19. PHOTOMETRIC EVIDENCE OF INSTABILITY IN ECLIPSING SYSTEMS

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Many eclipsing variables exhibit characteristics that indicate lack of stability. Physically, these systems range from certain short-period dwarfs, which show irregular brightness fluctuations unexplainable by any normal eclipse hypothesis, to systems such as AO Cassiopeiae: class O super-giants whose periods, velocity curves, and light curves have shown remarkable variations. Included are systems having one Wolf-Rayet component and probably one old nova. Many eclipsing binaries show erratic changes of period that are difficult to explain on any concept of a stable system. Indeed, when we consider the physical conditions which must prevail when two stars are located with their surfaces only a few hundred thousand miles from each other, perhaps we should be surprised that the irregularities are not larger.

The general consideration of signs of non-stability shown by eclipsing systems is thus an extensive field. A great deal of evidence is now available, but some of it is unpublished, and the rest is rather widely scattered in the literature. The effort here will be to present a summary of photometric evidence that indicates instability is present in certain types of eclipsing systems.

Perhaps the first sound observational evidence that indicated that eclipsing systems could not always be represented by stable models was the visual observations of R Canis Majoris made in the years 1896–9 by Pickering [1] and by Wendell [2]. On certain nights in 1898, Wendell's observations showed clearly a 'hump', or region of increased brightness, at the end of primary minimum. This has been discussed in some detail by Dugan [3], who concluded that the hump undoubtedly had real significance, but could scarcely be a permanent feature of the curve. Indeed, observations Wendell made only a week later fell about 0·13 magnitude below the hump and agreed well with observations at maximum light elsewhere on the curve. Observations on one night in 1899 show a strikingly similar hump of slightly smaller amplitude. Pickering's mean curve shows a similar phenomenon. The scatter in Pickering's individual observations is larger than that in Wendell's, but he himself seemed fully convinced of the reality of the hump and speculated as to its cause. However, it is not found every cycle at this particular phase, and an analysis of Pickering's observations shows that it arises chiefly from observations made on two particular nights. Apparently by chance, these were nights on which Wendell also was observing the system and these are two of the nights in which the hump was clearly present in Wendell's observations. The degree of confirmation is thus rather strong.

No such phenomenon is found in Dugan's observations (1916–23) nor in any of the extensive later work. Dugan pointed out that the dispersion of Wendell's observations is much greater in the region of the hump than in the rest of the curve. This seems to reflect differences from night to night rather than increased scatter on a given night.

The observations by Pickering and by Wendell were made with polarizing photometers. In the hands of an experienced observer, this instrument is remarkably free from systematic error. The observer estimates when two nearby star images are equally bright, and is not aware of the actual values of the measures until he reduces his observations. Provision is made for interchanging the apparent positions of comparison and variable in the eyepiece and for reversing the optical parts of the photometer. Thus are eliminated the psychophysiological effects that caused strange phenomena to be reported by observers making visual estimates of brightness. We must conclude with Dugan that the hump was a real but non-permanent feature of the light curve.

Since several series of observations since 1916 have shown no trace of the hump, it seems pertinent to ask if evidence exists which indicated other unusual occurrences in the years preceding 1916. Two different kinds of evidence exist.

The first is the spectrographic work of Jordan [4] based on radial velocity measures made in the years 1908–12. Two features stand out in these observations. One is the internal inconsistency of the spectrographic observations in the individual years; the other is the large disagreement between the photometric and spectrographic orbit solutions. It seems reasonable to assume that these features are not disconnected and that whatever physical cause is responsible for the variation in the radial velocity measures is also responsible for this disagreement with the photometric elements.

In Hardie's [5] work on U Cephei a similar problem is discussed, and an answer is suggested that is now generally accepted: namely, that errors in estimating the central positions of the lines are caused by strong line asymmetry that varies with phase. The cause of the asymmetry is assumed to be absorption in circumstellar streams or shells of gas. But there is a significant difference between the radial velocity measures of R Canis Majoris and those of U Cephei. The U Cephei measures have always indicated a large—although not constant—eccentricity, in contradiction to the photometric evidence. This is not the case with R Canis Majoris, for later radial velocity observations by Sitterly [6] (1929–31) and by Struve and Smith [7] (1948–9) do not require the large eccentricity indicated by Jordan's observations. Indeed, both later series indicate that the orbit may well be circular.

