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Combined structural and topological stability are equivalent to Axiom A and the strong transversality condition

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Abstract. The purpose of this paper is to develop necessary conditions for a diffeomorphism to be topologically stable (lower semistable). Our results combine with a recent theorem of R. Mañé and with earlier results of J. Robbin, C. Robinson, and Z. Nitecki to give a complete characterization of diffeomorphisms of compact manifolds that are both topologically and structurally stable: they are precisely the Axiom A diffeomorphisms that satisfy the strong transversality condition.

0. *Introduction*

We consider diffeomorphisms of a compact Riemannian manifold M. f is (C^1) *structurally stable* if there is a neighbourhood $\mathcal{U}$ of f (in Diff(M) with the C^1 topology) such that for each g in $\mathcal{U}$ there is a homeomorphism $h = h_g$ on M satisfying $f \circ h = h \circ g$. A related notion is that of *topological stability*, also called *lower semi-stability*. f is topologically stable if for any $\delta > 0$ there is a neighbourhood $\mathcal{U} = \mathcal{U}_\delta$ of f (in the set of homeomorphisms of M with the uniform C^0 topology) such that for each g in $\mathcal{U}$ there is a continuous surjection $h = h_g$ from M to M satisfying $f \circ h = h \circ g$ and $d(h(x), x)$ is less than δ for all x in M. (Here and throughout this paper $d(\_\_, \_\_)$ denotes a metric on M that is induced by the Riemannian structure.)

J. Palis and S. Smale, in [**10**], made the conjecture that a certain set of conditions on f are necessary and sufficient for f to be structurally stable. These conditions are that f satisfy *Axiom A* and the *Strong Transversality Condition* (the definitions are given below). J. Robbin, [**11**], and C. Robinson, [**13**], succeeded in showing that these conditions are indeed sufficient for structural stability. There has also been much work done on the neccessity of these conditions for structural stability. In a recent paper, [**8**], R. Mañé gives a summary of much of this research; we refer the reader to the introduction of that paper for a history of this research. In the same paper Mañé establishes the fact that if f is structurally stable and if there is a uniform lower bound on the distance between any two periodic points of f with different indices, then f satisfies Axiom A and the strong transversality condition. (The index of a periodic point p is the dimension of the unstable manifold of p; that is, the number of eigenvalues of $Df^k(p)$ of modulus greater than one, where k is the period of p under f.)

The study of topological stability has had a development paralleling that of structural stability. The earliest results were due to P. Walters, [**16**], who showed that Anosov diffeomorphisms are topologically stable. Later, Z. Nitecki, [**9**], showed that Axiom A and the strong transversality condition are sufficient to imply topological stability.

As for necessary conditions for topological stability, note that it is immediate from the definitions that topological stability is a conjugacy invariant, so it is not hard to construct examples of diffeomorphisms that are topologically stable but do not satisfy Axiom A. Recently, however, it has been shown that topologically stable diffeomorphisms on the circle and topologically stable flows on compact surfaces must be conjugate to Morse–Smale dynamical systems, which are the Axiom A, strong transversality condition systems of these types [**17**], [**2**]. More generally, it has been shown that any topologically stable diffeomorphism shares a large number of qualitative features with Axiom A diffeomorphisms [**7**]. One of the purposes of the current paper is to establish further similarities of this type. Our main result is:

THEOREM A. *A diffeomorphism on a compact manifold is both topologically and* (C^1) *structurally stable if and only if it satisfies Axiom A and the strong transversality condition.*

As was mentioned earlier, the facts that Axiom A and the strong transversality condition imply topological stability and C^1 structural stability have been established by Nitecki and Robinson, respectively. In order to prove the converse, we use the assumption of topological stability to establish the following:

PROPOSITION. *If f is topologically stable, then the chain recurrent set of f is composed of a finite number of chain components* (*basic sets*). *If in addition f is Kupka–Smale, then there is a unique value of the index of the periodic points of f that lie in any given chain component of f.*

Combining this proposition with the following theorem of R. Mañé gives the proof of theorem A.

