II. RADIO EMISSION MECHANISM

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THE SLOT GAP MODEL OF PULSARS

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ABSTRACT

A general picture of pulsar magnetospheres is outlined which suggests polar current flow should proceed within a magnetic flux tube with conducting boundaries. The dynamics of this current flow is considered, including the effect of formation of pair plasma on the potential. A slot gap surrounding the stream of pair plasma is described, and the electric fields and $\underset{\sim}{J}\cdot\underset{\sim}{E}$ work done in the gap are calculated. The inapplicability of curvature emission to the formation of γ-ray pulses is discussed. The dynamics of pair plasma flow is described, including electrostatic trapping of low energy positrons, polar cap bombardment and heating, and the emission of soft X-rays from the cap. The relation of this current flow and polar cap heating to the formation of fluctuations on the radio subpulse time scale is also described. The importance of the boundary layer between the pair plasma and the slot gap is illustrated by calculations of the boundary layer structure and microscopic instability, which show that this region may be the source of pulsar emission.

I will describe a number of quantitative results and qualitative aspects of work in progress on the "slot gap" model for the emission regions of a pulsar, proposed by Arons and Scharlemann (1979, hereafter AS) and by Arons (1979b, hereafter I). The results and ideas concern the dynamics and emission physics of relativistic plasma and current flow along the polar field lines of an isolated, rotating magnetized neutron star with the axis of its dipole moment oblique to the rotation axis - a pulsar. The quantitative modelling is confined to the physics within a polar flux tube and is therefore "local". Full detail will be given in papers now being written. However, development of this type of model requires the use of boundary conditions which follow from a qualitative

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view of the whole magnetosphere. Because this view involves some qualitative ideas not widely discussed in the literature, I describe it briefly.

I. BOUNDARY CONDITIONS FOR POLAR FLOW

Let the star's angular velocity be $\Omega_* = 2\pi/P$ pointed along the +Z axis of an inertial frame, with P = rotation period. I assume the electromagnetic fields dominate the energy density at least through radii $\gtrsim c/\Omega_* = 48000$ P km except possibly in small singular regions of the magnetic field. This is equivalent to assuming the rate of loss of rest mass satisfies $\dot{M} \ll \Omega_*^4 \mu^2/c^5$, where μ = magnetic moment. I assume the magnetic moment subtends an angle i with respect to Ω_*, with $\theta_d \ll \tan i \ll \theta_d^{-1}$, $\theta_d = (\Omega_* R_*/c)^{1/2} = 1°(R_{10}/P_{600})^{1/2} \cong$ opening angle of the polar flux tube at the stellar surface. Here, R_* = stellar radius = $10\, R_{10}$ km, $P_{600} = P/600$ ms. The rotation of the magnetic field creates a displacement current of magnitude $(\Omega_* r/c)(B/P)$ whose sense and magnitude is to distort the magnetic field into the topology shown in Figure 1 at radii $\sim c/\Omega_*$. I assume the star does supply a quasi-neutral plasma along the field lines from the poles, with density $\gg B/Pce$, to the region of field reversal shown, and that the global conduction current density associated with this plasma has a sense and magnitude similar to the displacement current.

This will turn out to be the case as a consequence of the theory of plasma supply by pair creation outlined below. I explicitly assume magnetic dissipation energizes particles in the vicinity of the neutral points, so as to cause a flow of electrons and precipitating plasma across B into the formally closed regions. The sense of this flow is consistent with the field line pattern shown in Fig. 1, but will occur only if particles are accelerated to energies higher than are found in the creation of the pair plasma at low altitude (see below). The observation of pulsed γ-rays of energy $> 10^{11}$ eV from the Crab pulsar (eg., Grindlay et al. 1976) is consistent with the existence of this accelerator, since magnetic conversion prevents the escape of such photons from the inner magnetosphere and these pulses are not in phase with the rest of this pulsar's emissions.

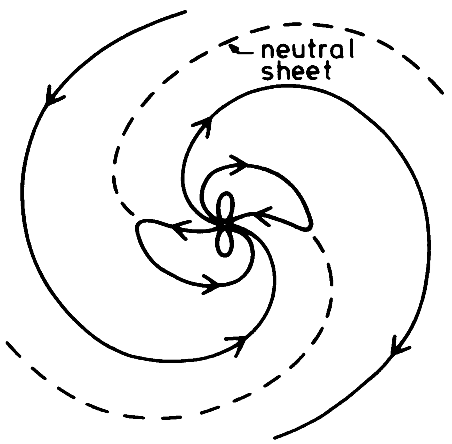

Fig. 1: Field lines of a vacuum-like rotator, in the rotational equator. Ω_* points out of the paper.

This electron inflow and plasma precipitation will be called the return current, and is needed to maintain a quasi-steady state in the

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corotating frame. Such precipitation may involve either direct flow from the neutral point, or radial diffusion in the transition region between fully closed and fully open zones; these field lines are then an "auroral zone", as in the terrestrial magnetosphere. The return current, driven by magnetic dissipation, requires a primary current to match it. The existence of a quasi-steady state is assumed; pulsars which show prolonged nulls may be in their null state when the current flow is off.

This view is obviously related to previous pictures of the magnetosphere, especially those of Sturrock (1971) and of Kennel et al. (1979). So far as I know, however, the essential point here is new, except for a brief mention in Paper I. This is that the displacement current of the oblique rotator puts the field into an essentially open configuration with (almost) singular points and lines, independent of the global plasma stress. This essential fact leads to several related points which serve to distinguish the physics likely in the oblique rotator from what can occur in the aligned rotator models popular in formal pulsar theory.

