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ABSTRACT Energy is transported from the central regions of a star to its surface. Generally this transport is in certain layers carried on by convective motions. Because of the structure, which these motions have due to the influence of the overall rotation, the star becomes electromagnetically unstable, i.e. a large magnetic field grows from small seed fields as a result of the dynamo process. The internal structure, especially the symmetries of a star, will be, at least to some extend, reflected by the spatial structure and the time behaviour of the excited magnetic field. In this sense observations of the magnetic field on a surface of a star and the related activity phenomena can provide insight in the internal structure of the star, since characteristic parameters like thickness of the convection zone, mixing length, turnover time, profile of the differential rotation, etc. strongly influence the dynamo process.

The actual magnetic field of a star is a product of a nonlinear process. Models elaborated on the kinematical (i.e. linear) level provide insight in the excitation conditions and the linear field modes. The marginal mode, i.e. the mode which is easiest to excite, reflects properties of the nonlinear solution in case the system operates not far from the margin to the dynamo unstable region. Here the solutions show symmetries with respect to the axis of rotation and the equatorial plane, properties which are, for example, to a large extend fulfilled for the solar average magnetic field. For systems operating far from this margin irregular or even chaotic behaviour has to be expected. From observations there is a strong indication that these theoretical possibilities find their realizations within the sample of late-type stars.

INTRODUCTION

Observations of the solar surface reveal a manifoldness of phenomena closely connected with magnetic fields. At first glance these phenomena are of a small scale nature in space as well as in time. However, long time statistical studies suggest the existence of a large scale background field. This is best known from the sunspot phenomenon which has now been studied in detail over more than one century. It suggested an odd parity magnetic field - antisymmetric with respect to the equatorial plane - which consists of ring fields of opposite orientation on both sides of the equatorial plane. This toroidal magnetic field is accompanied by a dipol-type poloidal field indicated by magnetograph records

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of the polar caps and by structures in the corona which can be seen at solar eclipses. Both field parts form the large scale magnetic field of the Sun which oscillates with a period of about 22 years.

The oscillatory character of this magnetic field of the Sun exludes a fossil origin. An explanation has to present a mechanism which continously provides for a supply of energy. Already J. Larmor (1919), who thoroughly analyzed possible explanations in his famous paper "How could a rotating body such as the Sun become a magnet?", ruled out all alternatives and leaves the dynamo as the only one.

Since the presentation of the first conclusive spherical model of the Solar dynamo by Steenbeck and Krause in 1969 a great number of such models have been elaborated by several authors. On proper assumptions concerning the internal motions inside the Sun (i.e. convection, differential rotation, meridional motions) all relevant observational facts find an explanation.

It should, however, be noted here that the internal motions inside the Sun originate in the transport process by which the energy released in the core is brought to the solar surface. The magnetic field being produced by the induction action of these internal motions appears thus embedded in the whole complex of stellar constitution. In this general sense the dynamo problem proves to be a highly nonlinear problem which is beyond of acces even to the best to-days computers. The problem "simulating the solar dynamo" is still unsufficiently solved.

From the above adopted standpoint the global magnetic field of a star proves to be an entity which reflects by its spatial structure and its temporal behaviour the individual features of this star. Consequently, it has to be expected that stars of the same stellar type must have about the same magnetic field, if we, for the moment, refrain from the fact that the stars may differ with respect to their rotational velocities, which have a rather strong influence on the internal motions.

In this sense is the idea of the dynamo origin of the solar magnetic field strongly supported by the observational fact that late type stars also show activity cycles, i.e. they show the same magnetic activity phenomena as the Sun, in many cases also with a certain periodicity.

The situation is different for the magnetic Ap-stars: These stars have fields which significantly deviate from axial symmetry with respect to the axis of rotation, i.e. the magnetic signals recorded on the Earth are strictly periodic, thus reflecting the rotational period. Till now there are no clear indications of time variations of these fields in a co-rotating coordinate system, even if the observations of certain peculiar surface structures, which exist since the begin of this century, are included in the considerations. Thus the possibility of a frozen-in field from the birth of a star cannot be ruled out, although dynamo excitation in the star's convective core offers an alternative process (Krause 1983).

