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Predicted time-scales for GISP2 and GRIP boreholes at Summit, Greenland

CHRISTINE SCHØTT,
Department of Glaciology, Geophysical Institute, University of Copenhagen, DK-2200 Copenhagen N, Denmark

E. D. WADDINGTON AND C. F. RAYMOND
Geophysics Program AK-50, University of Washington, Seattle, Washington 98195, U.S.A.

ABSTRACT. Two deep-drilling projects (GISP2 and GRIP) in central Greenland will provide ice cores for paleoclimate studies. Drilling decisions and preliminary interpretations require age–depth curves (time-scales). Using a finite-element momentum-balance model, we calculate the modern ice-flow pattern on the flowline through the two drill sites. Our model appears to require relatively soft ice either throughout the ice sheet or below the Wisconsinan–Holocene transition in order to match the modern geometry and mass balance. By scaling the ice velocity to an assumed mass-balance history throughout the past 200 000 years, we estimate the time-scales at both sites. At GISP2, a flank site, we place the 10 000 years BP isochrone (representing the Wisconsinan–Holocene transition) at 1535 m ice-equivalent depth. At GRIP, on the ice divide, the corresponding depth is 1377 m. Our calculations show ice older than 200 000 years at 100 m above the bed at both coring sites. The time-scale calculation can be used for drilling decisions and preliminary interpretations. It should be refined as more regional-survey and ice-core data become available.

INTRODUCTION

Two deep ice cores are presently being drilled in the vicinity of Summit, Greenland (71° N, 37° W), with the expectation of recovering ice of age up to 250 000 years. A European group (GRIP) is coring at the present summit of the ice sheet, while a U.S. group (GISP2) is drilling at a site 28 km to the west (Fig. 1). Plans call for completing both ice cores by 1992. Because of current interest in global change, these ice cores will be important records of paleoclimate.

Ultimately, we hope that profiles of physical or chemical variations measured in the ice core will permit reliable dating of the stratigraphic record; however, during drilling it is necessary to have an estimate of the expected time-scale (depth–age curve) in order to choose rational ice-sampling strategies. For this purpose, we present preliminary time-scale estimates based on ice-flow-model calculations. Comparison with absolute dates, when they become available, will lead to an understanding of the relative roles of transient ice dynamics and climatic transients on the stratigraphic record.

ICE-FLOW CALCULATIONS

The ice thickness, and the surface and bedrock topography have been determined by radio-echo sounding (Hodge and others, 1990). The recent regional mass balance has been measured (personal communication from J. Bolzan; personal communication from H. Clausen) and cores to about 200 m depth in 1989 at GISP2 and at GRIP give the variability over the past few hundred years. Temperature has been measured in the 1989 hole (Alley