(1) So long as plasma is supplied in sufficient amounts to the neutral point region, this limited zone can be energized by magnetic dissipation, even if the plasma pressure is otherwise small, simply because the distorted B field is part of the vacuum (non-potential) field. Unpublished work indicates that tearing mode dissipation may occur in this zone; whether this is sufficient to do the required heating is unknown. However, one could approach the construction of a global model by assuming plasma stress is unimportant except in small zones (represented by current sheets), then proceed by using the magnetic field of the vacuum rotator as the initial guess for a relaxation scheme, with the plasma current supply theory as in § II and III below.

(2) The conduction current loop closes near the star, in this picture. In the aligned rotator, this would mean no torque. In the oblique rotator, this is no problem, since the conduction current modifies B, which modifies the displacement current and thus modifies the angular momentum loss. In essence, conduction currents and displacement currents are a linked current system, with conduction current important interior to $r \sim c/\Omega_*$ and displacement current more important in the exterior region. Of course, an external conduction current system can be significant also.

(3) Current sheets with dissipation are likely to be unsteady, even as observed in the corotating frame (Kennel et al. 1979). This leads to fluctuations of the torque with time scale $\sim P$, when such "substorm" activity is in progress.

The magnetosphere always tries to reconstruct the neutral sheet, since it exists in the "vacuum" field, suggesting a quasi-steady flickering may be the appropriate state, so long as the system is far from being aligned (such flickering of the torque is of obvious relevance to fluctuations in $\dot{P}$, for which observational evidence exists (Helfand et al. 1980); another mechanism for torque flickering, asso-

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ciated with subpulse formation, is outlined below). When the object is close to alignement, the distortions imposed by displacement current disappear and other, more quiescent states are possible (cf Michel and Pellat 1980). The transitions between such states offer a possible explanation for the long nulls observed in the emission of long period pulsars close to cut off (Ritchings 1976).

The above scenario assumes the star does supply a quasi-neutral plasma to the outer magnetosphere. The only way I know how it might do this is through pair creation in relatively large electric fields parallel to B above the polar caps.

II. PAIR CREATION ABOVE PULSAR POLAR CAPS: STARVATION ELECTRIC FIELD AND SLOT GAPS

The above scenario implies the polar cap region to be a flux tube surrounded by dense plasma precipitating from above along the "auroral" field lines. As a simple working hypothesis, I assume these transition field lines can be represented as perfect conductors due to the dense plasma falling from above. The polar flux tube contains the outward flow of electrons needed to close the current loop. Since gravity is large ($g \sim 10^{14}$ cm s^{-2}) and the surface is cold ($T_* <$ a few x 10^6 K, Giacconi 1979), this current must be formed by electrical acceleration of electrons emitted from the surface (thermionic emission supplies the particles, given the temperature found below). Even if the outer magnetosphere is temporally variable in the corotating frame, the transit time for relativistic flow over distances $\sim$ length of the pair creation region (length $\sim R_*$, transit time $\sim$ 30 μs) is short compared to the variability time for the global structure ($\sim P/2\pi$). This fact allows a fairly quantitative development. The basic picture of what happens is shown in Figures 2 and 3. Above the surface marked PFF = "pair formation front" in Figure 2, γ-rays (emitted by ultrarelativistic electrons from the star) form a dense $e^\pm$ plasma. Positron bombardment of the polar caps, combined with the ability of the surface atmosphere to emit particles freely and bring $E_\parallel = \underset{\sim}{E}\cdot\underset{\sim}{B}/B$ to 0 at the surface, enforce the extraction of an electron beam of flux $F_e = -f\,\eta_R(R_*)\,ce^{-1}(B/B_*)$ with B_* = surface magnetic field, $1 \leqq f \lesssim 3$ (f varies with position over the cap) and $\eta_R = -\underset{\sim}{\Omega}_*\cdot\underset{\sim}{B}/2\pi c$ is the corotation charge density (if the only backflow of positrons is due to pair creation <u>below</u> the PFF, $|f-1| << 1$, as shown by Fawley et al. (1977) and more generally by Arons (1980c); when the $e^\pm$ flow includes positrons trapped at higher altitudes, $f-1 \sim 1$ but the charge density in the region below the PFF is still very close to η_R). The origin of this $E_\parallel$ is the progressive "starvation" of the part of the polar flux tube which bends toward the rotation axis; in the pure $e^\pm$ beam region, the charge density η varies ∝B while $\eta_R \propto B_z = \underset{\sim}{B}\cdot\underset{\sim}{\Omega}_*/\Omega_*$, with the result that $\eta/\eta_R < 1$ above the surface. This starves the tube of sufficient charge to completely wipe out the vacuum EMF, leaving a residual "starvation electric field" (Arons 1979a, 1981). If the PFF (a surface of $E\cdot B \cong 0$ also) has a specified (arbitrary) location in the long skinny flux tube, the potential Φ whose gradient

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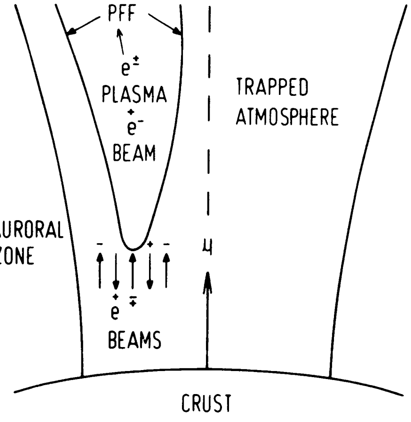

