THE JOURNAL OF SYMBOLIC LOGIC
Volume 88, Number 3, September 2023

THE AUTOMORPHISM GROUP OF THE FRAÏSSÉ LIMIT OF FINITE HEYTING ALGEBRAS—ADDENDUM

KENTARÔ YAMAMOTO

The original article [3] was erroneously published without an adequate proof of Theorem 3.6, which may be found in the following. The author expresses his gratitude to Dugald Macpherson for pointing out this gap that was present in an earlier draft of this article.

Recall that structures M_1 and M_2 sharing the same domain M but of possibly different languages are *definitionally equivalent* if subsets of M^n are 0-definable in M_1 if and only if it is 0-definable in M_2 for every $n < \omega$.

LEMMA 0.1. *Let M be a countable ultrahomogeneous structure. Suppose that M is a definitionally equivalent expansion of a bounded semilattice. If* $\mathrm{Age}(M)$ *has the superamalgamation property, then the ternary relation $\mathop{\smile\hspace{-0.9em}\mid\hspace{0.5em}}'$ among finite sets of M defined by*

$$A \mathop{\smile\hspace{-0.9em}\mid\hspace{0.5em}}_{B}' C \iff \langle AB\rangle \mathop{\smile\hspace{-0.9em}\mid\hspace{0.5em}}_{\langle B\rangle} \langle BC\rangle,$$

where $\mathop{\smile\hspace{-0.9em}\mid\hspace{0.5em}}$ is as in the original article, i.e.,

$$S \mathop{\smile\hspace{-0.9em}\mid\hspace{0.5em}}_{U} T \iff \forall a \in S\, \forall b \in T \begin{Bmatrix} a \le b & \implies & \exists c \in U\ a \le c \le b \\ b \le a & \implies & \exists c \in U\ b \le c \le a \end{Bmatrix},$$

and $\langle S\rangle$ denotes substructure generated by S, is a stationary independence relation in the sense of Tent and Ziegler [2].

PROOF. By [2, Example 2.2.1], the ternary relation $\mathop{\smile\hspace{-0.9em}\mid\hspace{0.5em}}'$ satisfies the axioms Existence, Invariance, and Stationarity. By the shape of the definition of $\mathop{\smile\hspace{-0.9em}\mid\hspace{0.5em}}'$, we have Monotonicity and Symmetry. It remains to show the axiom Transitivity. Since M is ultrahomogeneous and definitionally equivalent to its partial order reduct, whenever $A, B, C \subseteq M$ are finite, any minimal S witnessing the superamalgamation property for the diagram $\langle AB\rangle \hookleftarrow \langle B\rangle \hookrightarrow \langle BC\rangle$ belong to the same $\mathrm{Aut}(M)_{(B)}$-orbit. This orbit only depends on the orbit of A and that of B. Moreover, $A \mathop{\smile\hspace{-0.9em}\mid\hspace{0.5em}}_B' C$ if and only if the $\mathrm{Aut}(M)_{(B)}$-orbit O of AC is such that the orbit of S above is that of $\langle ABC\rangle$, which only depends on O. That $A \mathop{\smile\hspace{-0.9em}\mid\hspace{0.5em}}_{BC}' D$ and $A \mathop{\smile\hspace{-0.9em}\mid\hspace{0.5em}}_B' C$ imply $A \mathop{\smile\hspace{-0.9em}\mid\hspace{0.5em}}_B' D$ follows easily from this characterization. ⊣

With this preparation, Theorem 3.6 of the original article can be proved in the following manner, with a slightly stronger assumption.

0022-4812/23/8803-0016
DOI:10.1017/jsl.2023.15

THEOREM 0.2. *Let M be as in the lemma. If* $\mathrm{Age}(M)$ *has the superamalgamation property, and* $\leq$ *is dense, then the abstract group* $\mathrm{Aut}(M)$ *is simple. In fact, for any nontrivial* $g \in \mathrm{Aut}(M)$, *every element of* $\mathrm{Aut}(M)$ *is the product of at most* 16 *conjugates of* g *and* g^{-1}.

PROOF. Let $g \in \mathrm{Aut}(M)$ be nontrivial. Then $\operatorname{supp} g$ is infinite by density. Indeed, take $a \in M$ such that $g(a) \neq a$; the interval $(a \wedge g(a), a)$, which is infinite by density, is included in $\operatorname{supp} g$. One can then see that there is no type over a finite set whose set of realizers is infinite and fixed pointwise by g. This follows from $|\operatorname{supp} g| = \aleph_0$ by arguing in the same manner as in [1, Corollary 2.11]. Finally, since if $A \mathop{\smile\!\!\!\!\!\!\mid}_X{}' B$, $X' \subseteq X$, and $(A \cup B) \cap X \subseteq X'$, then $A \mathop{\smile\!\!\!\!\!\!\mid}_{X'}{}' B$, by [2, Lemma 5.1], the claim follows. ⊣

REFERENCES

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POD VODÁRENSKOU VĚŽÍ 271/2
LIBEŇ
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THE CZECH REPUBLIC
E-mail: yamamoto@cs.cas.cz