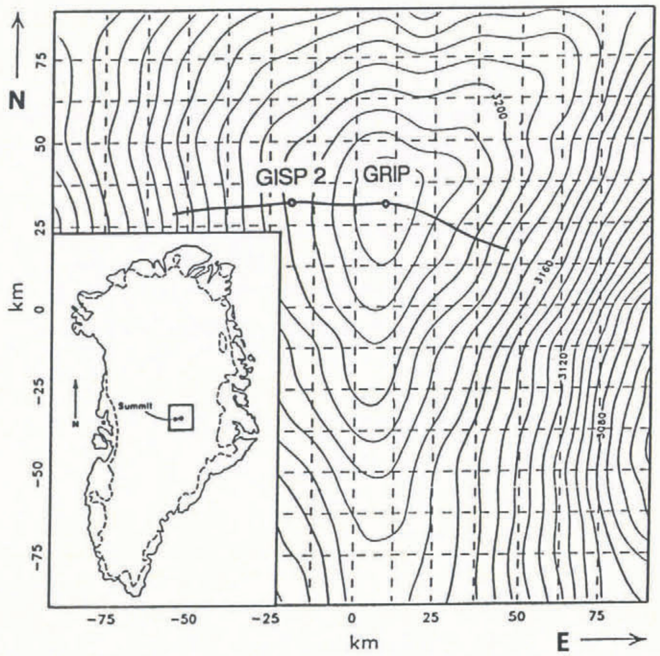
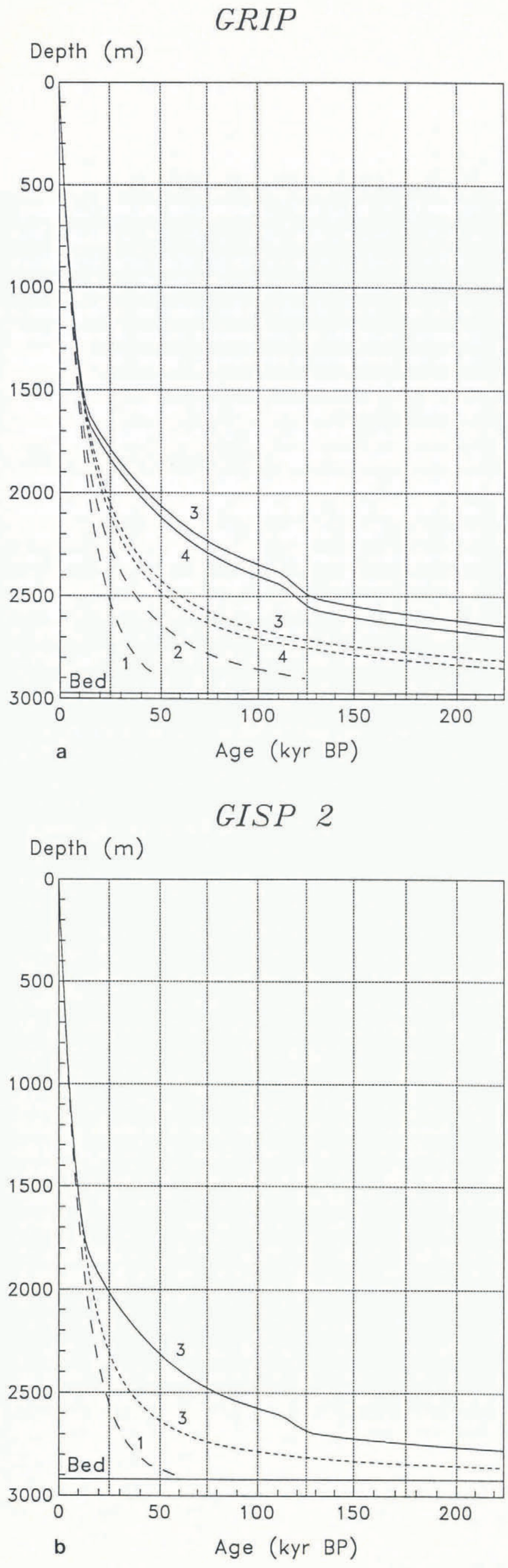


Fig. 1. Location map (inset) and surface-elevation contours in the Summit region showing model flowline and GISP2 and GRIP core-hole locations. The topographic information is from Hodge and others (1990).



and Koci, 1990). Thus, sufficient data for a preliminary ice-flow calculation are available.

First, we show the time-scales for these sites using simple analytical models (Fig. 2). The Nye–Haefeli time-scale (Haefeli, 1961; Dansgaard and Johnsen, 1969; Paterson, 1981, p. 328) assumes a constant vertical strain rate with depth. The Dansgaard–Johnsen (1969) time-scale is more realistic, because it allows the horizontal velocity to vanish where the ice is frozen to bedrock. Vertical strain is assumed to be constant in the upper part of the ice sheet, then to decrease linearly to zero at the bed. Dansgaard and Johnsen (1969) chose the depth of the break in the slope of the horizontal velocity–depth curve based on features in the stable-isotope record in the core. We fit the Dansgaard–Johnsen model to annual-layer thicknesses measured by ECM (electrical conductivity) and to identifiable volcanic eruptions that provide dating back to 3600 BP in the top 770 m at GRIP. The Dansgaard–Johnsen time-scale places the Wisconsinan–Holocene transition at 1532 m true depth at GRIP (personal communication from D. Dahl-Jensen and H. Clausen). This time-scale, which is based on data, can be used to date the GRIP core above the Wisconsinan–Holocene transition. An adequate time-scale for the deeper ice at both sites must reflect the different pre-Holocene climatic conditions.

We have used a finite-element momentum-balance model derived from that used by Raymond (1983) to calculate the present ice-flow pattern on a flowline joining the two coreholes.