Table 1. *Photometric Elements of R Canis Majoris*

	Wendell (vis 1898)	Dugan (vis 1928)	Wood (pe 1939)
$1-\lambda_1$	0·081	0·028	0·035
$1-\lambda_2$	0·426	0·373	0·405
L_b	0·834	0·943	0·928
L_f	0·166	0·057	0·072

A value of $\alpha_0 = 0{\cdot}489$ was used in computing the L's.

The second unusual occurrence at about the epoch of Jordan's observations was a large, and apparently relatively sudden, period change. Although the period of R Canis Majoris had remained constant or nearly so over the preceding thirty years, a change of nearly a second occurred very close to the time when Jordan's spectrographic observations were showing fluctuations in velocity.

As a working explanation, we might assume the presence of the hump to be evidence of a developing unstable condition—a condition that terminated in ejection of material which distorted Jordan's spectrographic measures and which caused the period change by means of a physical mechanism suggested previously [8]. A further shortening of the period and the continued existence of discrepancies between the two later sets of spectrographic elements suggest that the final story may not yet have been told. Neither the photometric nor the spectrographic observations of the system since 1916 show the marked irregularities existing before that date.

If a catastrophic phenomenon of this kind did occur shortly before Dugan's observations so that the ejected material was still in the general neighbourhood of the stars, we might expect to notice some effect upon his light curve. A comparison of Dugan's observations with those of Wendell and with later photo-electric observations [9] shows serious discrepancies in the observed depths of minima. This is illustrated by the elements listed in Table 1. Even though the photo-electric observations were not available at the time of his study, Dugan was well aware of the difference between the

https://doi.org/10.1017/S0074180900018738 Published online by Cambridge University Press

two sets of visual observations, and recognized it as a real discrepancy which could not be explained at that time. The later photo-electric observations (made at an effective wave-length of about λ 4500) indicated that the depths of minima found by Wendell were in better agreement with the visual depths inferred from the photo-electric work, and it was pointed out that the conventional comparison of the 1920 visual observations with the 1939 photo-electric observations led to unacceptable conclusions concerning the relative surface brightnesses. Yet the most critical examination of these three sets of observations gives no suggestion that observational error can be responsible for the discrepancy. The accidental scatter is much too small for such an explanation, and systematic errors (such as the Purkinje effect) would affect the depth of primary minimum much more than that of secondary.

Table 2. *Corrected Photometric Elements of R Canis Majoris*

	Wendell (adjusted and rectified)	Dugan (adjusted)	Dugan (corrected for shell)	Wood
$1-\lambda_1$	0·053	0·038	0·046	0·035
$1-\lambda_2$	0·412	0·363	0·435	0·405
L_b	0·892	—	0·906	0·928
L_f	0·108	—	0·094	0·072

If we assume, however, that the ejected material was incandescent (from analogy with novae, Wolf-Rayet stars, P Cygni stars and similar objects), quite a different interpretation is possible. We are now in a case similar to that which occurs when a third body contributes to the light of the system, and the observed light curve representing the combined light of the three bodies must be corrected to obtain the true depths of minima that would be found if we could observe the two bodies alone. The chief difference in this case is that, instead of observing the light of the ejected material, we must infer it by asking what its brightness must be in order to bring Dugan's observations into agreement with the other sets.