Fig. 2: Cross-section of the starvation zone in the Ω_*-μ plane

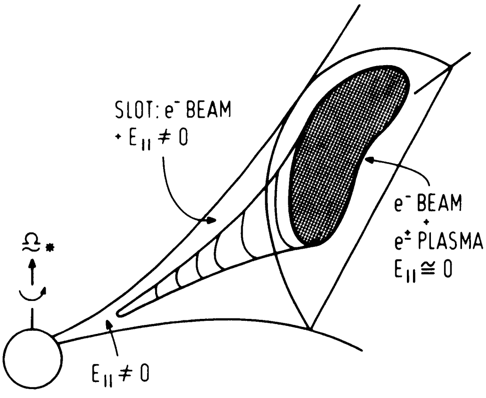

Fig. 3: Slot gap at high altitude in the favorably curved flux tube

gives the departure from the corotation electric field, and which satisfies the boundary conditions $E_\parallel = 0$ on the stellar surface and on any surface $H = H(\tilde{\omega},\phi_*)$, is

$$\Phi = 2.7\times10^{13}\,\mu_{30}\,P_{600}^{-2}\,(\theta_d f_\rho)\,\sin i\,\sin\phi_*\,\frac{\tilde{\omega}}{\tilde{\omega}_c}\left(1-\frac{\tilde{\omega}^2}{\tilde{\omega}_c^2}\right)\left[\left(\frac{r}{R_*}\right)^{1/2}-1-F\right]\ \text{Volts.} \tag{1}$$

Here μ_{30} = magnetic moment in units of 10^{30} cgs, $\tilde{\omega}$ = local cylindrical radius with respect to the magnetic axis, $\tilde{\omega} = \theta_d R_*(r/R_*)^{1/2}$ = boundary radius of the polar flux tube, ϕ_* = magnetic azimuth, f_ρ = ratio of dipole radius of curvature to actual radius of curvature; $1 \leqq f_\rho \leqq \theta^{-1}$, and F is a messy integral over angles, heights and the location of the PFF. f_ρ is unknown, but may be subject to observational investigation (Paper I). I have adopted the simple fudge factor $f_\rho \geqq 1$ to represent more realistic models of the surface magnetic field (Barnard and Arons 1980) whose distribution of pair creation is similar to that found here.

Since $\Phi = 0$ at the stellar surface, no γ-rays are emitted at the surface itself. At sufficient altitude, $\Phi \geqq 1$ TV, and GeV γ-rays are emitted by the curvature process. Above a critical energy $\propto \Phi^3$, the emission spectrum is exponentially cut off, but because $|\eta-\eta_R| \ll |\eta_R|$ in the starvation zone, it is possible to show that $E_\parallel$ is poisoned by the magnetic conversion of photons emitted on this exponential tail. Direct calculation (Arons 1980b) shows that the number of pairs created by a single electron accelerating in (1) is $\kappa(s) \propto s^6 \exp[-(\mathrm{const}/s^4)]$, where s = height of the electron above the star. This expression applies for $0 \leqq s \leqq (4/3)H$, H = height of the PFF. The pair density changes from essentially 0 to >> beam density in a thin layer. The height above H where $E_\parallel = 0$ is fixed by requiring the starvation zone to

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merge smoothly into the "force free" region above H, or $\nabla_{\parallel}\cdot E_{\parallel} = 0$ at $s = H$. This requires the trapping of only a small number of e^+ below the PFF; I find (Arons 1980b) $\kappa(H) \cong 10^{-4} P^{-1} R_{10}$ positrons are trapped below the PFF for each primary electron, so as to change the charge density to the value required by $E_{\parallel} \cong 0$ for $s \geq H$.

A simple way to see what is going on is to note that the potential is poisoned when the critical energy for curvature emission $\varepsilon_\gamma \propto \Phi^3$ at some emission point s_e has risen to a fixed fraction (~ 0.1) of the photon energy $\varepsilon_{abs}(s_e)$ such that a photon emitted at s_e reaches optical depth unity at $s = (4/3)s_e$ (expressions for ε_γ and ε_{abs} are given in AS and in Arons 1980d). This procedure yields essentially the same result as obtained by the more rigorous method outlined above, and yields an equation for the PFF in the form

$$\left(\frac{\tilde{\omega}_p}{\tilde{\omega}_c}\right)^5 \left[1 - \left(\frac{\tilde{\omega}_p}{\tilde{\omega}_c}\right)^2\right]^3 \sin^3\phi_* = \text{const.} \left(\frac{P}{P_D}\right)^{17/2} g(H,\tilde{\omega}_p,\phi_*) \ . \tag{2}$$

(2) turns out to be easier to solve in the form $\tilde{\omega}_p = \tilde{\omega}_p(H,\phi_*)$, and const. really is a slowly varying logarithmic function of P, μ, etc. The preliminary model in AS used a form for g which was independent of $\tilde{\omega}_p$ itself, an approximation which does not correctly describe the shape of the surface. In (2),

$$P_D = 1^s\!.7\ \mu_{30}^{8/17}\ (f_\rho/50)^{10/17}\ R_{10}^{-3/17}\ (\sin i)^{6/17} \tag{3}$$

and is approximatively twice the maximum period at which the model has sufficient opacity to form a dense pair plasma at all. P/P_D is the basic parameter of the theory, since its value fixes the width of the asymptotic starvation slot around the pair plasma shown in Figure 3, which in turn controls the amount of energy available for excitation of the various emission processes in the model. P/P_D can be reexpressed in terms of the observables P, $\dot{P}$, the moment of inertia and the unknown f_ρ by using the vacuum angular momentum loss rate for the vacuum rotator, roughly consistent with § I. Then

$$\frac{P}{P_D} = 0.4(I/10^{45}\,\text{g cm}^2)^{-4/17}\,(\dot{P}/10^{-15}\,\text{ss}^{-1})^{-4/17}\,P_{600}^{13/17}\,R_{10}^{3/17}\,(f_\rho/50)^{-10/17} \tag{4}$$

Above heights $s \sim R_*$ (radii $\sim 2R_*$), the magnetic field runs out of opacity, leaving a slot gap around the tube of pair plasma. The flow of the pair plasma proceeds with $E_{\parallel}$ << in the slot gap. In addition, the pair plasma has a density which varies strongly with distance across the magnetic field, which causes substantial variations of $E_{\parallel}$ within the plasma. The result is the model for the plasma flow at radii $r \gg R_*$ shown in Figure 4. My task is now to peel this onion and describe the different physics in each region.

