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Helicity of the solar magnetic field

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Abstract. Helicity measures complexity in the field. Magnetic helicity is given by a volume integral over the scalar product of magnetic field **B** and its vector potential **A**. A direct computation of magnetic helicity in the solar atmosphere is not possible due to unavailability of the observations at different heights and also due to non-uniqueness of **A**. The force-free parameter α has been used as a proxy of magnetic helicity for a long time. We have clarified the physical meaning of α and its relationship with the magnetic helicity. We have studied the effect of polarimetric noise on estimation of various magnetic parameters. Fine structures of sunspots in terms of vertical current (J_z) and α have been examined. We have introduced the concept of signed shear angle (SSA) for sunspots and established its importance for non force-free fields. We find that there is no net current in sunspots even in presence of a significant twist, showing consistency with their fibril-bundle nature. The finding of existence of a lower limit of SASSA for a given class of X-ray flare will be very useful for space weather forecasting. A good correlation is found between the sign of helicity in the sunspots and the chirality of the associated chromospheric and coronal features. We find that a large number of sunspots observed in the declining phase of solar cycle 23 do not follow the hemispheric helicity rule whereas most of the sunspots observed in the beginning of new solar cycle 24 do follow. This indicates a long term behaviour of the hemispheric helicity patterns in the Sun. The above sums up my PhD thesis.

Keywords. Sun: atmosphere, Sun: magnetic fields, Sun: sunspots

1. Introduction

Magnetic helicity is a physical quantity that measures the degree of linkage and twistedness in the magnetic field lines (Moffatt 1978). It is given as

$$H_m = \int \mathbf{A} \cdot \mathbf{B} \, dV \tag{1.1}$$

The term magnetic helicity was introduced by Elsasser (1956) and many of its important characteristics were studied by Woltjer (1958); Taylor (1974); Berger & Field (1984) etc.

The handedness associated with the field is defined by ‘chirality’. Helicity is closely related to chirality. If the twist on the surface is clockwise, the chirality is negative and the field bears dextral chirality. The sunspot twist direction is decided by the curvature of sunspot whirls (Martin 1998; Tiwari 2009). If the twist is counterclockwise (when we go from sunspot center towards outside), the chirality is sinistral and sign of helicity is positive. Reverse is true for the dextral chirality. These definitions of chirality have been used to study the hemispheric patterns of the active regions (Tiwari *et al.* 2008, 2010b).

One of the main motivations of the thesis (Tiwari 2009) was to use the helicity or related parameters to help in predicting the severity of the solar flares. If done so, this would contribute in improving the space-weather forecasting. We have found the parameter signed shear angle (SSA) to be very useful in this context (Tiwari *et al.* 2010a).

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Some of the important results of my Ph.D. thesis are summarized very briefly in the following sections.

2. Estimating magnetic parameters

Physical meaning of α. We arrive at the following depiction of α (for details, please see Appendix A of Tiwari *et al.* (2009a)):

$$\alpha = 2\,\frac{d\phi}{dz} \tag{2.1}$$

From Equation 2.1, it is clear that the α gives twice the degree of twist per unit axial length. If we take one complete rotation of flux tube i.e., $\phi = 2\pi$, and loop length $\lambda \approx 10^9$ meters, then

$$\alpha = \frac{2 \times 2\pi}{\lambda} \tag{2.2}$$

comes out to be of the order of 10^{-8} per meter.

Correlation between sign of H_m *and that of* α. Vector potential in terms of scalar potential ϕ can be expressed as (for details, please see Appendix B of Tiwari *et al.* (2009a))

$$\mathbf{A} = \mathbf{B}\alpha^{-\mathbf{1}} + \nabla\phi \tag{2.3}$$

which is valid only for constant α. Using this relation in Equation 1, we get magnetic helicity as

$$\begin{aligned} H_m &= \int (\mathbf{B}\alpha^{-1} + \nabla\phi)\cdot\mathbf{B}\;dV \\ &= \int B^2\alpha^{-1}dV + \int (\mathbf{B}\cdot\nabla)\phi\;dV \sim (\int (\phi\;\mathbf{B})\cdot\mathbf{n}\;dS) \end{aligned} \tag{2.4}$$

showing that the force free parameter α has the same sign as that of the magnetic helicity iff $\mathbf{n}\cdot\mathbf{B} = \mathbf{0}$ i.e., no field lines cross the boundary, which is not the case with the Sun.