For a proper comparison we must take one other step: Wendell's observations must be corrected for ellipticity. If we do so, and adjust the depths of minima of each of the sets of visual observations by about 0·01 magnitude in the direction necessary to produce agreement—an adjustment easily permitted by the degree of precision of the observations—and assume that the shell contributes 0·2 of the total light of the system at the time of Dugan's observations, then we find the results shown in Table 2. Reasonable agreement is obtained between the visual observations, and a difference between the visual and photo-electric results is in the direction to be expected from the relative wave-lengths. The entire story may not be quite this simple; nevertheless, the concept of a developing unstable condition, followed by either continuous or intermittent ejection of material for several years does promise at least a partial solution of old and hitherto unresolved discrepancies. It is of interest to note that, in order to rationalize his observations

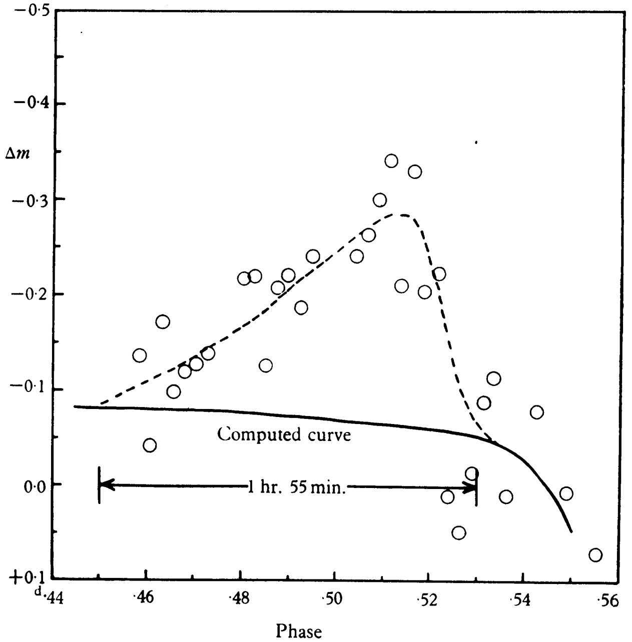

Fig. 1. Photographic observations of SV Camelopardalis on JD 2429287. (In *Princeton Contr.* No. 21, note that the values of $a-v$ at times 0·7480 and 0·7503 should read $0^{m}\cdot 13$ and $0^{m}\cdot 14$, respectively.)

of W Ursae Majoris, Kwee has found it necessary to postulate a contribution from some other source amounting to 0·25 of the total light of the system.

We ask whether other systems have shown similar photometric humps, and if so, what is their history. A similar effect, but in this case at a phase just preceding primary minimum, has been found in SV Camelopardalis [10], and is illustrated in Fig. 1. The plotted points represent photographic observations taken on one night in 1939. The plates were deliberately taken out-of-focus and the image densities were measured photo-electrically. Each plate was calibrated by impressing on it a series of standard squares whose densities were also measured photo-electrically.

A check star was measured at least once on each plate; at the time of the hump, the comparison-check star differences showed only the usual observational scatter. There are available scattered observations on one other night (about a week earlier) at this part of the light curve. These observations also indicate the existence of a hump of about the same amplitude, but suggest that it was then located somewhat closer to primary minimum.

The hump observed in SV Camelopardalis is in many ways similar to that of R Canis Majoris. It was not clearly present on earlier series of observations such as those of Detre [11] and Pierce [12]. In later years, various photo-electric observers covered the curve without reporting any trace of it. Yet, gradually, the later observers began to uncover evidence of other irregularities. Nelson [13] noted asymmetry in the light curve. A Lick Observatory report [14] stated that photo-electric observations indicated evidence of spots similar to those described for AR Lacertae. Hiltner [15] found no hump, but did observe the secondary minimum to be significantly shallower than was found photographically to be the case in 1939, and he also reported irregular surface brightness variations. One spectrogram—but only one—showed weak Ca II K emission. Finally, in an as yet unpublished investigation based on extensive photo-electric observations, H. van Woerden finds variations in the depth of secondary minimum by as much as 0·15 magnitude and also variations in the depth of primary.* Van Woerden also finds evidence of plateaux on the ascending branch of secondary minimum which change in length and slope within a few days. He has investigated thoroughly the older observations, and finds in them evidence of irregularities that had previously escaped attention. He concludes that the light curve has probably displayed irregularities for a long time without our being aware of it. Again, this is a system which has shown erratic period fluctuations. After almost 20,000 epochs of constant period, a sudden change of period occurred, followed by another interval of period constancy, and then by another apparently sudden change at or shortly after the time the hump was observed. More recent photo-electric observations by van Woerden have indicated even larger changes. While the system seems more erratic than R Canis Majoris, the same general picture appears of a temporary hump, period changes, and peculiarities in the light curve.