The slot gap is the simplest zone. Here, solution of the free

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boundary value problem describing the PFF yields the asymptotic gap width

$$\delta\,\tilde{\omega}_g = 6.4\, R_{10}^{3/2}\, P_{600}^{-1/2}\, (r/100\, R_*)^{3/2}\, (P/0.3\, P_D)^{5/3}\ \text{km}\ . \qquad (5)$$

In the "corners" $\psi \sim 125°, 235°$, $(P/P_D)^{5/3}$ is replaced by $(P/P_D)^{5/6}$. If no dissipation occurs in the boundary layer (the "sheath"), a perfect conductor model for the potential in the gap yields the minimum energy released in $\tilde{J}\cdot\tilde{E}$ work by the starvation electric field. The current flow in this zone has $f = 1$, and the only photon emission process possible (with a stable sheath) is curvature emission. The total $\tilde{J}\cdot\tilde{E}$ work is then given by

$$\pi \frac{d\lg}{d\psi} = \frac{2}{3}\, \Omega_*^4\, \mu^2\, c^{-3}\, \sin^2 i \left[0.03 \left(\frac{\Omega_* r}{c}\right)^{1/2} \cot i\right] f(\psi) \left(\frac{P}{P_D}\right)^{\beta} . \qquad (6)$$

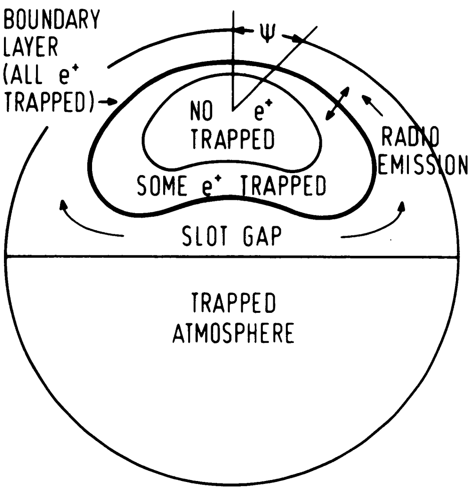

Fig. 4: Cross-section of the plasma flow at high altitude

The power β is 5 for $|125° - \psi| \gg (P/P_D)^{5/3}$. The variation with ψ is shown in Figure 5; most of the work is done in the "corner" $\psi \sim 125°$ (with the high altitude field represented as a pure dipole, the system is mirror symmetric across the plane $\psi = 0, \pi$). Clearly, $(P/P_D)^{\beta}$ is the efficiency of conversion of stellar spindown energy loss into the energy of accelerated electrons in the slot.

For parameters appropriate to the Crab pulsar, this model, with infinitely conducting boundary conditions, is short of enough total energy to explain the pulsed X-ray and γ-ray observations, by about a factor of 3. For Vela, the total particle acceleration power is sufficient to explain the γ-ray emission, but the efficiency of conversion into curvature radiation is too small to explain the total emission (see Kanbach et al. 1980 for data). This failure is fortunate for the general picture (although it shows that the model with perfectly conducting boundary conditions cannot explain the data). Despite the fact that the slot gap localizes the acceleration to a thin sheaf of field lines and delivers the energy at high altitude, both desirable properties for a gamma ray pulsar model (Arons 1981), curvature gamma rays are emitted over a wide range of radii. Due to field line curvature, an observer then sees photons for a wide longitude range, giving rise to broad modulation instead of sharp

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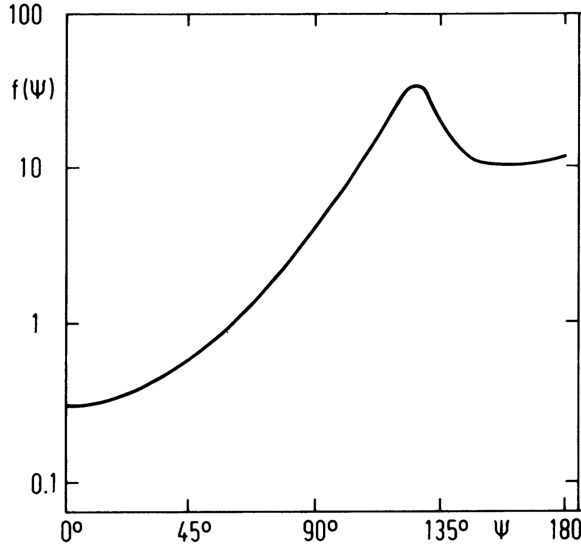

Fig. 5: Particle acceleration efficiency as a function of ψ around the slot gap. Here P = 0.3 P_D.

