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ABSTRACTS OF PAPERS ACCEPTED FOR THE SYMPOSIUM BUT NOT PRESENTED

MATHEMATICAL MODELS OF ICE SHELVES

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Abstract. The flat form and high ice velocity of floating glaciers are explained by the absence of shear stresses at the lower surface. In orthogonal co-ordinates with one axis normal to the upper and lower surfaces shear stresses in these glaciers are absent. Another important peculiarity of ice shelves is their essential non-isothermality. Among them two dynamically different types are distinguished, which are described by different models.

A. *External ice shelves* join the coast at one edge and at some distance from it can expand freely in all directions. They can be considered with sufficient accuracy as flat plates without any physical differences between the directions of the horizontal plane, except that strains lead to the movement relative to the fixed edge. Thus the problem of thermodynamics becomes one-dimensional. In the affine dimensionless system of co-ordinates, the equations of the dynamics are simplified and together with the rheological equation lead to the non-linear integro-differential equation involving the reduced temperature. For the quasi-steady case, the boundary problem for this equation is solved by means of the method of sewing together of asymptotic expansions. It is shown that the stability of the thermodynamic regime in external ice shelves takes place only when the stream lines pass through both glacier surfaces, because in this case the advection removes the dissipative heat. In the case of ice coming from the upper and lower surfaces in opposite directions, the regime is unsteady because of the internal accumulation of heat.

B. *Internal ice shelves* are limited by coasts from various sides and interact with them dynamically. In the boundary zone coasts provide a braking action and outside it they prevent sideways spreading. The relation between horizontal stresses is conditioned by the configuration of coasts, therefore the angle between the coasts, or between the directions of ice flow from them, is included essentially into the equations of thermodynamics. Another complication is connected with the considerable change of the temperature and of the accumulation–ablation rate at the upper and lower surfaces of the glacier along flow lines. Integro-differential equation for the temperature in this case is more complicated, but its solution is analogous to the case above. In the coastal zone the thermodynamics are described by other equations in connection with the predominance of the shear stress in the plane parallel to the coast.

ELECTRICAL RESISTIVITY PROFILES AND TEMPERATURES IN THE ROSS ICE SHELF

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Abstract. During the 1973–74 Antarctic field season, two electrical resistivity profiles were completed along directions perpendicular to each other at a site in the south-eastern

part of the Ross Ice Shelf. These profiles differ from each other only at short electrode spacings (less than 10 m) indicating no measurable horizontal anisotropy below the uppermost firn zone. The shape of the apparent resistivity curves is similar to that found by Hochstein on the Ross Ice Shelf near Roosevelt Island, but is displaced toward lower resistivities despite the colder 10 m temperature (−29°C instead of −26°C) at the more southerly site. Some factor other than temperature must therefore be effective in determining the overall magnitude of the resistivities in the shelf, although the variation with depth can still be expected to be primarily a temperature phenomenon.

A computer program has been written to calculate apparent resistivities based on Crary's analysis of temperatures in an ice shelf. Results are not yet available; when completed they should indicate the sensitivity of the resistivity measurements to differences in the temperature–depth profile, and hence their usefulness in estimating bottom melt/freeze rates.

MATHEMATICAL MODEL OF A THREE-DIMENSIONAL NON-ISOTHERMAL GLACIER

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Abstract. In the case of a non-isothermal glacier it is necessary to integrate the equations of dynamics together with the equation of heat conduction, heat transfer, and heat generation because of the interdependence (1) of strain-rate of ice on its temperature, and (2) of ice temperature on the rate of heat transfer by moving ice and on the intensity of heat generation in its strain. In view of the complexity of the whole system of equations, simplified mathematical models have been constructed for dynamically different glaciers. The present model concerns land glaciers with thicknesses much less than their horizontal dimensions and radii of curvature of large bottom irregularities, so that the method of a thin boundary layer may be used. The principal assumption is the validity of averaging over a distance of the order of magnitude of ice thickness.

Two component shear stresses parallel to the bottom in glaciers of this type considerably exceed the normal stresses and the third shear stress, so the dynamics are described by a statically determined system of equations. For the general case, expressions for the stresses have been obtained in dimensionless affine orthogonal curvilinear coordinates, parallel and normal to the glacier bottom, and taking into account the geometry of the lower and upper surfaces. The statically undetermined problem for ice divides is solved using the equations of continuity and rheology, so the result for stresses depends considerably on temperature distribution. In the case of a flat bottom the dynamics of an ice divide is determined by the curvature of the upper surface.

The calculation of the interrelating velocity and temperature distributions is made by means of the iteration of solutions (1) for the components of velocity from the stress expressions using the rheological equations (a power law or the more precise hyberbolic one) with the assigned temperature distribution, and (2) for the temperature with the assigned velocity distribution. The temperature distribution in the coordinate system used is determined by a parabolic equation with a small parameter at the principal derivative. Its solution is reduced to the solution of a system of recurrent non-uniform differential equations of the first order by means of a series expansion of the small parameter: the right part for the largest term