I know of no other system showing precisely this type of instability. If we look at UX Monocerotis, however, we see evidence of instability on a considerably greater scale. Actually we do not really know whether this

* I am indebted to Dr van Woerden for sending this information in advance of publication.

is a system that is intrinsically more 'unstable', or whether we have been fortunate in catching with all the power of modern observational astronomy a representative system at a particularly unstable phase in its development. The history of this system goes back to 1926. Even in the earliest work based on photographic estimates [16], the dispersion of the observations suggested to the authors that maximum light was not constant. Wyse [17]

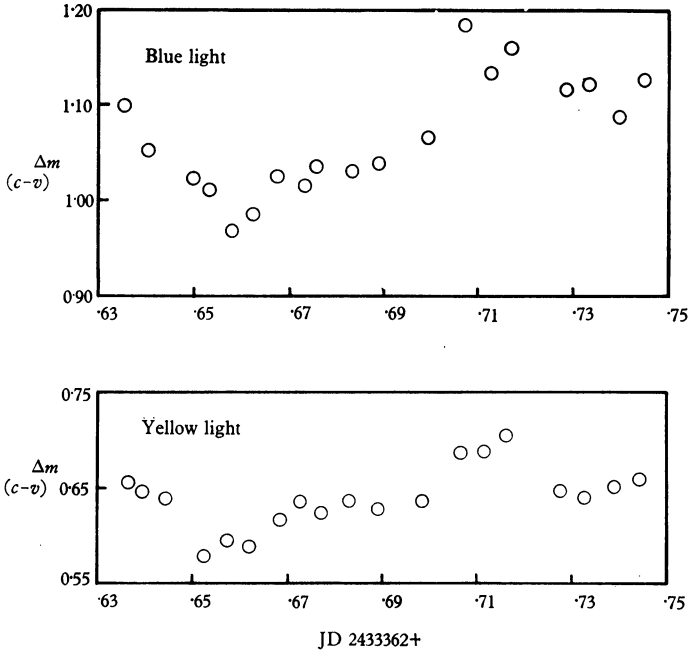

Fig. 2. The light variations of UX Monocerotis in two colours.

classified the spectra as A5 and dG1p and pointed out that the fainter component apparently had a dwarf spectrum and the density of a giant star. Further spectrographic peculiarities, first pointed out by Gaposchkin [18], have been discussed in detail by Struve [19]. The spectral changes are extremely complicated and involve changes in the strength of both the emission and the absorption lines. The irregular light variations were detected quite independently by photo-electric observations at the Steward Observatory [20] and at the McDonald Observatory [21] in the spring of 1950. While the out-of-eclipse light is often relatively stable, it sometimes exhibits surprisingly large fluctuations.

A typical fluctuation is shown in Fig. 2. Over about $2\frac{1}{2}$ hours, variations of almost 0·2 magnitude in the blue and about half this much in the yellow were observed.* On some nights, even more rapid changes were indicated, as illustrated in Fig. 3. On that particular night, the system was observed both at the McDonald and the Steward Observatories. The McDonald observations, corrected to the comparison star used at Tucson, are included. Unfortunately, the McDonald observations were interrupted at precisely the times when the rapid changes were occurring, so the strong confirmation which might be hoped for from simultaneous observation with two telescopes is lacking.