The boundary conditions are as follows. By extending the flowline by ten ice depths beyond both holes, we can impose simple velocity-boundary conditions at both ends without introducing significant error into the velocities calculated at the boreholes (Waddington and others, 1986). The upper surface is stress-free. Because the basal ice is probably frozen to the bed at present (Firestone and others, 1990), we impose a zero velocity-boundary condition there. Paterson and Waddington (1986) and Dahl-Jensen (1989) also concluded that the ice sheet is cold-based in central Greenland.

Our approximations in the preliminary calculation

Fig. 2. Time-scales for (a) GRIP and (b) GISP2 sites. All depths are given in ice equivalent. Curves labelled (1) show the steady-state Nye–Haefeli time-scales for the two sites. Curve (2) shows a Dansgaard–Johnsen time-scale for GRIP; the parameters were derived from measurements on the 1990 ice core. Curves labelled (3) and (4) were derived from the finite-element model; dashed curves are for steady state. Solid curves were derived using the mass-balance history in Figure 4. (3) Uniform flow-law constants, $n = 3$, $A_0 = 1..6 \times 10^{-17}\,year^{-1}\,Pa^{-3}$. (4) Flow-law constants, $n = 3$, $A_0 = 0.71 \times 10^{-17}\,year^{-1}\,Pa^{-3}$, and enhancement factor $E = 3$ for ice older than 10 000 years. At GISP2, (4) is virtually indistinguishable from (3), and is therefore omitted.

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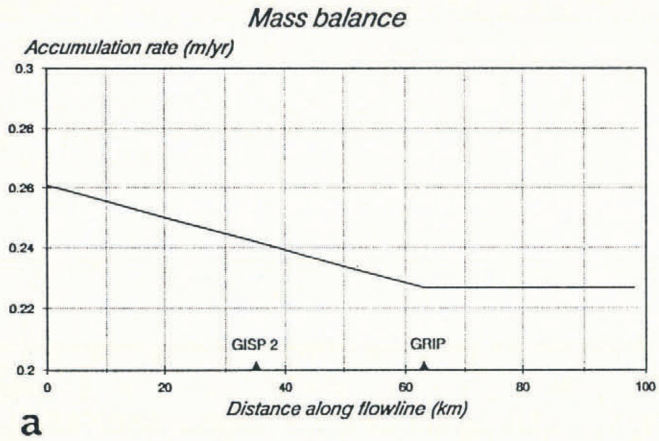

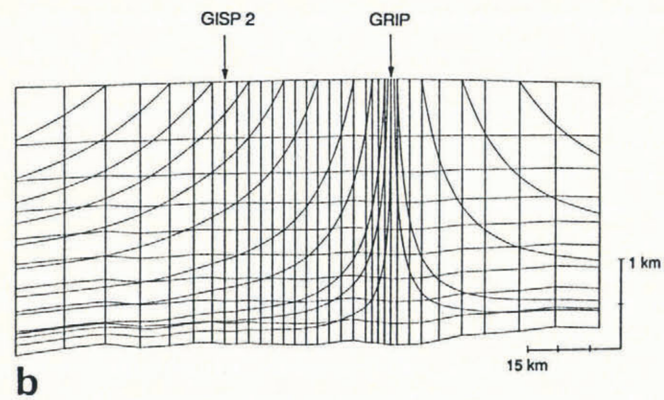

Fig. 3. (a). Modern mass-balance profile along GISP2–GRIP flowline (personal communication from J. Bolzan; personal communication from H. Clausen). (b) Longitudinal finite-element representation of the flowline in Figure 1, with representative calculated ice-particle trajectories for the model with uniform rheological parameters n and A_0. The GISP2–GRIP separation is 28 km. The vertical exaggeration is × 15.

are listed below. Later, we will discuss the impact that these assumptions have on the derived time-scales.

(1) We assume that the ice deforms in plane strain.

(2) The upper 3% or less of the ice depth consists of compressible firn. We make no attempt to incorporate firn compaction. Instead, we assume an initial ice-equivalent thickness which is 24 m (Clausen and others, 1988) less than the true thickness at all points. If the density–depth relation does not vary along the flowline, the shear stresses at depths below 100 m should be accurately represented.