The peculiar A-stars are a small group of stars which might have a frozen-in field, the overwhelming number of stars do not show a sign of such fields. Thus it is not probable that stars get a field from the interstellar field by the condensing matter, or, if they do, the field will be destroyed by the strong turbulence during the star forming processes. However, dynamo theory revealed that turbulence and rotation make a star (or a protostar) to work as a dynamo. Consequently a star can hardly escape from being a dynamo, at least during some evolutionary phase, and will thus form its own individual magnetic field.

Magnetic fields of stars will so reflect certain internal structures, the most obvious of which is the convection zone, its thickness and its position. Further the differential rotation is of influence because of its strong amplification action. Details have to be studied with models which are as close to reality as possible.

THE BASIC PROBLEM

According to our view is the ability of a star to work as a dynamo due to internal motions which originate in the transport process which brings the energy produced in the star's core to the surface. The basic equations for describing this process are given by the conservation laws of mass, momentum and energy

$$\frac{\partial \ln \rho}{\partial t} + (\boldsymbol{u} \cdot \nabla) \ln \rho + div\ \boldsymbol{u} = 0 \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\frac{p}{\rho}\nabla \ln p + \boldsymbol{g} - 2\boldsymbol{\Omega} \times \boldsymbol{u} + \frac{1}{\rho}\, \boldsymbol{j} \times \boldsymbol{B} + \frac{1}{\rho}\, div\ \tau \ , \tag{2}$$

$$\frac{\partial e}{\partial t} + (\boldsymbol{u} \cdot \nabla)\, e = -\frac{p}{\rho}\, div\ \ \boldsymbol{u} + \frac{\kappa}{\rho}\Delta e + Q_{visc} + Q_{joul} \ , \tag{3}$$

and the induction equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = curl\ (\boldsymbol{u} \times \boldsymbol{B}) + \eta \Delta \boldsymbol{B} \ . \tag{4}$$

Here denotes ρ the mass density, $\boldsymbol{u}$ the velocity, p the pressure, $\boldsymbol{g}$ the gravity, $\boldsymbol{\Omega}$ the angular velocity, $\boldsymbol{B}$ the magnetic field, $\boldsymbol{j}$ the current density, e the specific internal energy, κ the heat conduction, τ the viscosity tensor and η the magnetic diffusivity. Finally define Q_{visc} and Q_{joule} the heat production by viscosity and electric resistivity. These quantities are quadratic expressions in the velocity gradient and the current density.

The equations (1) to (4) have to be completed by an equation of state. Furthermore, appropriate boundary conditions have to be formulated. Physically clear conditions we have in the case where the star is considered as embedded in an empty, electrically insulating space, although this does not fully correspond to the real conditions: stars generally have a corona, i.e. the environment is electrically well conducting.

Difficulties originate in the boundary conditions since the magnetic field as a far reaching quantity cannot be described by local conditions in a correct way. Nevertheless, a certain number of models were based on the local boundary condition $\boldsymbol{B}_{tan} = 0$. It provides for some mathematical simplification, however, it has to be noted that a physical justification is not possible. Especially, no electromagnetic energy, which is produced inside by the dynamo, is transported through the surface in the environment: The pointing vector $\boldsymbol{E} \times \boldsymbol{B}$ is tangential.

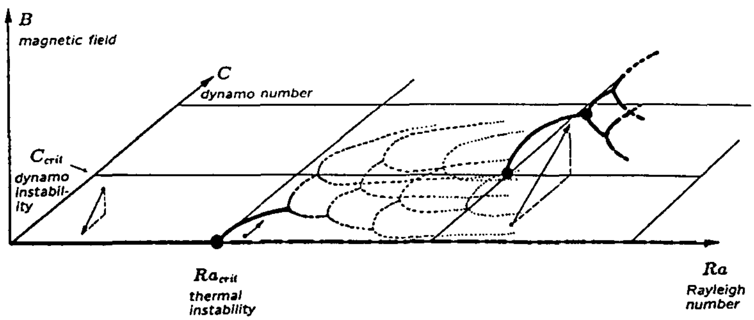

FIGURE I Stability diagram of a convection-driven dynamo. The axis to the right characterizes the temperature difference in units of the Rayleigh number Ra, the axis to the back the dynamo-active part of the convective motion by the dynamo number C. The arrows indicate the attractors, to which an arbitrary initial field developes.