THEOREM (Mañé). *A C^1 structurally stable diffeomorphism satisfies Axiom A and the strong transversality condition if and only if $\Lambda_i \cap \Lambda_j$ is empty for every $0 \le i < j \le$* $\dim(M)$, *where Λ_i is the closure of the set of periodic points of f of index i.*

(This result is essentially the same as theorem B of [**8**]; see the discussion on pages 504–505 of [**8**].)

Before going on, we should mention that there have been other papers showing that certain hypotheses stronger than structural stability are equivalent to Axiom A and the strong transversality condition. Among these results are those of J. Franks, [**4**], [**5**], and J. Guckenheimer, [**6**].

I. *Preliminaries*

Given a diffeomorphism g on M, an *α-chain* for g is a sequence (finite, infinite, or bi-infinite), (x_i) with the property that $d(g(x_i), x_{i+1})$ is less than α for all relevant values of i. A bi-infinite α-chain (x_i) is *periodic* if $x_{i+k} = x_i$ for all i and some $k \geq 1$. A point p in M is *chain recurrent* for g if for each $\alpha > 0$ there is a periodic *α-chain* (x_i) for g with $x_0 = p$; the collection of all such points is the *chain recurrent set* of g, denoted $\mathrm{CR}(g)$. It is easy to see that $\mathrm{CR}(g)$ is both closed and g-invariant.

A subset X of $\mathrm{CR}(g)$ is called *chain transitive* if for any pair of points p, q in X and any positive constant α there is a periodic *α-chain* (x_i) with p, q contained in $\{x_i\}$. X is a chain *component* of g if X is chain transitive and no set that contains X as a proper subset is chain transitive. (Equivalently, a chain component is an equivalence class in $\mathrm{CR}(g)$ under the relation $p \sim q$ if and only if for every positive α there is a periodic α-chain for g containing both p and q.) Note that each chain component of g is closed and g-invariant. Note also that if x is any point of M then the alpha-limit set and the omega-limit set of x with respect to g are chain transitive sets. (These sets are the maximal g-invariant subsets of clos $\{g^{-n}(x) | n \geq 0\}$ and clos $\{g^n(x) | n \geq 0\}$, respectively.)

If g is a homeomorphism on M and x is in M, then we can define the *stable set* of x with respect to g, $W^s(x; g)$, and the *unstable set* of x with respect to g, $W^u(x; g)$ by

$$W^s(x; g) = \{y \text{ in } M | d(g^n(y), g^n(x)) \to 0 \quad \text{as } n \to \infty\}$$
$$W^u(x; g) = \{y \text{ in } M | d(g^{-n}(y), g^{-n}(x)) \to 0 \quad \text{as } n \to \infty\}.$$

When the identity of g is clear from the context we will at times abbreviate $W^\sigma(x; g)$ to $W^\sigma(x)$, $\sigma = s$ or u. A similar definition is that of the *γ-local stable set* of x with respect to g, $W^s_\gamma(x; g)$, which is the set of points y in $W^s(x; g)$ satisfying $d(g^n(x), g^n(y)) < \gamma$ for all $n = 0, 1, 2, \ldots$. The *γ-local unstable set* of x with respect to g is defined analogously and is denoted $W^u_\gamma(x; g)$.

A diffeomorphism g is said to satisfy *Axiom A* if $\mathrm{CR}(g)$ has a hyperbolic structure. A hyperbolic structure for g on $\mathrm{CR}(g)$ is a continuous splitting $E^u_x \oplus E^s_x$ of the tangent space TM_x at each point x in $\mathrm{CR}(g)$ satisfying

(1) $Dg(x)(E^\sigma_x) = E^\sigma_{g(x)}$, $\quad \sigma = s$ or u;

(2) there are constants $C > 0$, $0 < \lambda < 1$, such that for $n = 1, 2, 3, \ldots$,

$$\|(Dg^n(x))| E^s_x\| \leq C\lambda^n \quad \text{and} \quad \|(Dg^{-n}(x))| E^u_x\| \leq C\lambda^n$$

for all x in $\mathrm{CR}(g)$.