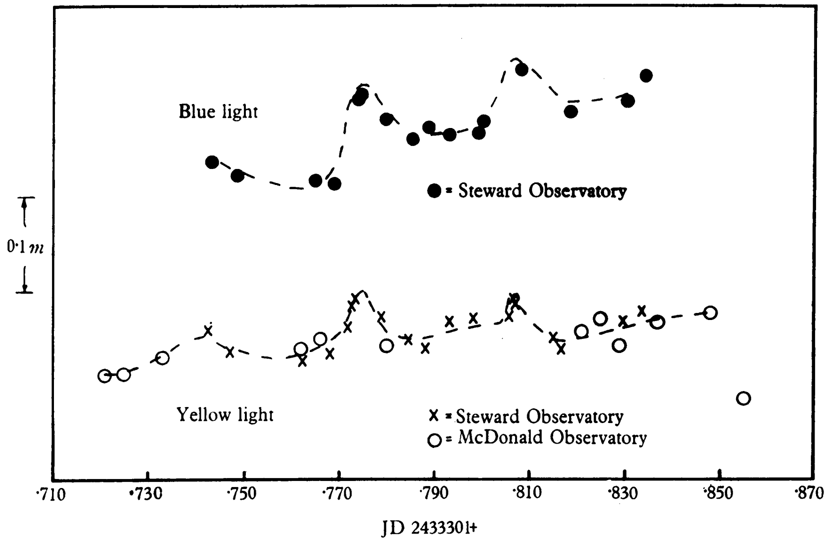

Fig. 3. Observations of UX Monocerotis made at the McDonald and Steward Observatories on the same night.

Recently, C. R. Lynds has observed this star in three colours at the Lick Observatory: his findings strongly confirm this general type of irregular fluctuations. Lynds and B. S. Whitney [22] have independently detected period variations; Whitney has suggested that, since Struve's spectrographic study was made when the period was increasing, it might be valuable to carry out a similar study at a time of decreasing period.

An example even more extreme than UX Monocerotis is shown by AE Aquarii. There is as yet no definite evidence that this is an eclipsing

* Publication of these observations has been delayed in the hope of completing the entire curve. This hope has been temporarily abandoned, and I plan to publish the observations in the near future.

variable but Joy's spectrographic work indicates that it is a close double system. The erratic light fluctuations, studied chiefly by Lenouvel [23], are similar in general form to those just described, but are more rapid and much greater in amplitude. In two or three minutes the system has been observed to double in brightness in the spectral range in which it was observed.

Whether there exists a continuous sequence from the stars showing humps on rare occasions (R Canis Majoris) to those exhibiting almost continuous variation (AE Aquarii) and whether, if such a sequence exists, it has evolutionary significance are matters that can be decided only after extensive series of precise observations have been obtained of many stars.

Quite a different system from any of the above is AR Lacertae. This system also shows irregular fluctuations, but of a noticeably different sort. From an extensive series of photo-electric observations, Kron [24] infers that these are caused by patches of varying brightness on the star's surface. These patches cover at maximum as much as 20 % of a given hemisphere, with the sizes of the individual patches ranging from 3–5 % of the area. The spectra of both components are peculiar, and an analysis by Struve [25] on the basis of spectra taken by Sanford and by himself interprets many of the peculiarities in terms of turbulent spottedness. Struve also concludes that the phenomenon is variable. Thus a strikingly similar picture is built up quite independently by both the photometric and the spectroscopic data. This system also has shown a large and apparently sudden period variation.

Another system of interest is AO Cassiopeiae, which consists of two extremely bright and massive O-type stars. The system is obviously quite young, and it may not be surprising to find evidence of unstability. Variations are found in the light curve, in the velocity curve, and especially in period. The changes prior to 1947 have been summarized [26]. In 1946–7 the system showed an asymmetric light curve. Observations made in 1947–8, however, indicated nearly equal maxima. The general picture, as yet not strongly established, seems to be that of a normally symmetric curve which is often significantly disturbed. The distortion apparently remains more or less constant during an observing season; at least, shorter term variations have not been firmly established. In 1948 Hiltner [27] observed the star in two colours. His chief conclusions were that the maxima were then nearly equal, the secondary was apparently displaced, and that the two minima were of markedly different shapes. The significant spectral changes of this system have already been discussed [28]. They include both changes in shape of the velocity curve and variation in line intensities; the latter variation was present also in spectroscopic observations made in 1916–17. Contrary to some of the other systems discussed, AO Cassiopeiae has shown recurrent instability for over thirty years. The period changes are also well established [29]; they are certainly not periodic, and seem to occur at irregular intervals. Instead of being of the order of less than a second, as is usual in systems showing such changes, the magnitude of these period variations is of the order of 15 sec. or more. Certainly, whatever is happening, the system is now in process of evolution in an unstable manner.