(3) We assume an isotropic power flow law for ice of the form (e.g. Paterson, 1981, p. 27, 39),

$$\dot{\varepsilon} = A(T)\tau^n \tag{1}$$

relating effective strain rate $\dot{\varepsilon}$ to effective shear stress τ. We use the same value of n everywhere, and A depends on temperature T. Ice-age ice is probably softer than interglacial ice (Dahl-Jensen and Gundestrup, 1987) and fabric development could also affect the layer-thinning rate. These effects would introduce perturbations to our derived time-scale.

(4) Ice temperature affects rheological properties of the ice through the exponential Arrhenius factor (e.g. Paterson, 1981, p. 27) in the flow-law parameter $A(T)$, i.e.

$$A(T) = A_0 \exp(-Q/RT) \tag{2}$$

where $Q = 60\,\mathrm{kJ\,mol^{-1}}$ is activation energy for creep, and $R = 8.314\,\mathrm{J\,mol^{-1}\,deg^{-1}}$ is the gas constant. The ice-flow model assumes a simple analytical form (one-quarter cycle of a cosine curve) for the temperature variation with depth. The vertical temperature gradient vanishes at the surface where temperatures equal the measured mean annual values. The gradient at the bed was chosen so that the temperatures agreed within a few degrees at all depths with one-dimensional calculations of temperature beneath Summit (Schøtt, 1990).

(5) We assume that the central part of the ice sheet is close to a steady state at present. Then, by also assuming that the flow divide is located at the present topographic summit for our flowline, we can calculate the ice flux at each lateral boundary by integrating the modern mass-balance profile (Fig. 3a; personal communication from J. Bolzan; personal communication from H. Clausen). These fluxes determine the velocity-boundary conditions.

We calculate the present ice flow in the following way. We first choose a finite-element mesh representation (Fig. 3b) of a longitudinal profile for the flowline following the steepest path down the surface slope as measured by Hodge and others (1990). The profile extends far enough beyond each borehole so that simplified boundary conditions do not propagate errors back into the region of interest (Raymond, 1983; Waddington and others, 1986). Secondly, we use the calculated ice flux at each end of the flowline to set simple isothermal laminar-flow boundary conditions on the horizontal velocity profile. Thirdly, we assign a temperature to each node and choose flow-law parameters n and A_0. Fourthly, we solve for the velocity pattern. We will discuss this step in greater detail.

Because the input geometry, temperature, rheology and plane-strain state do not perfectly reflect the physical state of the ice sheet, the model gives a spatially variable vertical velocity pattern that cannot be in equilibrium with the smoothly varying measured mass balance (Fig. 3a; see Raymond, 1983, for discussion). Near an ice divide where the horizontal velocity is small, the age–depth relationship is determined largely by the vertical velocity. We expect that a vertical-velocity solution incompatible with the observed mass balance is one of the largest sources of error for the calculated time-scale. Because we do not yet know the actual complex physical conditions causing the discrepancy between the model velocity and the mass balance, we obtain a compatible vertical-velocity field in the following way. We let our relatively simple ice-flow model evolve through time, adjusting its surface elevation until it achieves a steady-state velocity solution in equilibrium with the smooth modern mass balance. During this evolution, we maintain the constant ice-flux-boundary conditions at both ends. With appropriate

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rheological parameters, we can achieve a steady-state profile that has a surface elevation within about 5 m of the initial estimate based on Hodge and others (1990) in the zone of interest between and within at least five ice depths downstream from the two boreholes. The ice sheet thinned by 2 m at 15 km downstream from GISP2, thickened by 5 m near GISP2, and thinned by 2 m at the divide (GRIP site). This is comparable to the relative accuracy of the Hodge and others' (1990) elevation data.

Our preferred model with uniform flow-law constants throughout the cross-section used $n = 3$, and A_0 chosen to give $A = 1.6 \times 10^{-17}$ year^{-1} Pa^{-3} at −20°C in Equation (2). This corresponds to the value recommended by Paterson (1981, p. 39), combined with an enhancement factor of $E = 3$ (e.g. Dahl-Jensen and Gundestrup, 1987). This suggests that the soft Wisconsinan ice still exerts a strong influence on the ice-sheet flow in the Summit area. Having determined an acceptable estimate of the present flow pattern at Summit, we then calculated the particle paths, isochrones and time-scales that would result if the present flow pattern were maintained for 225 kyear. Figure 2 shows these time-scales as dashed curves labelled 3 for both the GISP2 (Fig. 2a) and the GRIP (Fig. 2b) sites.