(This definition of Axiom A is not the one originally given by Smale. However, in the presence of the strong transversality condition the original definition and the one we are using are equivalent; see [**3**] for more details.)

When g satisfies Axiom A and x is in $\mathrm{CR}(g)$ the stable and unstable sets of x with respect to g are smoothly immersed submanifolds of M, with $\dim W^\sigma(x; g) = \dim E^\sigma_x$, $\sigma = s$ or u. These stable and unstable manifolds enjoy several other interesting properties, among them

(3) invariance: $g(W^\sigma(x)) = W^\sigma(g(x))$, $\quad \sigma = s$ or u.

(4) $W^\sigma(x)$ is tangent at x to E_x^σ, $\sigma = s$ or u.

(5) M is the union of the sets $W^\sigma(x)$ for x in $\mathrm{CR}(g)$, with σ equal to either s or u.

Because of (5) it makes sense to talk about the stable or unstable manifold of x with respect to an Axiom A diffeomorphism g regardless of whether or not x is in $\mathrm{CR}(g)$.

An Axiom A diffeomorphism g is said to satisfy the *strong transversality condition* if $W^s(x; g)$ and $W^u(y; g)$ are everywhere transverse for every pair of points x, y in M. Much is understood about the structure of $\mathrm{CR}(g)$ when g satisfies Axiom A and the strong transversality condition; see [1] for example. One particular fact that we will be making use of is the existence of a 'local product structure' near the chain recurrent set.

PROPOSITION 1. *Suppose that g is a diffeomorphism satisfying Axiom A. Then there is a constant $\gamma_0 > 0$ such that if $0 < \gamma < \gamma_0$ then there is a constant $\delta = \delta(\gamma) > 0$ with the property that if x, y are in M, within δ of each other, and at least one of them is in $\mathrm{CR}(g)$, then $W^s_\gamma(x)$ and $W^u_\gamma(y)$ intersect in a non-empty set which we denote by $[x, y]$. Moreover, δ can be chosen small enough that $[x, y]$ is a single point that is contained in $\mathrm{CR}(g)$ as long as both x and y are in $\mathrm{CR}(g)$.*

Proof. See §§ 3.A and 3.B of [1].

II. *Properties of topologically stable diffeomorphisms*

In [7] it is shown that any topologically stable diffeomorphism shares a large number of qualitative features with Axiom A diffeomorphisms. In this section we build upon the techniques developed in that paper.

LEMMA 2. *If f is topologically stable and X is a chain component of f, then $f|X$ is topologically transitive; that is, there is a point in X whose alpha-limit set and omega-limit set are each dense in X.*

Proof. The first statement is proved in [7]; that the second follows from the first is a standard exercise in point-set topology. □

PROPOSITION 3. *Suppose that f is topologically stable and that X is a chain component of f. Let g be a homeomorphism of M and h a semiconjugacy from g to f, $hg = fh$. Then*

(a) *$h(\mathrm{CR}(g))$ is contained in $\mathrm{CR}(f)$;*

(b) *$h^{-1}(X)$ contains every chain component of g that it meets;*

(c) *there is a chain component Y of g with $h(Y) = X$. (Consequently* (a) *can be strengthened to $h(\mathrm{CR}(g)) = \mathrm{CR}(f)$.)*

Proof. Since h is uniformly continuous, given $\alpha > 0$ there is a $\beta > 0$ such that if (x_i) is a β-chain for g then $(h(x_i))$ is an α-chain for f. Hence if Z is chain transitive for g, then $h(Z)$ will be chain transitive for f. This establishes both (a) and (b).