In addition specify the boundary conditions that heat energy flows from the cental part of the star to the surface and leaves there by the emission of radiation.

The above presented equations have the simple solution

$$u = B = j = 0 , \quad \frac{\partial \rho}{\partial t} = 0 , \tag{5}$$

with e determined by

$$\frac{\partial e}{\partial t} = \frac{\kappa}{\rho} \Delta e \quad . \tag{6}$$

In this case the transport of energy is simply carried on by molecular conduction.

However, this solution generally proves to be unstable. First the velocity field $u = 0$ becomes unstable, and energy transport occurs by convective motions - hot material rises towards the surface, cold material sinks down. In addition, since the medium in the convection zone of the Sun or a star is electrically conducting, the magnetic field $B = 0$ also becomes unstable and dynamo excitation of a magnetic field sets in.

The 3-D-scheme in Fig. 1 shall illustrate the physical situation: The axis directed to the right characterizes the heat conduction process by the Rayleigh number R_a. At a critical value of R_a we have the first bifurcation at the thermal instability, a velocity field $u \neq 0$ is formed, the magnitude is indicated by the axis, which is directed to the background. The representation shall illustrate that first, i.e. close to the bifurcation point, the velocity shows a rather regular, periodic structure, in a greater distance it generally becomes turbulent. The dynamo number C now characterizes the electromagnetic action of this velocity

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field: At a critical value we have the second basic bifurcation where the magnetic field $B = 0$ becomes unstable and dynamo excitation sets in.

It is now well known that systems like the Sun or stars generally ly beyond the critical values of the parameters R_a and C. It is so clearly to be seen that the considered problem requires the solution of the full set (1) - (4) of nonlinear equations, and that we especially have to consider their irregular solutions, which describe the turbulence phenomena.

The problem appears so to be of unmanagable complexity. During the last three decennia succesfull research has been carried out on the basis of the mean-field concept (Krause and Rädler 1980). Equations for the mean fields have been developed which describe the collective phenomena of the fluctuating fields by certain averages. Parallel to this development there was also the try to calculate dynamo processes by numeric simulations (e.g. Gilman 1972, 1983, Glatzmaier 1984, 1985a,b, Chan et al. 1982, Nordlund and Stein 1989, Brandenburg et al. 1990b). Although models which describe situations close to reality are not achieved till now these investigations will be promising for the future with more and more powerfull computers (Krause 1991).

In the following we will present an overview of the results which have been achieved by the research of stellar dynamos. Most of them have been gained by using the fact that the backreaction of the magnetic field on the motion - the Lorentz force $j \times B$ - is of second order. Consequently, it makes sense first to consider the kinematical problem, especially, if the question for the origin is raised. From that platform a certain insight in the highly nonlinear dynamo problem has been elaborated in the recent years.

THE KINEMATIC PROBLEM

We will now specify the problem according to the natural conditions. As a model of a star we consider a sphere which consists of an electrically conducting fluid (ionized gas). It rotates about a certain axis. Due to gravity and the overall rotation the model shall show symmetry with respect to the equatorial plane and the axis of rotation in all non-magnetic properties. As far as it concerns turbulent quantities this requirement shall be fulfilled in the average.