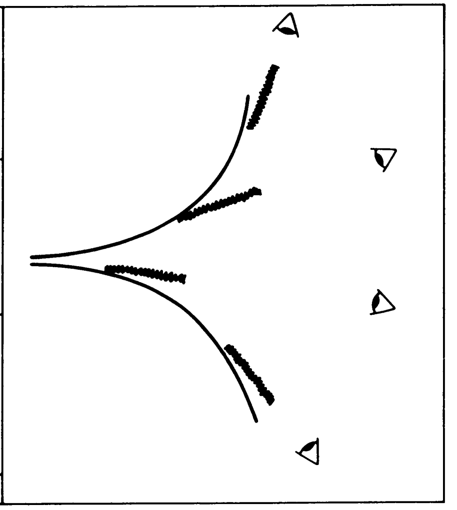

Fig. 6: Accessibility of emitting field lines of the slot gap to a distant observer

pulses as shown in Figure 6. While such properties may be of interest in modelling some of the unidentified COS B sources, they do not explain the Crab and Vela pulsars. Nor does the curvature spectrum match the steepness of the observed spectra. This situation is a general problem of all the curvature models proposed so far (see, for example, Fig. 2 of Ayasli and Ögelmann 1980), while "surface" curvature emission models, such as that of Salvati and Massaro (1978), have an upper cut off in the spectrum due to magnetic conversion below the energies of photons seen from Vela. They also have great difficulty in producing enough total energy. All this points to the gamma rays being a consequence of another mechanism, to which I return below.

III. FLOW OF PAIR PLASMA: SOFT X-RAYS FROM THE POLAR CAPS AND SUBPULSE FLUCTUATIONS

The results described in the last section are largely complete and in the process of being written up. The work described in this and the next section is still in progress; therefore, my discussion is preliminary.

Asymptotically, the PFF becomes a flux tube containing the outflowing pair plasma. Once an emitting electron crosses the PFF, the electric field is greatly reduced. Within heights $H < s \lesssim R_*$ where the magnetic field still has large conversion opacity, the electric field in the plasma can be neglected. Then the spectrum of pairs created on each

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field line within the flow tube is easy to compute, either by the method of generations used by Tademaru (1973) or by the method of air shower theory (Arons 1980e). The results are easy to understand. On field lines interior to the boundary layer between plasma and gap, curvature emission makes pairs until the emitting electron passes into the optically thin part of the magnetosphere. The gamma rays have a broad spectrum, being exponentially cut off above the energy

$$\varepsilon_\gamma(\Phi_{PFF}) = 550\, \mu_{30}^{-.12}\, P_{600}^{-1.4}\, R_{10}^{2.2} \left(\frac{f_\rho}{50}\right)^{.1} (\sin i)^{-.67} \left(\frac{\tilde{\omega}_c}{\tilde{\omega}}\right)^{9/4} \left(1 - \frac{\tilde{\omega}^2}{\tilde{\omega}_c^2}\right)^{9/4} \times (\sin \phi_*)^{9/4}\ \mathrm{MeV}\ . \tag{7}$$

This energy decreases as the boundary layer is approached. All the photons emitted at radius r_e with energy $> \varepsilon_\infty(r_e)$ are absorbed, with

$$\varepsilon_\infty = 20\, \mu_{30}^{-1}\, P_{600}^{1/2}\, R_{10}^{5/2}\, (50/f_\rho)\, (r_e/R_*)^{5/2}\, (\tilde{\omega}_c/\tilde{\omega})\ \mathrm{MeV}\ . \tag{8}$$

Since the pairs radiate their perpendicular momenta as soon as they are created, the final particle spectra have zero energy of gyration, but have a large momentum dispersion parallel to B. The spectrum is fairly flat, with momentum distribution function $\propto p^{-4/15}$ between $p_{min} = \varepsilon_\infty(H)/2c(1 + K(H))^{1/2}$ and $p_{max} = \varepsilon_\gamma(H)/2c(1 + K(H))^{1/2}$, $K \sim 1 - 10$. Above p_{max} and below p_{min}, the spectra are exponentially cut off. The total number of pairs created by each electron of the primary beam through absorption of curvature photons is

$$\kappa_c \cong 4 \times 10^4\, \mu_{30}\, P_{600}^{-3}\, R_{10}\, (f_\rho/50)^2 \sin i \left(\frac{P}{0.3\, P_D}\right)^{1.6} \left(\frac{\tilde{\omega}}{\tilde{\omega}_c}\right)^{1/6} \times \left(1 - \frac{\tilde{\omega}^2}{\tilde{\omega}_c^2}\right)^{3/2} (\sin \phi_*)^{3/2}\ G\ . \tag{9}$$

G is a cut off function which reduces the pair density exponentially with distance from the axis of the plasma flow, for field lines close to the boundary layer between plasma and gap, while deep in the plasma interior, $G \cong 1$. In the plasma interior ($\tilde{\omega}_c - \tilde{\omega} \gg \delta\tilde{\omega}$), there is sufficient energy in the synchrotron photons emitted by the pairs as they lose their perpendicular momentum to create more pairs. This leads to a total multiplication factor κ typically ~ 10 times greater than κ_c (in a special case like the Crab pulsar, this factor is $\sim 10^2$, leading to a total particle flux $\sim 10^{39}$ pairs/s from this object). At low energy, these tertiary particles give a distribution function $\propto p^{-3/2}$.

