Black Holes from Stars to Galaxies – Across the Range of Masses
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Rotation curves, dark matter and general relativity

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Abstract. We study the possibility of pure general relativistic models without exotic matter to describe the observed flattening of the rotation curves for stars moving in circular orbits in a galaxy disk. In particular we consider the dragging of inertial frames (rotation of the source) and the presence of a Taub–NUT (Newman–Unti–Tamburino) "charge", the gravitational equivalent to a magnetic monopole in electrodynamics.

Keywords. Thin disks – rotation curves – dragging of inertial frames – Taub–NUT parameter

1. Introduction

Thin disk models in General Relativity can be constructed using the "Displace, Cut and Reflect method" (image method). This method amount to take a solution to the Einstein vacuum field equations in cylindrical coordinates and perform the transformation $z \to |z| + d$, where d is a constant. From the Einstein field equations we find $G_{\mu\nu} = -T_{\mu\nu}\delta(z)$, where $\delta(z)$ is the usual Dirac distribution. See for instance González & Letelier (2000).

The rotation curves associated to a particular disk are found using the geodesic equation computed from the particular metric that represents the gravitational field of the disk: $ds^2 = g_{ij}dx^i dx^j + f(dr^2 + dz^2)$, were $(x^i) = (t, \varphi)$. For a circular geodesic we find that the tangential velocity is $v^2 = -det(g_{ij})\frac{d\varphi}{dt}/(g_{tt} + g_{t\varphi}\frac{d\varphi}{dt})^2$.

2. Rotation curves dragging of inertial frames and Taub–NUT mass.

The Kerr metric that represents a rotating black hole has two parameters the mass m and the rotation parameter a, once we apply the image method me add the new parameter d, so for the Kerr disk we have (m, a, d) as parameters. Thin Kerr disks are studied in some detail in González & Letelier (2000), see also Vogt & Letelier (2005).

For these disk we have two classes of circular orbits the ones co-rotating with the disk and the ones moving in counter rotation. For Newtonian disks both rotation curves are the same, but in the general relativistic context we have different behaviour due to the fact that the rotating bodies drag the spacetime around them. In Fig. 1 we show the profiles of the rotations curves for a Kerr thin disk with $m = 1$, $d = 3$ and $a = 0$, 0.5, 9.98, (co-rotation) and $a = 0$, -0.5, -0.98 (counter-rotation).

We find the the counter-rotation does help a little the flattening of the rotation curves co-rotation does not. So dragging of inertial frames has a very little effect on the flattening of the rotation curves. Note that the limit value of the parameter a is one.

The Taub–NUT metric has also two parameters the mass, m and the Taub–NUT parameter b. Loosely speaking we can associate mass to the electric part of the Riemann–Christoffel curvature tensor and the Taub–NUT parameter to its the magnetic part. So one can say that this parameter represents a kind of "magnetic mass". In Fig. 2 we

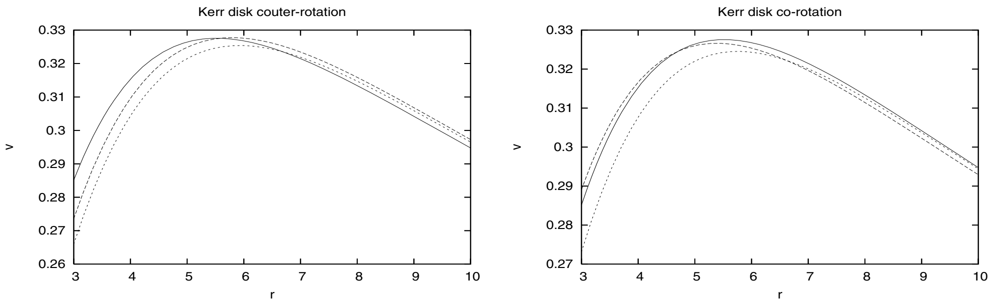

Figure 1. Rotation curves for a Kerr disk with parameters $m = 1$, $d = 3$ and $a = 0,\ 0.5,\ 9.98$, (co-rotation) and $a = 0,\ -0.5,\ -0.98$ (counter-rotation).

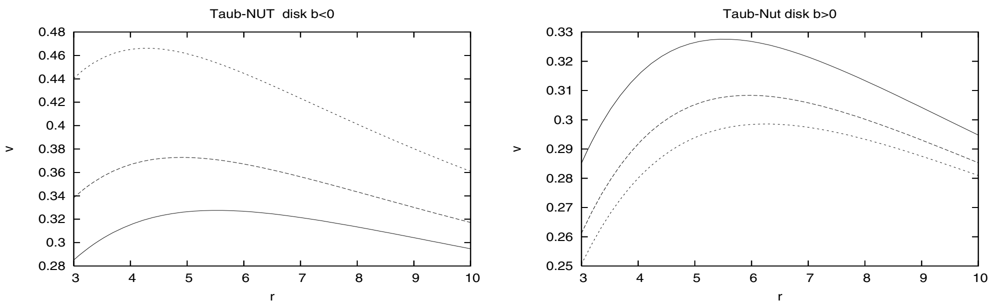

Figure 2. Rotation curves for a Taub-NUT disk with parameters $m = 1, d = 3$ and $b = 0, 0.5, 1$ and $b = 0, -0.5, -1$.

show the the rotation curves for a Taub-NUT disk with parameters $m = 1, d = 3$ and $b = 0, 0.5, 1$ and $b = 0,\ -0.5,\ -1$. We see that the Tab-NUT charge $b > 0$ helps a little the flattening of the rotation curves, but when $b < 0$ we have the opposite effect; see also González & Letelier (2000).

The disk constructed from the Kerr metric can have a perfect fluid as a source. The Taub–NUT disk has heat flow and tensions, it has a rather not astrophysical source.

One can fit actual galaxy rotation curves with disks constructed from the van Stockum solution Cooperstock & Tieu (2005), but again the source has hoop tensions and heat flow Vogt & Letelier (2005). Also it is possible to construct disks with more general solutions like Kerr–NUT or Tomimatsu–Sato solutions. We do not foresee a different behaviour of the rotation curves as the one presented above.

In summary, pure relativistic effects like dragging of inertial frames and Taub-NUT charges do not have a big effect on the flattening of rotation curves. To flatten the rotation curves within GR we need exotic matter, not a very appealing solution.

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