The following considerations rest on the assumption of a prescribed velocity field. We study the fate of a magnetic field which exists at a certain time ($t = 0$) in the star for some reason by solving the initial value problem of the induction equation (4). We have so to determine a magnetic field $B(x, t)$ as a solution of the equations

$$\frac{\partial B}{\partial t} = curl\ (u \times B)\ + \eta \Delta B\ ,\quad div\ B = 0 \tag{7}$$

for $r < R$ and

$$curl\ B = 0\ ,\quad div\ B = 0\ ,\quad B\ = O(1/r^3)\quad if\ r \to \infty\ , \tag{8}$$

for $r > R$. r denotes the distance from the centre and R the radius of the sphere which represents the star.

We enter with the ansatz

$$B(x,t) = e^{\gamma t}\, B(x) \quad , \tag{10}$$

into the equations (7), (8). According to general mathematical rules we find a denumerable set of eigen modes which are characterized by a set of eigen values $\gamma_1, \gamma_2, ..., \gamma_n, ...$ and corresponding eigen fields $B_1(x), B_2(x), ..., B_n(x), ...$. The real parts of the eigen values γ_n, $Re\{\gamma_n\}$, describe growth, resp. decay of the eigen modes. Generally we have

$$Re\{\gamma_n\} \to -\infty \;\; if \;\; n \to \infty \tag{11}$$

i.e. nearly all eigen modes decay due to Ohm's dissipation. Only a finite number of them may have non-negative growth rates: in this case the system represents a dynamo. The imaginary part of a γ_n characterizes a periodic behaviour if unequal zero.

There are in astrophysics important types of motions like differential rotation and compression (e.g. due to accretion) which provide for amplification but not for dynamo excition. In this cases the system (7), (8) degenerates and we have to consider also the ansatz

$$B(x,t) \; = t\, e^{\gamma t}\, B(x) \quad , \tag{12}$$

in order to complete the system of eigen modes. For the motions mentioned above $R_e\{\gamma\}$ would be negative: the field decays, however, first it grows proportional to time.

Research in dynamo theory over the last three decennia revealed that basically any rotating cosmical object should be a dynamo in case it shows internal motions like convection. This statement is crucial: As a consequence the large scale part of a magnetic seed field grows exponentially and the cosmic object forms its own individual magnetic field, the memory of the seed fields got lost.

The ability of being a dynamo is due to the influence of the Coriolis forces on the internal motions. They get a helical structure where generally in a certain region one kind of helicity, either lefthanded or righthanded, dominates. For example, in the convection zone of the Sun are lefthanded helical motions dominating in the northern hemisphere and righthanded in the southern.

The induction action of these motions used to be described in the frame of mean-field magnetohydrodynamics. The correlated action of the small-scale fields u', B' is described by the turbulent electromotive force $\mathcal{E}$ defined by

$$\mathcal{E} = \overline{u' \times B'} \quad , \tag{13}$$

where the overbar denotes the average. Ohm's law for the mean fields reads so

$$j = \sigma(E + u \times B + \mathcal{E}) \quad , \tag{14}$$

where from now j, E, u, B describe the mean values of these fields.

The induction action of small-scale motions with helicity leads to a turbulent emf $\mathcal{E}$ of the form

$$\mathcal{E} = \alpha B - \beta curl\; B \quad , \tag{15}$$

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The first term on the righthand side represents the α-effect, i.e. an emf parallel to the magnetic field. The second one provides for an enhanced decay by turbulent diffusion. It describes the cascade of magnetic energy down to smaller and smaller scales due to the turbulent motions.

Relation (15) corresponds in this form to the idealized case of a homogeneous isotropic turbulence. α is connected with the helicity, the average of the scalar product of the fluctuating velocity field $\boldsymbol{u}'$ with its *curl*:

$$\alpha = -\frac{\tau_{cor}}{3}\,\overline{\boldsymbol{u}'\ curl\ \boldsymbol{u}'}\ , \tag{16}$$

where τ_{cor} denotes the correlation (overturn) time (Steenbeck and Krause 1969) and β is connected with the turbulence intensity by

$$\beta = \frac{\tau_{cor}}{3}\,\overline{\boldsymbol{u}'^2}\ . \tag{17}$$

In real conditions the dominance of one kind of helical motions is due to the stratification by gravity $\boldsymbol{g}$ and the influence of the rotation represented by the angular velocity $\boldsymbol{\Omega}$. According to these highly anisotropic conditions (15) has to be exchanged by the tensorial connection.