We recognize that steady state is improbable. Due to its long response time, the Greenland ice sheet probably does not maintain equilibrium with the changing climate. We assume, however, that in response to a change in accumulation rate, the ice sheet maintains the same geometry and flow pattern with all velocities scaled to the new accumulation rate. This assumption was used previously by Paterson and Waddington (1986), Dahl-Jensen and Johnsen (1986) and Firestone and others (1990) to model ice-age-cycle temperature changes in Greenland. For unchanging ice stiffness, this assumption is not completely compatible with momentum conservation; the ice-sheet geometry must change in order to generate a different flow rate. In reality, changes in ice temperature (Whillans, 1978), ice fabric (Fisher, 1987) and impurities (Dahl-Jensen and Gundestrup, 1987) probably alter the rheological properties over a glacial cycle. Model calculations (Dahl-Jensen, 1989) have shown that the depth–age profile close to an ice divide is not affected seriously by either the distribution of the enhancement factor or by the temperature, but changes in the accumulation rate imply an equivalent scaling of the vertical-velocity profile.

In a finite-difference simulation of the whole Greenland ice sheet, including temperature effects but not intrinsic ice-flow changes, Letréguilly and others (1991) have found that the ice-sheet thickness at Summit did not vary by more than a few hundred metres over an ice-age cycle, even though the thickness changes could be large elsewhere. Those results indicate that our assumption of constant surface height is reasonable.

Following Dahl-Jensen (1989), we have assumed that the precipitation rate follows a simplified periodic cycle consisting of 100 kyear of Wisconsinan conditions followed by 10 kyear of Holocene conditions (Fig. 4). Various estimates of the accumulation rate in Greenland during the last glaciation suggest it was reduced to one-half to one-fifth or less of the modern value (see Reeh (1990) for discussion). Following Paterson and Waddington (1986),

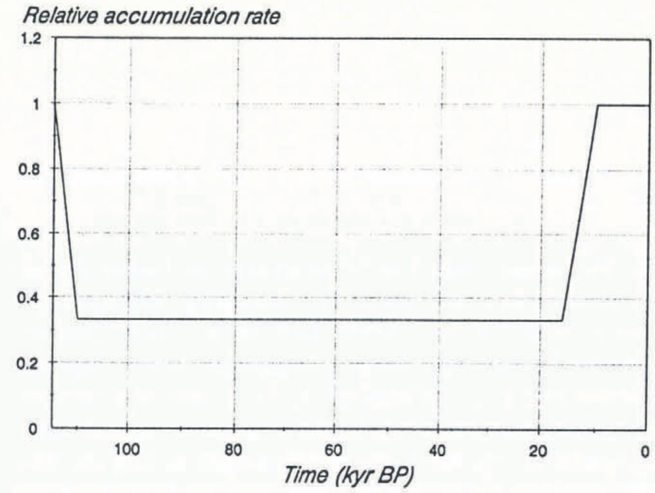

Fig. 4. The smoothed mass-balance history during the past 115 kyear glacial cycle used to derive the time-scales.

Dahl-Jensen and Johnsen (1986) and Firestone and others (1990), we assume that the precipitation at Summit during glacial periods was one-third its present value. We have then corrected the steady-state time-scales for the periods of reduced precipitation (and ice velocity) to derive the time-scales shown as solid curves labelled 3 in Figure 2. The derived time-scales are sensitive to temporal changes in the accumulation rate. Since the accumulation rate during the last glaciation and earlier is not well known, the simplified precipitation history and our corrections to the steady-state time-scales are only approximate.

EFFECTS OF APPROXIMATIONS

(1) The ice flow is probably not strictly plane strain, especially near the GRIP site, where the ice surface slopes radially outward. Cunningham and Waddington (1990), using a finite-element model with adjustable transverse strain rates, found that deformation on this flowline through Summit was best modeled by very near-plane-strain geometry. Reeh (1989) showed that divergence or convergence along a flowline upstream from a borehole do not affect the time-scale where mass balance is uniform; however, divergent flow upstream can amplify the influence of thickness and mass-balance gradients. Mass balance varies by about 10% between GRIP and GISP2 (Fig. 3; personal communciation from H. Clausen; personal communication from J. Bolzan), but the ice-thickness gradient is very small.