A direct method for calculating global α. We prefer to use the second moment of minimization (Tiwari *et al.* 2009a) leading to the following expression:

$$\alpha_g = \frac{\sum(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y})B_z}{\sum B_z^2}. \tag{2.5}$$

This formula gives a single global value of α in a sunspot and is the similar to $\alpha_{av}^{(2)}$ of Hagino & Sakurai (2004). We do not use direct mean (0^{th} order moment) as it leads to singularities at neutral lines where $B_z \sim 0$. First order moment will also lead to singularities when flux is balanced.

Estimating the effect of polarimetric noise in the measurement of field parameters. Using the analytical bipole method (Low 1982), non-potential force-free field components B_x, B_y & B_z in a plane have been generated. We calculate the synthetic Stokes profiles for each B, γ and ξ in a grid of 100 x 100 pixels, using the He-Line Information Extractor "HELIX" code (Lagg *et al.* 2004). We add random noise of 0.5 % of the continuum intensity I_c (Ichimoto *et al.* 2008) to the polarimetric profiles as observed in SOT/SP aboard Hinode. In addition, we also study the effect of adding a noise of 2.0% level to Stokes profiles as a worst case scenario. We add 100 realizations of the noise of the orders mentioned above to each pixel and invert the corresponding 100 noisy profiles using the "HELIX" code. The effect of polarimetric noise in the derivation of vector fields and other parameters such as α_g and magnetic energy is found to be very small. We have

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done similar investigations as a second step using real data (Gosain *et al.* 2010). As a third step we plan to use MHD simulated data to check the inversion codes including the effect of optical depth corrugation.

3. Global twist of sunspot magnetic fields

Introduction of signed shear angle (SSA). To emphasize the sign of shear angle we introduce the signed shear angle (SSA) for the sunspots as follows: choose an initial reference azimuth for a current-free field (obtained from the observed line of sight field). Then move to the observed field azimuth from the reference azimuth through an acute angle. If this rotation is counter-clockwise, then assign a positive sign for the SSA. A negative sign is given for clockwise rotation. This sign convention will be consistent with the sense of azimuthal field produced by a vertical current. This sign convention is also consistent with the sense of chirality (Tiwari *et al.* 2009b). The SSA is computed from the following formula (Tiwari *et al.* 2010a):

$$SSA = \tan^{-1}\left(\frac{B_{yo}B_{xp} - B_{yp}B_{xo}}{B_{xo}B_{xp} + B_{yo}B_{yp}}\right) \tag{3.1}$$

where B_{xo}, B_{yo} and B_{xp}, B_{yp} are observed and potential transverse components of sunspot magnetic fields respectively. A spatial average of the SSA (SASSA) gives the global twist of sunspot magnetic fields at observed height irrespective of the force-free nature of the field and shape of sunspots (Venkatakrishnan & Tiwari 2009).

Fine structures in terms of J_z and α. Local J_z and α patches of opposite signs are present in the umbra of each sunspot. The amplitude of the spatial variation of local α in the umbra is typically of the order of the global α of the sunspot. We find that the local α is distributed as alternately positive and negative filaments in the penumbra. The amplitude of azimuthal variation of the local α in the penumbra is approximately an order of magnitude larger than that in the umbra. The contributions of the local positive and negative currents and α in the penumbra cancel each other giving almost no contribution for their global values for whole sunspot. The data sets used in the analysis are taken from ASP/DLSP and Hinode (SOT/SP). See for details: Tiwari *et al.* (2009b). Most of the data sets we studied are observed during the declining minimum phase of solar cycle 23. All except 5, out of 43 sunspots observed, follow the reverse twist hemispheric rule, while 5 follow the conventional helicity rule. Also, α_g has same sign as the SASSA and therefore the same sign of the photospheric chirality of the sunspots, but the magnitudes of SASSA and α_g are not well correlated. This lack of correlation could be due to a variety of reasons: (a) departure from the force-free nature (b) even for the force-free fields, α is the gradient of twist variation whereas SASSA is purely an angle. The missing link is the scale length of variation of twist.