This plasma is "hot", since the spectrum extends from a few MeV/c to a few GeV/c. Since it is quite dense, one expects $E_\parallel$ to be very small, at least in comparison to the starvation $E_\parallel$ outside the PFF. However $E_\parallel \neq 0$. Just as in the starvation zone, the curvature of the field lines requires the charge density to vary in a manner not achiev-

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able by steady flow of particles along B at constant speed. Then $E_\parallel$ develops so as to decelerate one species and accelerate the other always so as to make the charge density more negative (closer to η_R) than would be the case for $E_\parallel = 0$. If the momentum dispersion were small (<< momentum), the resulting "supersonic" flow would be that discussed by Scharlemann (1974), in which $E_\parallel > 0$ decelerates e^- and accelerates e^+. In the present hot ("subsonic") flow, this type of solution applies to the central high density region of Figure 4, a region which vanishes when κ is sufficiently small. In the lower density regions, the hot plasma instead has $E_\parallel < 0$ and forms the necessary excess of electrons by trapping the lowest energy positrons, with the flow regions being joined by a curious form of Debye sheath. For small κ, this solution occupies the whole polar cap. The plasma in this region then consists of untrapped e^+ and e^-, as well as e^+ trapped in the ambipolar potential which supports $E_\parallel \neq 0$ in the plasma. The trapped e^+ flow back down to the stellar surface, providing the positron flux accelerated through the starvation zone below the PFF.

The total number of positrons trapped on each field line depends on the maximum value of the ambipolar potential as well as the distribution function. I use a waterbag model for the distributions and adopt a very simple view of the outer magnetosphere, in which η_R remains proportional to $\underset{\sim}{\Omega}\cdot\underset{\sim}{B}$ alone, but the poloidal field lines are taken to be straight for $r \gtrsim r_A \cong R_*/\theta_d^2 = c/\Omega_*$. For the regions of the flow where $\kappa < p_{max}/p_{min}$, one then finds a positron flux incident on the stellar surface of

$$F_+^{(-)} \cong 10^{22}\, \mu_{30}\, P_{600}^{-2}\, R_{10}^{-3} \cos i \sin i \quad cm^{-2}\, s^{-1} , \qquad (10)$$

which greatly exceeds the flux of positrons trapped in the starvation zone below the PFF.

The bombardment of the polar cap by TeV positrons creates soft X-ray emission from the cap, since the positrons do not give up their energy until they pass through more than $10\,g\,cm^{-2}$ of crustal material (corresponding to a physical depth $\gtrsim$ 10 microns). When the whole polar cap is exposed to the flux (10), the polar cap X-ray luminosity is

$$L_{cap} \cong 4\times 10^{28}\, \mu_{30} \left(\frac{3\ sec}{P}\right)^{2.3} \left(\frac{100}{f_\rho}\right)^{0.3} R_{10}^{0.9}\, (\sin i)^{1.2} \cos i \ erg\, s^{-1} \quad (11)$$

with effective temperature

$$T_{eff} \cong 10^6\, \mu_{30}^{1/4} \left(\frac{3\ sec}{P}\right)^{0.32} \left(\frac{100}{f_\rho}\right)^{.08} R_{10}^{-.53}\, (\sin i)^{.3}\, (\cos i)^{.25}\ K . \quad (12)$$

(11) applies only to objects which have $\kappa < p_{max}/p_{min}$ everywhere, which occurs only for long period, weak field stars. For shorter periods, the no trapping zone in Fig. 4 appears and covers an area of the cap

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which increases with decreasing P/P_D, reducing the rate of increase of L_{cap} below that predicted by (11).

Several other effects related to this positron backflow are important and of possible observational interest.

(1) Because the number of pairs created by any primary lepton introduced into the starvation zone is very small, the bombardment of the polar cap by the relatively high flux of e^+ does not create a dense plasma at the stellar surface which disrupts the extraction of the electron beam. On the contrary,

(2) the extra positive charge of the positrons is added to the effective positive charge density of the vacuum $-\eta_R(R_*)$ and causes the extraction of more electrons than is the case if no e^+ backflow occurs (in the equilibrium models, the electron current density is between 2 and 3 times the no backflow value $\eta_R(R_*)c$). Thus, the total current density is explicitly tied to the nature of the electric fields in the outer magnetosphere, which determine the amount of e^+ trapping, and time dependence of these electric fields (in the corotating frame) introduces time dependence into the conduction current component of the torque. Indeed,

(3) time dependence seems to be an intrinsic feature of the current flow in the e^+ backflow zone. The time scale for the electric field to cause the electron emission to respond to a variation of $F_+^{(-)}$ is short, $\sim$ transit time over a polar cap height $\sim 0.6\ P_{600}^{-1/2}\ R_{10}^{1/2}$ μs. The time for the change in the positron flux $F_+^{(-)}$ to form in response to the change in the injection rate is long, $\sim$ transit time to radii $\sim c/\Omega_*$ or times $P/2\pi$. This mismatch in the response times can lead to intrinsic instability in the current flow, with time scale for current fluctuations $\sim P/\pi$ and amplitude of fluctuation $\delta J_{\parallel} \sim J_{\parallel}$ itself. This fluctuation causes both fluctuations in the torque and variations in the beaming direction of photons radiated parallel to the polar magnetic field whose magnitude and time scale of variation is just right to explain the fluctuations seen as subpulses. It is easy to show that the polar field lines at radius r change their latitude and longitude with respect to the star by an amount $\Delta\theta \sim 2f(\Omega_* r/c)\delta J_{\parallel}/J_{\parallel}$ where $f = J_{\parallel}(R_*)/\eta_R(R_*)c \sim 2$. At the altitudes appropriate for the radio emission ($\sim 10^2\ R_*$, Cordes 1979 and below) and with $f \sim 2$, $\delta J_{\parallel}/J_{\parallel} \sim 1$, then $\Delta\theta \sim 10° - 20° \sim$ width of pulse wave forms. The fact that the subpulse longitude itself has a longitude width much less than that of the waveform will be attributed below to the emission region being the thin boundary layer between the plasma and the slot gap. Then the fact that one sees ~ 1 narrow subpulse within the pulse window at any time is explained by our looking at the instantaneous position of the boundary layer, while the change in beaming angle by amounts $\sim \Delta\theta$ during one rotation period is explained by this being the time for the current to vary due to the backflow instability. If correct, this model predicts that variations in L_{cap} should occur on the subpulse variation time scale $\sim P$, in this scheme where subpulse variation is beaming fluctuation of a <u>thin</u> emitting layer. The model also appears to be rich enough to encompass regular marching of subpulses, as a coherent limit cycle of the current flow.