To prove (c), we use lemma 2 to find a point x in X whose omega-limit set is all of X. Since h is a surjection, we can find a point p in $h^{-1}(x)$. Let Y be the chain component that contains the omega-limit set of p with respect to g. To see that

$h(Y)=X$, let q in X be given. By the way x was chosen, there is a sequence of integers n_k going to infinity with $f^{n_k}(x)\to q$. By compactness there is a subsequence (which we also label n_k) with $g^{n_k}(p)$ converging to some point y. By definition of Y, y is in Y. Finally,

$$h(y)=\lim_{k\to\infty} h(g^{n_k}(p))=\lim_{k\to\infty} f^{n_k}(h(p))=\lim_{k\to\infty} f^{n_k}(x)=q. \qquad \square$$

A diffeomorphism g is said to be *in phase* if for each x in M there are points $y=y(x)$ and $z=z(x)$ in $\mathrm{CR}(g)$ with x contained in the intersection of $W^s(y;g)$ and $W^u(z;g)$. If g is Axiom A then g is in phase (this is the content of (5) in the previous section); see 3.10 of [**1**].

LEMMA 4. *If f is topologically stable then f is in phase.*

Proof. By Shub's density theorem [**14**] we can find an Axiom A diffeomorphism g that is C^0 close to f; consequently there is a semiconjugacy h from g to f, $hg=fh$. Let x in M be given, and pick p in $h^{-1}(x)$. Since g is in phase there are points q, r in $\mathrm{CR}(g)$ with p in the intersection of $W^s(q;g)$ and $W^u(r;g)$. Let $y=h(q)$ and $z=h(r)$. Then y, z are in $\mathrm{CR}(f)$ by proposition 3. It is an easy consequence of the continuity of h that $h(W^\sigma(w;g))$ is contained in $W^\sigma(h(w);f)$ for $\sigma=s$ or u, so x is in $W^s(y;f)\cap W^u(z;f)$ as desired. $\square$

If A, B are closed non-empty subsets of M, let

$$\mathrm{dist}(A,B)=\inf\{d(a,b) \mid a\in A,\ b\in B\}.$$

LEMMA 5. *If h is a continuous surjection from M to itself and if Y is a closed non-empty subset of M, then the map $x\to h^{-1}(x)\cap Y$ is upper semicontinuous. In particular, if x is in M and $\beta>0$ then there is a constant $\alpha=\alpha(x,\beta)>0$ such that if z is within α of x, then*

$$\mathrm{dist}(h^{-1}(x)\cap Y,\ h^{-1}(z)\cap Y)<\beta.$$

Proof. We argue by contradiction; if the conclusion of the lemma is false then there are sequences $x_n\to x$ and $y_n\to y$ with y_n in Y and $h(y_n)=x_n$ for each n, but with y not in $h^{-1}(x)\cap Y$. Since Y is closed and h is continuous, this is absurd. $\square$

PROPOSITION 6. (Non-uniform local product structure) *Suppose that f is topologically stable and that X is a chain component of f. Let x in X and $\beta>0$ be given. Then there is a constant $\alpha=\alpha(x,\beta)>0$ such that if z is in M and within α of x, then there are points v, w in M such that*

(1) *v is in $W^s_\beta(x;f)\cap W^u_\beta(z;f)$;*

(2) *w is in $W^s_\beta(z;f)\cap W^u_\beta(x;f)$.*

Moreover, if z is in X then so are v and w.

Proof. As in the proof of lemma 4, choose an Axiom A diffeomorphism g and a semiconjugacy h from g to f, $hg=fh$. By proposition 3 there is a chain component Y of g with $h(Y)=X$. Now let $\gamma>0$ be small enough that

(i) $d(y_1,y_2)<\gamma$ implies $d(h(y_1),h(y_2))<\beta$ for any y_1, y_2 in Y.

(ii) $\gamma<\gamma_0$, where γ_0 is the constant given by proposition 1.

