ if the initial orbit is taken as the system of elements in 1875 and $\Delta T = 1^{d}.3$ if the initial orbit is taken as the 1939 system of elements.

TABLE II

Examples of Comets with variable values of non-gravitational parameters

Comet	Style	Intervals of constants A1 and A2	A1	A2
P/Biela	1	1805-1833	+0.28	-0.0250
		1826-1846	+0.39	-0.0254
		1832-1852	+0.36	-0.0260
		1800-2000	+0.39	-0.0254
P/Tempel 2	2	1873-1915	+0.08	+0.0021
		1915-1956	-0.04	+0.0012
		1930-1967	-0.04	+0.0008
		1956-1978	+0.08	+0.0022
		1800-2000	-0.01	+0.0008

TABLE III

Influence of non-gravitational effects on the accuracy of the perihelion passage moments of some comets

NN	Comet	Epoch	T_{max} (A1, A2≠0)	T_{max} (A1, A2=0)
1	P/Biela (1772)	1832[1/]	$0^d.99$	$18^d.38$
2	P/Wolf-Harrington (1924 IV)	1952	0.03	5.83
3	P/Giacobini-Zinner (1900 III)	1959	0.72	2.65
4	P/Tempel-Swift (1869 III)	1891	0.10	1.55
5	P/Kopff (1906 IV)	1951	0.24	0.74
6	P/Forbes (1929 II)	1961	0.03	0.44
7	P/Wirtanen (1947 XIII)	1967	0.04	0.27

1/ Epoch denotes an apparition of a comet using whose elements one gets the best results of numerical integration.

the above given sense) non-gravitational effects. In column 5 of the Table for comparison purposes are indicated the highest possible values of ΔT, derived without consideration of non-gravitational effects. Thus, this Table gives a partial proof for the necessity of taking into account the non-gravitational forces when studying the evolution of cometary orbits. The more comprehensive answer is provided by Tables IV and V. Table IV illustrates the

TABLE IV

Influence of non-gravitational effects on the evolution of the cometary orbits in the absense of the close approaches to Jupiter ($\Delta_{\mathcal{J}min} > 0.33$AU)

1800	π	Ω	i	e	q	P	ΔT_{max}
	P/Johnson		A1=0.78			A2= -0.0266	
A1,A2=0	317°	127°	15°	0.35	2.35AU	6.9yr	$0^d.01$
A1,A2≠0	317	127	15	0.35	2.34	6.9	0.03
Discrepancies	-	-	-	-	0.01	-	0.02
	P/Jackson-Neujmin			A1=0.8		A2=-0.45	
A1,A2=0	3°	184°	14°	0.65	1.49AU	8.6yr	$7^d.02$
A1,A2≠0	2	180	11	0.64	1.56	9.1	0.01
Discrepancies	+1	+4	+3	+0.01	-0.07	-0.5	+7.01

influence of the non-gravitational effects upon the orbital evolution of P/Johnson and P/Jackson-Neujmin whose motions did not reveal any evidence of the close approaches with Jupiter back to 1800. For each comet, integrations have been run back to 1800 to provide orbital elements for cases both with and without consideration of the non-gravitational forces, the third line representing discrepancies between the elements. Table V, using the same scheme, presentsdata for two comets which experienced close approaches with Jupiter - P/Schassmann-Wachmann 2 and P/Honda-Mrkos-Pajdusakova. The data presented above and these examples show that non-gravitational forces can produce substantial perturbations in cometary orbital elements and, to a certain extent, affect the final results when studying the dynamic evolution of cometary orbits. This influence may be rather appreciable even in the absence of close approaches of the comet to Jupiter under investiga-

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TABLE V

Influence of non-gravitational effecs on the evolution of cometary orbits in presence of close approaches ($\Delta\gamma_{min} < 0.33$ AU)

1800	π	Ω	i	e	q	P	ΔT_{max}
P/Schwassmann-Wachmann 2 A1=1.02, A2=-0.1801 1926(0.18)							
A1,A2=0	106°	123°	1°	0.20	3.52AU	5.23yr	$1^d.27$
A1,A2≠0	104	121	0.6	0.19	3.59	5.25	0.15
Differencies	-2	-2	-0.4	-0.01	+0.07	+0.02	-1.12
P/Honda-Mrkos-Pajd. A1=0.27, A2=-0.0420, 1876 (0.08) 1935 (0.079)							
A1,A2=0	52°	271°	3°	0.68	1.22AU	7.51yr	$0^d.99$
A1,A2≠0	53	258	11	0.69	1.09	6.86	0.15
Differencies	+1	-13	+8	+0.01	-0.13	-0.65	-0.84

tion but become very notable in the presence of such approaches. However, the extent of this influence on the final results, as is obvious from Tables IV and V, may be variable and, therefore, it is difficult to predict the result in advance. The general conclusion of the present investigation is as follows: if at all possible then one ought to take account of the non-gravitational forces when studying the dynamic evolution of cometary orbits. Our results point out that it is necessary to determine the average values of the non-gravitational parameters A1 and A2 over the time span under consideration if a researcher is facing the problem of their choice.

A complete account of the results obtained in the present investigation took place when studying orbital evolution of all short-period comets over the 200 yr time interval (1800-2000).

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