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IV. PLASMA-GAP BOUNDARY: INSTABILITY AND PHOTON EMISSION

The very hot plasma filling most of the flow tube appears to be microscopically stable, since the distribution functions never separate substantially. Thus, 2-stream instability of the type proposed by Cheng and Ruderman (1977) does not occur (the same calculations of the plasma distribution function as described here apply to their model too, so that the plasma flow model assumed by Cheng and Ruderman for their radiation theory is not a consequence of their electrodynamic model). However, the model illustrated in Figure 4 does have a region where free energy is made available for microscopic plasma instability and consequent radio and (in special cases) high frequency emission. This is the boundary layer, where the secondary plasma density drops exponentially with increasing distance across B from the center of the plasma flow and the electric field rises to the much larger values present in the slot gap. The basic effects are due to the fact that the plasma must shield itself against a large non-corotation electric field $\tilde{E}_\perp$ across B whose magnitude changes with height. In the conducting gap model (valid if the voltage across the boundary layer is small compared to that in the gap), the plasma must form a surface charge to shield itself against

$$E_\perp = -5\times 10^6 \mu_{30} P_{600}^{-2} (1000 \text{ km/r}) (P/0.3 P_D)^{5/3} \text{ Volts/meter}, \quad (13)$$

directed into the plasma. $E_\perp r^{3/2}$ increases with r, since the slot gap is a zone of progressively increasing starvation. Therefore, $d(E_\perp r^{3/2})/dr \neq 0$ implies a weaker $E_\parallel \neq 0$ which acts to decelerate and trap secondary positrons, forming a completely charge separated layer of accelerating ultrarelativistic electrons at the boundary of the flow tube. The plasma creation theory outlined above allows one to determine the injected and trapped particle flux on each field line through integration of Poisson's equation, thus simultaneously determining the potential, electron density and thickness of the sheath. Like everything else, the flow is inhomogeneous, with the highest density of electrons in the lowest voltage regions of the sheath. The average density and voltage are easily given, and are (in the "corners" $\psi \sim 125°, 235°$)

$$\langle\Psi\rangle = \langle\Phi - \Phi_{PFF}\rangle = 3\times 10^9 \mu_{30} P_{600}^{-2} (P/0.3 P_D)^{5/2} (\Omega_* r/c)^{1/2} \text{ Volts}, \quad (14)$$

$$\langle n_e\rangle = 10^9 \mu_{30} P_{600}^{1/2} R_{10}^{-5/2} (P/0.3 P_D)^{5/6} \sin i \, (R_*/r)^{5/2} \text{ cm}^{-3}, \quad (15)$$

and sheath thickness

$$\delta\tilde{\omega}_s \cong 8 P_{600}^{-1/2} R_{10}^{3/2} (r/100 R_*)^{3/2} (P/0.3 P_D)^{5/24} \text{ km}. \quad (16)$$

($-e\langle n_e\rangle \delta\tilde{\omega}_s$ is the surface charge needed to shield against $\tilde{E}_\perp$). The average voltage in the sheath is large compared to the initial injection energies: the sheath is a cold relativistic flow, and is a cold beam propagating <u>next to</u> a warm plasma. The total energy of this beam flow is somewhat in excess of the typical radio luminosities of pulsars, even in the context of the very restrictive conducting gap model which

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applies when the flow of the electrons in the sheath is still laminar.

Because of the relative streaming between sheath electrons and the neighboring plasma, the free energy of this flow can be released in a form appropriate for creating resistance in the boundary region, thus allowing the release of more of the gap's free energy, and for direct photon emission, radio and high frequency. In order to illustrate the physics, consider the following oversimplified model of the boundary layer. Let the boundary sheath have a uniform electron density $n_e = \langle n_e \rangle$, with all particles having the same velocity $v_s = c\,\beta_s\,b$ along $B = B\,b$ and all having the same energy $\gamma_s(r)\,m_e c^2 \simeq e\langle\Psi\rangle$. Let the thickness of the sheath be $\delta\tilde{\omega}_s(r)$. Assume the neighboring pair plasma is cold and streams with speed $v_\pm = c\,\beta_\pm\,b$ and relativistic energy/particle $\gamma_\pm m_e c^2$, and has a uniform density $n_\pm \gtrsim n_e$. Assume the plasma occupies the half space $-\infty < x \leqq 0$, the sheath electrons occupy the slab $0 \leqq x \leqq \delta\tilde{\omega}_s$, and a vacuum (the slot gap) fills the region $\delta\tilde{\omega}_s \leqq x < \infty$. Assume the magnetic field is strong enough to enforce completely one dimensional response by the plasma particles ("$B = \infty$"). Because of the density gradients present (the density jumps at $x = 0$ and $x = \delta\tilde{\omega}_s$) and the relativistic speeds, the electric fields are intrinsically electromagnetic. Finally assume the sheath flow is disturbed, with all disturbances of the form $h(x)\exp[i(k_\parallel z + k_\perp y - \omega t)]$ with z = distance along B. With a cold linearized fluid model for the plasma response, simultaneous solution of the dynamic equations and Maxwell's equations, with continuity of the fields across the density discontinuities, yields the dispersion relation

$$(\varepsilon_s + \varepsilon_p^{1/2})(1 - e^{-2K_s\delta\tilde{\omega}_s}) + \varepsilon_s^{1/2}(1 + \varepsilon_p^{1/2})(1 + e^{-2K_s\delta\tilde{\omega}_s}) = 0 \qquad (17)$$