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Use this value of γ in proposition 1 to obtain $\delta > 0$ as in the statement of that proposition. By lemma 5 we can find $\alpha = \alpha(x, \delta) > 0$ such that $d(x, z) < \alpha$ implies that

$$\operatorname{dist}(h^{-1}(x) \cap Y, h^{-1}(z) \cap Y) < \delta.$$

Thus if z is within α of x, then we can find points y_1 in $h^{-1}(x) \cap Y$ and y_2 in $h^{-1}(z) \cap Y$ with $d(y_1, y_2) < \delta$. By proposition 1 and the way the constants have been chosen the sets $[y_1, y_2]$ and $[y_2, y_1]$ are non-empty. Choose q_1 in $[y_1, y_2]$, q_2 in $[y_2, y_1]$ and let $v = h(q_1)$, $w = h(q_2)$. (1), (2) follow from the choices of the various constants involved. The final assertion of the proposition is a general fact about chain recurrence; if x, z belong to the same chain component, X, of f and p is in both $W^s(x; f)$ and $W^u(z; f)$ then p is also in X. □

COROLLARY 7. *Let f, X be as in proposition 6. Suppose p is in X and is periodic, $f^k(p) = p$ for some $k \geq 1$. Then*

$$\operatorname{clos}\left[\bigcup_{j=1}^{k} W^\sigma(f^j(p) \cap X)\right] = X, \qquad \sigma = s \text{ or } u.$$

Proof. We give the proof for $\sigma = s$; the other case is analogous. Choose $\alpha = \alpha(p, 1)$ as in proposition 6, and use lemma 2 to find z in X within α of p and with the alpha-limit set of z equal to X. Now apply the proposition to get a point y in $W^s(p) \cap W^u(z) \cap X$. Since y is in $W^u(z)$, y and z have the same alpha-limit set, so

$$\begin{aligned} X &= \text{alpha-limit set of } y \\ &\subset \operatorname{clos}\{f^{-n}(y) \mid n = 0, 1, 2, \ldots\} \\ &\subset \operatorname{clos}\{x \text{ in } X \mid x \text{ is in } f^{-n}(W^s(p)), n = 0, 1, \ldots.\} \\ &= \operatorname{clos}[\textstyle\bigcup W^s(f^j(p)) \cap X]. \end{aligned}$$

□

COROLLARY 8. *Let f, X be as in proposition 6 and suppose p, q are periodic points of f lying in X. Then $W^s(p) \cap W^u(q) \cap X$ is dense in X.*

Proof. Let x in X, $\beta > 0$ be given. Let $O(p)$, $O(q)$ denote the f-orbits of p and q, respectively. Use corollary 7 to pick y in $W^s(O(p)) \cap X$ with $d(x, y) < \beta/2$. Select $\alpha = \alpha(y, \beta/2)$ as in proposition 6 and choose z in $W^u(O(q)) \cap X$ with $d(y, z) < \alpha$. Applying the proposition gives a point v in X with $d(v, y) < \beta/2$ and

$$v \in W^s(y) \cap W^u(z) \cap X \subset W^s(O(p)) \cap W^u(O(q)) \cap X.$$

Finally,

$$d(x, v) \leq d(x, y) + d(y, v) < \beta.$$

□

A periodic point p of a diffeomorphism g is *hyperbolic* if none of the eigenvalues of $Dg^k(p)$ have modulus 1, where k is the period of p. When this is the case, $W^s(p)$ and $W^u(p)$ are immersed submanifolds of M of complementary dimension. The dimension of $W^s(p)$ is the number of eigenvalues (counting multiplicity) of modulus less than 1, and the dimension of $W^u(p)$ is the number of eigenvalues of modulus greater than 1. The diffeomorphism g is said to be *Kupka–Smale* if every periodic point of g is hyperbolic and if $W^s(p)$ and $W^u(q)$ are everywhere transverse whenever p and q are periodic points of g.

Recall that if p is a hyperbolic periodic point then the *index* of p is the dimension of its unstable manifold.