4. Net current in sunspots

Expression for net current. We consider a long straight flux bundle surrounded by a region of "field free" plasma following Parker (1996). Parker (1996) assumed azimuthal symmetry as well as zero radial component B_r, of the magnetic field. For realistic sunspot fields, we have already seen the ubiquitous fine structure of the radial magnetic field. Hence, we need to relax both these assumptions.

The vertical component of the electric current density consists of two terms, viz. $-\frac{1}{\mu_0 r}\frac{\partial B_r}{\partial \psi}$ and $\frac{1}{\mu_0 r}\frac{\partial (rB_\psi)}{\partial r}$. We will call the first term as the "pleat current density",

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j_p and the second term as the "twist current density", j_t. The total current I_z within a distance ϖ from the center is then given by

$$I_z(\varpi) = \int_0^{2\pi} d\psi \int_0^{\varpi} rdr(j_p + j_t) \tag{4.1}$$

The ψ integral over j_p vanishes, while the second term yields

$$I_z(\varpi) = \frac{\varpi}{\mu_0} \int_0^{2\pi} d\psi B_\psi(\varpi, \psi) \tag{4.2}$$

which gives the net currents within a circular region of radius ϖ. The transverse vector can be expressed in cylindrical geometry as

$$B_r = \frac{1}{r}(xB_x + yB_y) \tag{4.3}$$

$$B_\psi = \frac{1}{r}(-yB_x + xB_y) \tag{4.4}$$

The azimuthal field B_ψ is then used in Equation 4.2 for obtaining the value for the total vertical current within a radius ϖ.

No net current: an evidence for fibril bundle nature of sunspot magnetic field? As expected from the trend in Figure 3 of Venkatakrishnan & Tiwari (2009), the net current shows evidence for a rapid decline after reaching a maximum. Similar trends were seen in other sunspots. This can be interpreted as evidence for the neutralization of the net current. Table 1 of Venkatakrishnan & Tiwari (2009) shows the summary of results for all the sunspots analyzed. Along with the power law index δ of B_ψ decrease, we have also shown the average deviation of the azimuth from the radial direction ("twist angle $= tan^{-1}(B_\psi/B_r)$"), as well as the SASSA. The average deviation of the azimuth is well correlated with the SASSA for nearly circular sunspots, but is not correlated with SASSA for more irregularly shaped sunspots. Thus, SASSA is a more general measure of the global twist of sunspots, irrespective of their shape.

As is well known for astrophysical plasmas, that the plasma distorts the magnetic field and the curl of this distorted field produces a current by Ampere's law (Parker 1979). Parker's (1996) expectation of net zero current in a sunspot was basically motivated by the concept of a fibril structure for the sunspot field. However, he also did not rule out the possibility of vanishing net current for a monolithic field where the azimuthal component of the vector field in a cylindrical geometry declines faster than $1/\varpi$. While it is difficult to detect fibrils using the Zeeman effect, notwithstanding the superior resolution of SOT on *Hinode*, the stability and accuracy of the measurements have allowed us to detect the faster than $1/\varpi$ decline of the azimuthal component of the magnetic field, which in turn can be construed as evidence for the confinement of the sunspot field by the external plasma. The resulting pattern of curl **B** appears as a sharp decline in the net current at the sunspot boundary. Although the existence of a global twist in the absence of a net current is possible for a monolithic sunspot field (Baty 2000), a fibril model of the sunspot field can accommodate a global twist even without a net current (Parker 1996). A sunspot, made up of a bundle of magnetically isolated current free fibrils, can be given an overall torsion without inducing a global current. For details and more discussions please see Venkatakrishnan & Tiwari (2009); Tiwari (2009, 2010).