Turning from these systems which can easily be called unstable and each of which must be described separately, let us look at the close dwarf systems generally classified as of the W Ursae Majoris type. Here, intrinsic variability is the rule rather than the exception. Indeed, I know of no case where precise photometric observations at two or more different epochs have failed to indicate changes in the light curve. Two general types of such fluctuations, not necessarily mutually exclusive, seem to exist.

The first type of change apparently is characteristic of all the W Ursae Majoris variables. A most thorough study has recently been made by L. Binnendijk [30] of 44 Bootis B, a star that may be considered fairly typical of the class. The fluctuation of the relative heights of the quarter-points and the various degrees of asymmetry introduced thereby are clearly evident. It is possible that the displaced secondary minima reported in these systems are really caused by asymmetric light curves that can produce an apparent displacement of secondary.

Two rarer types of fluctuation have been described by Kron [31] for YY Geminorum (the only known dMe eclipsing binary) and by various authors for UX Ursae Majoris [32, 33, 34]. Preliminary reports on DQ Herculis [35] indicate that it may be similar to UX Ursae Majoris. In the case of YY Geminorum, Kron has explained the secondary light variation by a combination of rotation and the eclipse of a patchy, non-uniform surface. In the case of the last two systems, plateaux were found in the light curve which were not constant from night to night and were not explicable by any normal eclipse hypothesis. These systems have been described thoroughly in the recent literature.

At the other extreme of size, systems which have late-type super-giant components commonly show signs of unstability. The study of these has been a fruitful field in recent years, both photometrically and spectrographically, and variations other than those caused by eclipse are often truly remarkable. They range from large fluctuations, which in the case of VV Cephei and 32 Cygni may be as large as 0·3 to 0·5 magnitude (and which, except for a longer time scale, somewhat resemble the fluctuations of

UX Monocerotis), through the smaller changes in ζ Aurigae and ϵ Aurigae (which resemble slow surges of from a few hundredths to a tenth of a magnitude in range) to the constancy shown so far by 31 Cygni. The paucity of our knowledge of these systems is illustrated by the fact that we do not really know as yet whether the character and magnitude of the fluctuations is really dependent on the system itself or upon the particular epoch of

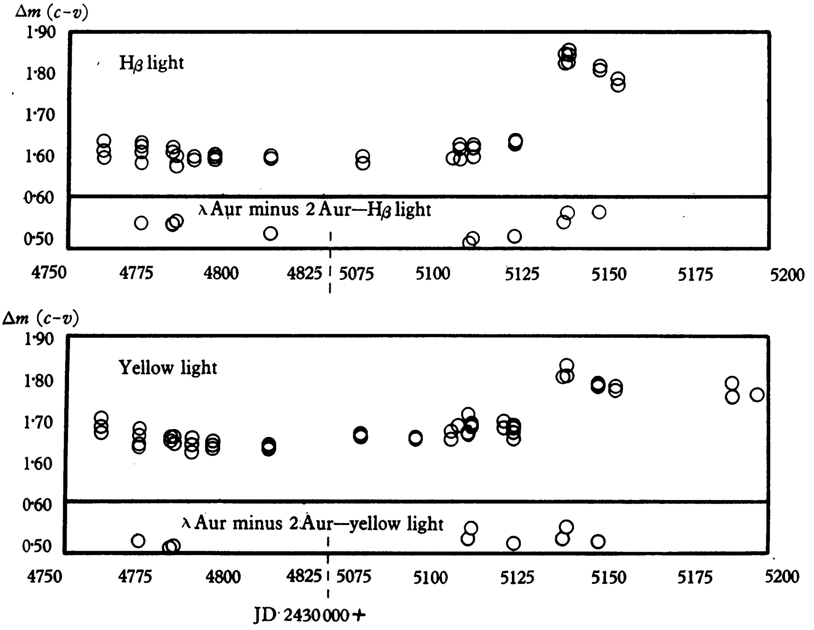

Fig. 4. Observations of ϵ Aurigae made in 1954 and 1955. The effective wave-lengths of the interference filters used are, approximately, λ 5240 for the yellow magnitudes and λ 4860 for the $H\beta$ observations. The comparison star was λ Aurigae.

observation. In other words, is 31 Cygni, so like others in many of its features but showing no intrinsic light fluctuations, really a stable system or do all these systems show varying degrees of unstability, 31 Cygni having merely been observed at a particularly stable stage? Certainly two solar observers would present somewhat different pictures of the Sun if they observed five years apart.