COROLLARY 9. *Let f, X, p, q be as in corollary 8. If f is Kupka–Smale then p and q have the same index.*

Proof. Since f is Kupka–Smale, the last corollary shows that $W^s(p)$ and $W^u(q)$ have a point of transversal intersection. Thus

$$\dim(W^s(p)) + \dim(W^u(q)) \geq \dim(M);$$

since $W^s(p)$ and $W^u(p)$ have complementary dimensions, we see that

$$\dim(M) - \dim(W^u(p)) + \dim(W^u(q)) \geq \dim(M),$$

so that $\dim(W^u(q)) \geq \dim(W^u(p))$. Reversing the roles of p and q gives the opposite inequality. □

Proof of theorem A. It is well known that any structurally stable diffeomorphism is Kupka–Smale; see [**12**] for a proof. Hence corollary 9 shows that there is a unique value of the index of the periodic points of a given chain component of f whenever f is both topologically and structurally stable. A topologically stable diffeomorphism has only finitely many chain components (this is easily deduced from proposition 3; see [**7**, theorem A] for more details). Now the theorem of Mañé quoted in the introduction can be applied to give the conclusion of theorem A. □

We finish with a result that shows that the non-uniform local product structure of proposition 6 can be taken to be uniform on large subsets of CR(f).

Let FM denote the collection of all non-empty closed subsets of M. For A, B in FM, let

$$d_H(A, B) = \inf\{\alpha > 0 \mid A \subset U_\alpha(B), \quad B \subset U_\alpha(A)\},$$

where $U_\alpha(X) = \{y \text{ in } M \mid \operatorname{dist}(y, X) < \alpha\}$. d_H is called the Hausdorff metric on FM; it is a standard fact that (FM, d_H) is a compact metric space.

The proof of lemma 5 shows that if h is a continuous surjection on M and Y is closed and non-empty, then the map from M into FM given by $x \to h^{-1}(x) \cap Y$ is upper semicontinuous. It is a well known fact that in this situation the set of continuity points of $h^{-1}(\_\_) \cup Y$ is residual in M; a proof may be found in [**15**].

PROPOSITION 10. *Suppose f is topologically stable and X is a chain component of f. Let $\beta > 0$ be given. Then there is an open and dense subset $G = G(\beta)$ of X (in the relative topology) such that if K is a compact subset of G then there is uniform local product structure on K. That is, there is a constant $\alpha = \alpha(K, \beta)$ such that if x_1, x_2 are in K and are within α of each other, then there are points v, w in K with v in $W^s_\beta(x_1) \cap W^u_\beta(x_2)$ and w in $W^s_\beta(x_2) \cap W^u_\beta(x_1)$.*

Proof. Choose g, h, Y, γ, and σ as in the proof of proposition 6. For x in X let $S(x)$ denote $h^{-1}(x) \cap Y$, and let Z be the set of continuity points of S as a map from X to FM. For each z in Z choose $\alpha(z) > 0$ such that $d_H(S(z), S(x)) < \delta/2$ whenever $d(x, z) < \alpha(z)$. Let

$$G(z) = U_{\alpha(z)}(z) \cap X \quad \text{and} \quad G = \bigcup_{z \in Z} G(z).$$

Clearly G is open and dense in X. Now suppose K is a compact subset of G. Let α be a Lebesgue number for K relative to the open cover $\{G(z)\}$. If any two points p, q of K are within α of each other then there is a point z_* in Z with both points contained in $G(z_*)$, so that

$$\begin{aligned}\operatorname{dist}(S(p), S(q)) &\le d_H(S(p), S(q))\\ &\le d_H(S(p), S(z_*)) + d_H(S(z_*), S(q)) < \delta.\end{aligned}$$

Now the rest of the proof is essentially the same as that of proposition 6. □

As a final remark, note that while one might expect a topologically stable diffeomorphism to have a uniform local product structure on all of its chain recurrent set, it is unlikely that this can be established by the type of argument that we have been using. The reason for this is that the mapping $x \to h^{-1}(x)$ generally has points of discontinuity. This is true even in the case when h is a semiconjugacy from some homeomorphism g to a topologically stable diffeomorphism f. As an example, suppose f has an attracting fixed point p. We can perturb f in a small neighbourhood of p to obtain a homeomorphism g which looks like f except that p has been enlarged to a closed ball B consisting entirely of fixed points of g. Any semiconjugacy close enough to the identity must map all of B to p. If g is constructed with a little care it is not difficult to construct a semiconjugacy h from g to f that is one-to-one on the complement of B. Here $x \to h^{-1}(x)$ will be continuous at every point except p.

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