Measurement of strain rates along the flowline should be given high priority if ice-flow models are to be utilized to their full potential in the interpretation of these ice cores.

(2) We assumed that the firn could be represented by a spatially invariant ice layer reduced in thickness by 24 m relative to the actual firn layer. Below the firn–ice transition, stresses in our model will match those in a model incorporating firn. Above the firn–ice transition, our model attributes the rheological properties of ice to the

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much softer, compressible firn. Since firn occupies less than 3% of the vertical column height, the effect is negligible for our study. We use mass balance in ice-equivalent thickness. Because the surface slopes are very small, the shear stresses in the upper 100 m are small. As a result, there is little error introduced in our calculated horizontal velocities.

(3) We expect that the Wisconsinan ice under central Greenland is softer than the overlying Holocene ice (Fisher and Koerner, 1986; Dahl-Jensen and Gundestrup, 1987). This may lead to boudinage-flow instabilities (Cunningham and Waddington, 1990) and to a different pattern of vertical velocity than the one we calculate. Although we do not know the exact pattern of the flow enhancement under central Greenland, we have tested the sensitivity of the time-scale to this factor with a model in which the ice of age less than 10 000 years (as determined by isochrones of our original model) had an enhancement factor of 1.0, and the older ice had an enhancement factor of 3.0. In order to get steady ice-sheet topography stable to within 5 m of the measured topography (Fig. 3b) over most of the profile, we used a flow-law parameter of $A = 0.71 \times 10^{-17}\,\text{year}^{-1}\,\text{Pa}^{-3}$ at −20°C. This lies between the unenhanced value determined at Dye 3 (Dahl-Jensen and Gundestrup, 1987) and that recommended by Paterson (1981, p. 39). Curves 4 in Figure 2b show the time-scale derived from this model at GRIP. The 10 kyear transition occurs at 1409 m depth. The dashed curve 4 results from steady state, while the solid curve 4 results from the mass-balance history in Figure 4. At GISP2, the time-scale was nearly indistinguishable from the uniform-rheology model (curve 3).

(4) A temperature-model study by Firestone and others (1990) found basal temperatures colder by about 8°C than those assumed by our flow model; however, because we adjust the flow-law parameter A_0 so that our model reaches a steady-state profile as close as possible to the profile measured by Hodge and others (1990), the effect of temperature differences is largely absorbed into the adjustable rheological constant A_0, and the effect on the time-scale is not large. To test the sensitivity to the assumed temperature, we also re-ran the model in (3) above with a temperature pattern similar to that derived by Firestone and others (1990) for the Summit flowline; the resulting time-scales differed only slightly from curves 4 in Figure 2. The 10 kyear transition occurred at 1405 and 1548 m, respectively, for the GRIP and GISP2 sites.

(5) We assumed that the surface elevation was invariant. The ice-sheet elevation may have changed by several per cent since Wisconsin time. The direction of the change is not clearly established for the central area of the ice sheet. Zwally (1989) found from satellite altimetry that the south dome of Greenland may be thickening at $0.23\,\text{m year}^{-1}$. However, Bolzan (personal communication, 1990) found from a mass-balance calculation that the surface elevation is changing at $0.0 \pm 0.04\,\text{m year}^{-1}$. The ice sheet may have been very much thinner in the previous interglacial (Koerner, 1989; Reeh, 1990) or it may have been close to the present value (Letréguilly and others, 1991). The magnitude of these dynamic changes can be assessed by further modeling coupled with measurements of ice-deformation rates and core chemistry.

(6) We assumed that the ice-flow pattern changed only in magnitude during the past 200 kyear. In fact, it is possible that the ice divide migrated by as much as 50 km between glacial and interglacial times (e.g. Reeh, 1984). Raymond (1983) showed that the vertical strain-rate pattern at an ice divide differs from the pattern at a distance of several ice depths to either side, mainly because the absence of horizontal shearing at a divide imposes a substantially different vertical variation of effective viscosity. Thus, the time-scale in a steady ice-divide zone differs in form from that on the flanks. If, however, the ice divide migrates by a distance comparable to or larger than the width of the divide zone, the actual time-scale will lie between the extremes for divide flow or flank flow. As a result of divide migration, it is probable that the measured time-scales at both Summit sites will lie between the two predicted curves in Figure 2. Bolzan (unpublished study for GISP2 site selection) has studied the impact of such a migration on the calculated time-scales.