An example of this type of fluctuation is shown by ϵ Aurigae. Observations by Huffer [36] preceding the last (1928–30) eclipse showed a hump of amplitude about 0·2 magnitude. Four-colour observations taken at the Cook Observatory of the University of Pennsylvania just before the present (1955–7) eclipse show quite a similar effect. The hump appears in all four colours (Figs. 4 and 5). Its amplitude is greatest in a colour region about

120 angstroms wide that is centred at the Hβ line. One difference between Huffer's observations and the modern results lies in the fact that Huffer reported almost continuous fluctuations while, except for the region of the hump, the Cook observations repeat well from night to night. This is an indication again that observations over many epochs will be required

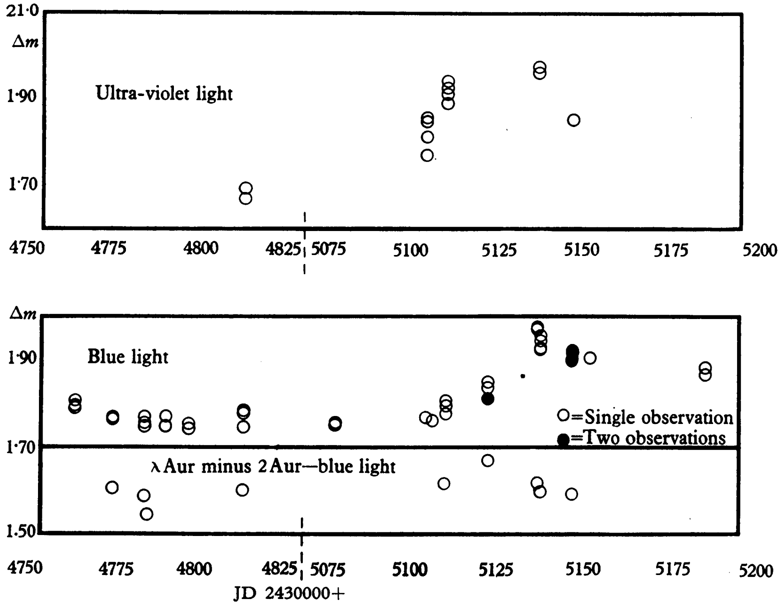

Fig. 5. Observations of ϵ Aurigae made in 1954 and 1955. The effective wave-length of the glass filter-cell-telescope combination used for the ultra-violet observations is approximately λ 3750. The effective wave-length of the filter used for the observations in blue light is approximately λ 4220. The comparison star was λ Aurigae.

before we can properly describe this system. (Gyldenkerne, at Copenhagen, has also reported irregular variations of this type.)

In addition to these irregular fluctuations, there are other features of these systems that may be connected with symptoms of instability. In an earlier paper [37], Roach and I showed from photometric observations of ζ Aurigae that the duration of the partial phases of the eclipse was a function of the wave-length of observation (as had been suggested by Fracastoro [38] from spectrophotometric results). Thus it was possible to determine the duration of the partial phases only when all the observations had been reduced to a common wave-length. When this was done, it became evident that the width of eclipse was different in 1947 from its value in 1939. (Welsh [39] has summarized the spectrographic observations that establish variations in the extent of the tenuous outer layers.) The change can most simply be explained by a change in size of the K-type component of about 1 %. ζ Aurigae has been intensively observed, yet we do not know if this type of fluctuation is irregular, or part of a periodic change, or even a secular change actually connected with the evolution of the system.