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5. Relationship between the SASSA of active regions and associated GOES X-ray flux

We find an upper limit of peak X-ray flux for a given value of SASSA can be given for different classes of X-ray flares. Figures 5(a) and 5(b) of Tiwari *et al.* (2010a) represent scatter plots between the peak GOES X-ray flux and interpolated SASSA and mean weighted shear angle (MWSA: Wang (1992)) values for that time, respectively. The cubic spline interpolation of the sample of the SASSA and the MWSA values has been done to get the SASSA and MWSA exactly at the time of peak flux of the X-ray flare. For details kindly see Tiwari *et al.* (2010a). We find that the SASSA, apart from its helicity sign related studies, can also be used to predict the severity of the solar flares. However to establish these lower limits of SASSA for different classes of X-ray flares, we need more cases to study. The SASSA already gives a good indication of its utility from the present four case studies using 115 vector magnetograms from Hinode (SOT/SP). Once the vector magnetograms are routinely available with higher cadence, the lower limit of SASSA for each class of X-ray flare can be established by calculating the SASSA in a series of vector magnetograms. This will provide the inputs to space weather models. Also, SASSA has shown a good correlation with the free magnetic energy computed by Jing *et al.* (2010).

The other non-potentiality parameter MWSA studied in Tiwari *et al.* (2010a) does show a similar trend as that of the SASSA. The magnitudes of MWSA, however, do not show consistent threshold values as related with the peak GOES X-ray flux of different classes of solar flares. One possible reason for this behavior may be explained as follows: The MWSA weights the strong transverse fields e.g., penumbral fields. From the recent studies (Su *et al.* 2009; Tiwari *et al.* 2009b; Tiwari 2009; Venkatakrishnan & Tiwari 2009, 2010) it is clear that the penumbral field contains complicated structures with opposite signs of vertical current and vertical component of the magnetic tension forces. Although the amplitudes of the magnetic parameters are found high in the penumbra, they do not contribute to their global values because they contain opposite signs, which cancel out in the averaging process (Tiwari 2009; Tiwari *et al.* 2009b). On the other hand, the MWSA adds those high values of shear and produces a pedestal that might mask any relation between the more relevant global non-potentiality and the peak X-ray flux. Whereas the SASSA perhaps gives more relevant value of the shear after cancelation of the penumbral contribution.

6. Solar cycle dependence

Helicity hemispheric rule. We compare the behaviour of magnetic helicity sign of AR's observed in the beginning of 24^{th} solar cycle with some AR's observed in the declining phase of 23^{rd} solar cycle. We find that the majority of active regions in the beginning of solar cycle 24 do follow the hemispheric helicity rule whereas those observed in the declining phase of solar cycle 23 do not (Tiwari, 2009; Tiwari, 2010).

Sign of magnetic helicity at different heights in the solar atmosphere. A good correlation has been found among the sign of helicity in the associated features observed at photospheric, chromospheric and coronal heights without solar cycle dependence (Tiwari *et al.* 2008; Tiwari, 2009; Tiwari *et al.* 2010b; Tiwari, 2010).

7. Conclusions

The magnetic field parameters can be derived very accurately using the recent data available (e.g. from *Hinode* (SOT/SP)) and advanced inversion codes. The SASSA is the best measure of the global magnetic twist of sunspot magnetic fields at observed

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height, irrespective of the force-free nature and the shape of sunspots. The sunspots with significant twist and no net currents show consistency with the fibril bundle nature of the sunspots. The study of evolution of SASSA of sunspots showed threshold values for different classes of X-ray flares. This is an important discovery which was being sought after for many decades. The magnetic helicity sign of active regions studied, has good correlation with the sign of chirality of associated features observed at chromospheric and coronal heights. The majority of sunspots observed in the declining phase of solar cycle 23 follow a reverse hemispheric helicity rule, whereas most of the AR's emerged in the beginning of solar cycle 24 follow the conventional helicity rule. This result indicates that revisiting the hemispheric helicity rule using data sets of several years is required.

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