By way of illustration, let 50 be the age at entry, and 60 the age at which the annuity (£1) is to be entered upon, and suppose it has been found that the premiums on one-twentieth of the policies in force at the beginning of any year are withdrawn during that year; then, using the Carlisle table of mortality, and 4 per cent. interest, we shall find the value of the expression (6) to be $\cdot 42556 = \text{Q}$; hence, from (7) we get $\text{P}_x = 9\cdot 4157$.

In the same case, if the premium were simply returnable at the end of the year of death, and no option of withdrawal were allowed, the single premium, by the ordinary formula, would be

$$\text{P}_x = \frac{\text{N}_{x+n}}{\text{D}_x - \text{M}_x + \text{M}_{x+n}} = 6\cdot 2631.$$

These numerical results show that a very moderate supposition as to the probability of withdrawal increases the premium more than 50 per cent.

It will be observed, in all that precedes, that $\log v$ denotes the *hyperbolic* logarithm of v.

I am, Sir,
Your obedient servant,
SAMUEL YOUNGER.

316, *Regent Street, London,*
30*th November*, 1865.

THE LATE MR. FINLAISON'S TABLES.

To the Editor.

SIR,—Reference having been made at the last meeting of the Institute to the discrepancy existing between the probabilities of living as computed by the late Mr. Finlaison, and those that would result from the methods of graduation he professes to adopt, I trouble you with a few details of these differences, with a view of promoting some inquiry into a matter which, having regard to the importance of the tables, and as bearing upon the points recently under discussion, may prove of interest to members.

The well-known tables of Mr. Finlaison are embodied in his Report, printed by order of the House of Commons, 31st March, 1829; and are founded on twenty-one "Observations" exhibiting the rate of mortality experienced under the Government schemes of tontines and other annuities, with an annuity table at 4 per cent. deduced from each. The logarithms of the probability of living one year at each age are tabulated, first as deduced from the data, and secondly as adjusted by Mr. Finlaison.

His formulæ for adjustment, written in Milne's notation, are embodied in the Report, and are as follows:—

$$\text{I.}\quad {}_1\overset{=}{a} = \sqrt[3]{\left\{\begin{array}{l} \sqrt[5]{{}_1a_3 \times {}_1a_2 \times {}_1a_1 \times {}_1a \times {}^1_1a\}} \times \\ \sqrt[5]{{}_1a_2 \times {}_1a_1 \times {}_1a \times {}^1_1a \times {}^2_1a\}} \times \\ \sqrt[5]{{}_1a_1 \times {}_1a \times {}^1_1a \times {}^2_1a \times {}^3_1a\}} \end{array}\right\}}.$$

$$\text{II.}\quad {}_1\overset{=}{a} = \frac{(5\,{}_1a + 4\,{}_1a_1 + 4\,{}^1_1a + 3\,{}_1a_1 + 3\,{}^2_1a + 2\,{}_1a_3 + 2\,{}^3_1a + {}_1a_4 + {}^4_1a)}{25}.$$

Mr. Finlaison informs us that nineteen of the observations were "all and each" of them adjusted by the first method above; and that two observations only, the earliest for each sex, which were completed in January, 1823, were adjusted by the second method, which he designates as "perhaps quite as good, but more laborious" than the first; and that by these last mentioned "the whole of the life annuities for the service of Government have been calculated." The two observations specified appear, by the repetition of the above notice in their titles, to be the 13th and 20th, being on female and male lives respectively. I shall take my examples from the latter, as in addition to its other uses it is that by which the succession duties are levied under Act 16th and 17th Vict., cap 51.

The logarithms of the unadjusted probabilities, as given by Mr. Finlaison, require correction. In two instances they can only result by the alteration of a figure in the number living; in these cases I have retained the logarithm given, assuming that the number entering on age 12 should be 4,904, and that at age 73 should be 1,323. At age 16, however, the alteration of one figure is demanded in the logarithm, which I have therefore read ·9977864, instead of ·9975864.

With these explanations, I now annex the probability of living one year at the ages indicated—as deduced from the exact mortality, as corrected by Mr. Finlaison, and as resulting from both his methods of adjustment above quoted; together with the probability that would be shown by treating the logarithms as numbers in the second formula, the alternative having suggested itself of that being the cause of the discrepancy.

The difference of each from Mr. Finlaison's is sufficiently great to require explanation.

Age.	Probability, from Data, that Life survive one Year.	Probability as adjusted by Mr. Finlaison.	Probability resulting from Mr. Finlaison's second Method of Adjustment.	Probability resulting from first Method.	Probability resulting from treating the Logs. by second Method.	Age.
15	·9951015	·9940560	·9942626	·9945517	·9942615	15
25	·9865307	·9860189	·9862160	·9859922	·9862152	25
35	·9895447	·9875697	·9878904	·9880562	·9878895	35
45	·9876687	·9863270	·9868670	·9869892	·9868667	45
55	·9768117	·9749987	·9760568	·9760392	·9760505	55
65	·9632754	·9592738	·9607349	·9616342	·9607157	65

You will not fail to notice that, with one trifling exception, the results *correctly* deduced by Mr. Finlaison's formulæ exhibit a greater probability of living than those given by himself; and it is a subject for legitimate speculation whether some unexplained reason, not appearing in his Reports, may not account for the great difference between his annuity values of the two sexes.

I may add that I have proved the correct application of the first method to the results as detailed in the 8th observation.

A few more discrepancies in the (20th) table under consideration are worthy of notice. The fraction measuring the probability of death is, in several cases, inconsistent with the number resulting from the published logarithm of the corrected probability of living. Thus, at age 15, for ·0059440, read ·0059493; at age 41, for ·0134731, read ·0133368; at age 71, for ·0656881, read ·0656860.

I am, Sir,

Your most obedient servant,

H. AMBROSE SMITH

7, Torrington Square, W.C.,
22nd December, 1865.