Finally, in this group of stars attention should be drawn to BL Telescopii. Two-colour photo-electric observations have been published by Cousins and Feast [40]. Clearly evident is a preliminary decline in brightness, which, from this evidence alone, could be identified either with the beginning phases of an atmospheric eclipse or with an intrinsic fluctuation of one of the components.

Another effect is shown by BL Telescopii when the light loss is expressed as fraction of eclipse. It is obvious that during ingress, the loss of light at any given time in the photo-visual region is greater than that in the photographic: an effect exactly opposite to that of ζ Aurigae. In the observed phases of the egress, however, the loss of light is the same in both colours to within the errors of observation.

It is tempting to try to explain the difference between BL Telescopii and ζ Aurigae in terms of the difference in spectral class of the eclipsing components, but such an attempt would be unwarranted by the observational evidence available to date. BL Telescopii shows a partial eclipse. Instead of forcing the two curves together at mid-eclipse, one should make solutions in each colour to find the maximum fraction of light lost by the eclipsed component, and compute the 'fractions of eclipse' from this. Two difficulties arise, however. The first is that solutions by conventional methods may not be appropriate in this case. The second difficulty is that the South African observers have discovered intrinsic fluctuations in the light of BL Telescopii which make it difficult to determine the uneclipsed brightness of the system. But any consistent treatment leads to the same qualitative conclusion, that there is an observable difference between ingress and egress at the same phases. This system warrants intensive investigation.

Time does not permit a complete discussion of other systems that have shown indications of non-stable features. In BM Cassiopeiae, for example, two-colour photo-electric observations by G. Thiessen have confirmed earlier reports by M. Beyer. Fluctuations in depth and shape of the minima and considerable fluctuations outside eclipse are found.

Many other unusual systems exist. One has only to name RT Andromedae, SX Cassiopeiae, RX Cassiopeiae, RT Coronae Borealis, W Serpentis, GO Cygni, V444 Cygni, V729 Cygni, V367 Cygni, VW Cephei, and

U Pegasi to demonstrate that already we know many systems that have shown unstable characteristics and which will repay further observation.

Let us turn finally to a feature found in many eclipsing systems: erratic period changes that do not fit any concept of a stable system. It seems to be significant that in almost every case where well-established irregular period changes exist in systems for which photometric solutions are available, one component of the system is near the limits of dynamical stability. The material available through 1950 has already been summarized [9]. I want to report my strong conviction that the study of these period changes may be of great significance in attacking the difficult problem of the evolution of close binary systems.

In summary, then, today we seem to be in the initial stages of exciting new developments in the study of eclipsing binaries. Two years ago, I pointed out that the study of eclipsing systems could roughly be divided into three phases. In the first, the theory was weak or non-existent and the observers frequently reported wonderful features in the light curves: features that we now know were usually characteristics of the observing technique rather than of the star. This phase ended with the theoretical papers of Russell in 1912–13, and, observationally, with the development of visual photometers, more accurate photographic techniques, and the use of photo-electric cells. For more than thirty years, we were in a second period in which the effort of the theoretical worker was directed toward explanation of the light fluctuations by more and more highly refined models, and the observer tried to produce curves of the highest possible precision, usually serene in the belief that the light variations of the star were repeating themselves uniformly cycle after cycle and year after year. About 1946 the stimulating spectrographic work of Struve and his collaborators on the one hand, and several series of photo-electric observations on the other, called attention strikingly to the fact that changes are occurring in these systems. We are now aware that few of the eclipsing systems now known are far removed from the limits of dynamical stability (although we should remember that observational selection gives preference to the discovery of such systems), and that we may have a chance to study evolutionary changes without waiting for millions of years.

In addition to using these ideas in planning our own programmes and interpreting the results, it may be of value to re-discuss the observations of the past in order to see what information this new approach may produce. We will of necessity be limited to cases where the observational technique was such as to minimize the systematic errors and where the individual observations were themselves published. In general, these will be systems observed photo-electrically, with the visual polarizing photometer or in some cases with the wedge photometer, or photographically when a sound objective technique was used in measuring the magnitude changes.

The importance of publishing the individual observations can scarcely be over-emphasized. Only thus will observations made now be available with their full potential to astronomers of the future.

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