$$\mathcal{E}_i = \alpha_{ij}B_j + \beta_{ijk}\frac{\partial B_j}{\partial x_k}\ , \tag{18}$$

where the tensor α_{ij} has the form

$$\alpha_{ij} = \alpha_1(\boldsymbol{g}\cdot\boldsymbol{\Omega})\delta_{ij} + \alpha_2(g_i\Omega_j + g_j\Omega_i)\ . \tag{19}$$

Especially recent investigations revealed that the contributions by the α_2- term in (19) may be remarkable (Rädler 1983). From a discussion of an anisotropic diffusivity tensor we will refrain here.

Numerous investigations have been carried out of models which are based on the induction equation

$$\frac{\partial \boldsymbol{B}}{\partial t} = curl\ (\boldsymbol{u}\times\boldsymbol{B} + \mathcal{E}) + \eta\Delta\boldsymbol{B}\ , \tag{20}$$

where $\boldsymbol{u}$ describes motions like differential rotation and/or meridional motions and $\mathcal{E}$ the influence of turbulent convection according to (15) or (18), (19) or even more complicated. Often simply (15) combined with a differential rotation has been considered which is general named $\alpha\Omega$-dynamo.

The results show that generally a few growing eigen modes, i.e. those with positive $Re\{\gamma\}$, do exist. They differ especially with respect to their symmetry properties: They are either symmetric (even parity) or antisymmetric (odd parity) with respect to the equatorial plane and axisymmetric with respect to the axis of rotation or depend on the azimuth φ according to $e^{im\varphi}$. The symmetry type will accordingly be designed by Sm or Am. For the three most important types $A0$, $S0$, $S1$ the shape is illustrated in Fig. 2.

The Sun's magnetic field has, especially because of the polarity laws of the sunspots, to be classified as $A0$. For magnetic Ap-stars it is not clear whether or not the fields show a dominating symmetric part, however, if they do, they with high probality belong to the type $S1$ (Oetken 1977, 1979). Also for late type stars

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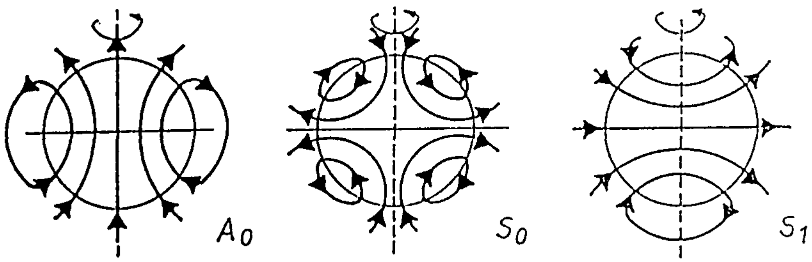

FIGURE II The three most important symmetry types of magnetic fields, which may be excited by a spherical dynamo showing symmetries with respect to the axis of rotation and the equatorial plane. In a multipole expansion the leading terms will be a dipole parallel to the axis of rotation for $A0$, a quadrupole for $S0$ and a dipole with its moment within the equatorial plane for $S1$.

the symmetry type is not clear, but significant deviations from the symmetry with respect to the axis of rotation seems to exist in some cases (Piskunov et al. 1990, Jetsu et al. 1990, Moss 1991).

Calculations of dynamo models within the kinematic theory show that fields of the three types $A0$, $S0$, $S1$ may have positive growth rates, or to say it in a different way, they take the places $B_1(x,t)$, $B_2(x,t)$, $B_3(x,t)$ within the sequence of eigen modes. So fields of these three symmetry types compete in being realized by a certain model, or a certain star.