DISCUSSION AND CONCLUSIONS

Our results put the 10 000 year BP isochrone (assumed to be the Wisconsinan–Holocene transition) at a depth of 1772 m and 1431 m (ice equivalent) at GISP2 and GRIP, respectively. The actual depths would be 24 m greater at both sites. This transition could be passed in both holes in the 1991 drilling season, providing tests of our time-scales. Until then, comparison with dating based on ECM (acidity) measurements in the upper 770 m of the GRIP core from 1990 provides the only check. Preliminary dates (personal communication from H. Clausen) suggest that our calculated ages at given depths at GRIP (solid curve 3; Fig. 2a) may be close to, but slightly older than, the true ages. Such an effect could be caused by a migrating ice divide during the past few thousand years, by a different enhancement factor at depth, or by an incorrect mass balance in our model during that time period.

Dahl-Jensen (1989) calculated the Holocene transition depth at Crête, approximately 100 km south of Summit, in the vicinity of the ice divide. Because the precipitation at Crête is 15–20% greater than at Summit (Clausen and others, 1988), the transition depths there are not directly comparable to our results. Nevertheless, Dahl-Jensen's model also gave shallower isochrones at the divide, where the depth of the transition was calculated to be 250 m less than at a flank position 28 km away.

In other ice cores, the climate interpretation near the bed has been complicated by basal melt and refreezing (e.g. Byrd core), and by unknown complicated flow histories around upstream basal bumps (e.g. Dye-3). Firestone and others (1990) found that Summit cores are unlikely to be disturbed by basal melt. Basal-flow disturbances, which scale with the height of the bedrock topography, could exist. If, for example, we estimated that the bottom 200 m at Summit may have been influenced by complicated basal-flow patterns, then the age of the ice at the 200 m level would estimate the length of the unambiguous climate history obtainable at Summit. Our results put this age at 120 000 years at GISP2 and 230 000 years at GRIP. The difference arises from the different character of the vertical-velocity pattern within a few ice-thickness units of an ice divide (Raymond, 1983). Ice-divide migration in the past may have increased the age at the 200 m level at GISP2 and has probably decreased the actual age at GRIP. If we assume that the stratigraphy can be interpreted reliably to 100 m above the bed, then the corresponding ages are well over 200 000 years at both sites. Future detailed radio-echo-sounding data near the coreholes should allow better assessment of the height of bedrock disturbances.

Although our search was not exhaustive, it is interesting to note that we were unable to fit the current ice-surface shape and mass balance unless either all the ice, or else the ice older than 10 000 years was softened relative to the standard values (Paterson, 1981, p. 37) by a factor of about 3. This implies that the Wisconsinan ice under Summit has bulk rheological properties similar to the Wisconsinan ice at Dye-3 (Dahl-Jensen and Gundestrup, 1987), which is softer than the overlying Holocene ice. Our model can duplicate the available data without requiring us to assign different rheological properties to the Wisconsinan and the pre-Wisconsinan ice. A more detailed pattern of rheological properties may be required in future models when data from the cores also become available.

The time-scales we present here should be considered only as preliminary. As climate data become available from the ice-core records, they should be incorporated into the model-forcing history. As surface-survey results (strain rate, ice velocity, internal radar echoes) become available, they should be used to test and constrain the ice-flow models. Temperature profiles, vertical strain rate and tilt measurements from the boreholes will allow better estimates of the ice rheology. With these new data as constraints, increasing confidence can be placed in the calculated time-scales.

The calculated time-scales can ultimately be used in two ways. First, if the chemical records in the cores cannot provide a time-scale, the model results will be the only dating tool. Secondly, if the chemical records can provide a time-scale even deep in the ice, then discrepancies between the observed and calculated time-scales will reveal transient flow behaviour of the ice sheet if the model assumptions are correct. Understanding the transient behaviour through modeling will provide additional constraints on the forcing climate and will strengthen interpretation of the ice-core stratigraphy.

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