In the frame of the kinematic theory one can find two marks for differentiating between the eigen modes: For the intensity of the induction action characterized by the dynamo number C exist a critical value, the marginal dynamo number C_{crit}, which is the border beyond of which the system is electromagnetically unstable. The related eigen mode, for which the growth rate $Re\{\gamma\} = 0$ if $C = C_{crit}$, we call the marginal mode. It is the mode which is easiest to excite. As a further mark we can use the largest growth rate. If C is in an immidiate neighbourhood of C_{crit} both these eigen modes are identical. However, for values of C which substantially exceed C_{crit} they may be different.

Our present findings allow for a solution of the initial value problem, i.e. we can describe the growth of a weak seed field. Clearly the eigen mode with the largest growth rate will dominate in a short time. That may be the marginal mode or not.

The actual magnetic fields we observe on the Sun and stars are clearly in a well developed state, in a state of saturation. This state cannot be described in the frame of a linear theory, we need information from the nonlinear investigations in order to understand the actual situation.

THE NONLINEAR DYNAMO PROBLEM

The treatment of the full dynamo problem needs the solution of the system of partial differential equations (1),.., (4), or, at least, the solution of the coupled system ,consisting of the Navier-Stokes equation and the induction equation. In order to avoid the mathematical difficulties simple modelling of the backreaction on the induction action of the motions have been introduced. The most common one is the so-called α-quenching, where a reduction of the α-effect is simply achieved by the ansatz

$$\alpha = \frac{\alpha_0}{1 + aB^k} \tag{21}$$

with a certain exponent k. In this way the induction action of the α- effect is reduced with the growing magnetic field.

Models of this kind allow the study of the extension of the eigen modes to the nonlinear regime in dependence on the dynamo number. The results allow for some general statements (Krause and Meinel 1989):

i The stability of a solution is decisive of which magnetic field will be maintained by the dynamo,

ii The extension of the marginal mode is stable in a certain neighbourhood of the marginal dynamo number,

iii The extension of a higher mode is first, i.e. in a neighbourhood of their bifurcation, unstable,

iv For values of the dynamo number that substantially exceed the marginal dynamo number bifurcations in the nonlinear regime lead to solutions of a complex structure.

According to (ii), (iii) the significance of the marginal mode is clearly to be seen.

To some extend the nonlinear extensions of the eigen modes possess the same symmetry properties, i.e. the extension is of even (odd) parity if the eigen mode is of even (odd) parity. The same is true for the symmetry with respect to the axis of rotation.

In this way the investigation of the linear problem provides insight in the symmetry properties of the nonlinear solutions, at least in some neighbourhood of the marginal dynamo number. The higher eigen modes are unstable, however, it may happen that some of them first dominate, because of their higher growth rates. Finally they decay thus demonstrating an inverse cascade of the energy (Gilbert and Sulem 1990). According to (iv) additional bifurcations leed to a break of symmetry in a certain distance from the marginal dynamo number and more irregular solutions will appear.

These findings allow for a possible scenario that predicts stable regular magnetic fields with certain symmetries (axisymmetry, equatorial symmetry) just beyond the marginal dynamo number, but for values of C that substantially exceed the marginal number, non-symmetric or even irregular fields have to be expected.

THE CONFRONTATION WITH OBSERVATIONS

The foregoing discussion revealed that it makes sense to investigate kinematic models even if the final state is investigated. Many models have so been considered (e.g. Roberts 1972, Roberts and Stix 1972, Yoshimura 1975a,b, Rädler 1980, 1986, see also the monographs Moffatt 1978, Krause and Rädler 1980, Zeldovich et al. 1983).

In connection with the explanation of the solar magnetic field many investigations concentrated on the $\alpha\Omega$-dynamo. In most cases an $A0$-type field was found as the marginal one. We thus understand the observed large-scale magnetic field of the Sun.

On the other side by changes of the internal structure also models have been found where a $S0$-type field, i.e. one with a quadrupol as leading term, are marginal (e.g. Roberts 1972). Such models may indicate that the magnetic field of the Sun not always during its history must have been of odd parity.

Furthermore, even models have been found, where fields of type $S1$ are marginal (Krause 1971, Stix 1971, Rüdiger 1980). These models provide the possibility to interprete the observational results concerning the magnetic *Ap*-stars in terms of dynamo theory (Krause 1983). Worth to note here that a field pattern as realized by meridional motions can excite a magnetic field of type $S1$ (Gailitis 1970, Moss 1990).

Very detailled investigations of spherical dynamo models have been carried out by Rädler (1980, see also Krause and Rädler 1980). Here stability maps are presented which provide insight how the magnetic fields of the different symmetry types become marginal in dependence on different influences like differential rotation, meridional motions and such turbulence parameters like α_1, α_2 in (18) and others.

A comparision with the solar magnetic field provides for a challenge to fit models best to the observed patterns. As well the modelling of the differential rotation as the different turbulence parameters provides for a diversity of possibilities.

However, things become more complicated since the differential rotation is also caused by the turbulent convection by the so-called Λ-effect (Rüdiger 1989). In this sense conclusive models require to determine all quantities like $\alpha, \alpha_1, \alpha_2, ...$ and those describing the Λ-effect out of the same root: the turbulent convection. In connection with reconciling the different aspects a number of difficile problems emerge (Brandenburg et al. 1990a).

Apart from theoretical determinations modern observations of solar oscillations provide for a possibility to derive the profile of differential rotation inside the Sun (Harvey 1988, Libbrecht 1988). In this way further constraints as well on the theory of the differential rotation as on the dynamo theory do now exists (Paterno, 1991). One misses the confirmation of the, already a long time existing forecasting of an increasing angular velocity with depth, which fits best to the polarity rules of the observed magnetic field. However, the oscillation data apparently allow for a radial jump of the angular velocity at the base of the convection zone. This already is sufficient for having the correct polarity relations and the migration of the toroidal field belts towards the equator (Steenbeck and Krause 1969, Roberts and Stix 1972, Belvedere et al. 1991).

BEYOND THE BREAK OF SYMMETRY

So far we discussed the dynamo from the view based on the linear theory, what means we considered the object in a state close to the dynamo instability, the realized field shows to a far extend the same properties as the marginal mode. In the following we will discuss results of nonlinear investigations and some relations to observational findings.

The break of symmetry at a certain dynamo number may appear in a way that the marginal mode of odd parity becomes unstable and the system migrates on a certain time scale to an even parity configuration, which again is unstable. This time scale proves to be much larger than the period of oscillation (i.e. the (solar) cycle period). The magnetic field has so the appearance of an odd parity oscillating field which changes after a large number of oscillations for some time the parity (Brandenburg et al., 1989a). A consideration of the energy of the excited field reveals a "grand minimum" just at the transition from one parity to the other (Brandenburg et al., 1989c). Hence one is tempted to relate this property of the model to periods of no, or minor solar activity in the past history, i.e. to the Maunder minimum.

Investigations of nonlinear models reveal a picture of high complexity (Rädler et al. 1990, Brandenburg et al. 1989b, Jennings and Weiss 1991, Jennings 1991). Jennings et al. (1990) found a model where the $A0$-type solution lose their stability to a mixed one with $A0$ and $S1$ contributions. So the possibility of modelling the observed sector structure on the Sun is feasible.

Apart from the observed field pattern there are characteristic observable parameters which may be studied in their mutual dependence. These investigations gain the more interest, since the observations of solar-type and late-type stars offer possibilities to study these dependences in a sample of objects. Those parameters are, for example, the rotational period, the cycle period and the intensity of the activity as a measure of the field strength of the excited magnetic field (Baliunas 1985, Schrijver 1991). It may so be possible in future to differentiate between different nonlinear models on a broad basis of observational data.

CONCLUSIONS

This is a try to present an insight in the development of the theory of dynamos in stars. It appears to be in a large part a well-ordered deductive theory, however, the more nonlinear processes are dominating and, consequently, nonlinear theory is needed for an appropriate description the picture becomes rather entangled. It is, indeed, the actual problem to build a passable street in the jungle of nonlinear phenomena by a cooperation of observation and theory.

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