where

$$\varepsilon_s = 1 - [\omega_{ps}^2/\gamma_s^3(\omega - k_\parallel v_s)^2], \quad \varepsilon_p = 1 - [\omega_p^2/\gamma_\pm^3(\omega - k_\parallel v_\pm)^2],$$

$$\omega_{ps}^2 = 4\pi e^2 n_e/m, \quad \omega_p^2 = 8\pi e^2 n_\pm/m, \quad K_s = K_v\,\varepsilon_s^{1/2} \text{ and}$$

$$K_v = (k_\parallel^2 - \omega^2/c^2)^{1/2}.$$

This dispersion relation is easily solved. As an example, in the thin beam limit $|K_s\delta\tilde{\omega}_s| << 1$, it has an unstable root, corresponding to the growth of space charge waves in the $e^\pm$ plasma (driven by the usual free energy source, in which bunching of the beam induces bunches of the plasma, whose electric field decelerates the beam when the frequency of the bunches is less than the plasma frequency of the plasma, thus leading to enhancement of the beam bunches and growth of the wave). The maximum temporal growth rate is

$$\omega_i = \frac{2}{3\sqrt{3}}\,\omega_{ps}\,\gamma_s^{-2}\,(\omega_p\,\delta\tilde{\omega}_s/c)^{1/2}\,\gamma_\pm^{1/4} \qquad (18)$$

with a corresponding real part of the frequency

$$\omega_r = (4/3)\,\omega_p\,\gamma_\pm^{1/2} \qquad (19)$$

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with wave number $k_{\parallel} = \omega_r/c\beta_s$. This simple solution holds when $k_{\parallel}(v_s - v_{\pm}) >> \omega_i$ and $k_{\perp}/k_{\parallel} << \gamma_s^{-1}$. Substitution of the values (14)-(16) yields rates of growth $\sim 10^7$ s^{-1} for typical parameters, but which decline with increasing height because of the increase of γ_s with r until at radii $\gtrsim$ 200 R_* (for a typical example), the growth of this two-stream instability would be convectively stabilized. In the linear regime, this uniform density model produces emission in a line; however, the rapid growth indicates one has to consider convectively saturated amplification, in order to actually model pulsar emission. This kind of instability automatically produces $\sim$ 100% linearly polarized electromagnetic waves; for the solution (19), the ratio of electrostatic to electromagnetic components is γ_s^{-1}, polarized along the direction normal to the sheath. The radiation is not curvature emission.

I do not wish to jump to conclusions about the relation of this two-stream instability to radio astronomical reality, for a number of reasons. The growth rate of this particular amplifier is controlled by the longitudinal mass of the beam particles $m\gamma_s^3$, and is largest when this is smallest. The actual sheath models contain a gradient of γ_s across the sheath, as well as of γ_s with altitude, so a realistic evaluation of growth rate requires inclusion of both the variation of γ_s across the sheath and of n_e across the sheath, since the sheath is densest where γ_s is smallest. The unstable waves propagate in the $e^{\pm}$ plasma; thus density gradients and the large momentum dispersion in the plasma must be incorporated in the linear theory, in order to obtain a good estimate of the width in longitude of the emitting region. Theoretical determination of this thickness, which should be $\sim$ dynamical width $\cong 8\varpi_s/r \cong 1°(r/100\ R_*)^{1/2}$ for typical parameters, determines the amount of polarization sweep expected in a subpulse, since the observer's line of sight transits the boundary layer in a time short compared to the variation time $\sim$ P for the beaming angle to change (if the sheet beam instability above dominates the emission, the radiated E vector lies along the density gradient in the sheath). The existence of the density and particle energy gradients across B and nonlinear coupling to other fluctuations allows this type of mechanism to be locally broad band; whether or not it is broad or narrow band requires much more work. The fact that the subpulse polarization samples the sweep of the waveform's polarization requires both nods (latitude variation) and wiggles (longitude variation) of the polar flux tube on the rotation time scale. The existence of both is guaranteed by variations $\delta J_{\parallel}$ of $J_{\parallel}$, in the oblique rotator, since $J_{\parallel}$ is then both a poloidal and a toroidal current, with respect to the rotation axis. Because the waves must propagate through the plasma, they are subject to a raft of radiative transfer effects, which I think are responsible for micropulse formation and orthogonal mode behavior (my own research has concentrated on nonlinear self-focusing, but this is by no means the only possible effect). Other aspects of the plasma physics can affect the emission process. The $B = \infty$ approximation forces the instability into a mode where the inertia is dominated by the longitudinal mass. Under some circumstances (at higher altitudes) it appears that whistler-like

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modes which exist in the charge separated regions can be more rapidly unstable than the 2-stream mode described here. Finally, when the sheath is sufficiently energetic (high altitude) a resonant beam cyclotron instability can occur, which excites particles into finite Landau levels and causes synchrotron emission, thus giving rise to a possible model for X-ray and γ-ray emission which may be more successful than curvature emission models.

The relevance of these various processes can be and is being studied by linear stability theory applied to the slot gap model with conducting boundary conditions. (I have mentioned only the effects which seem to work, in oversimplified calculations.) In a more speculative vein, the nonlinear saturation may cause important modifications to the basic model, as well as being crucial to a fully quantitative theory of emission. In particular, the low frequency instability of the boundary may lead to a resistive model of the sheath being more appropriate than the inertially limited conducting model outlined above. This has two effects.

(1) The boundary of the slot gap is then better described as an insulator than as a conductor, enhancing the total luminosity made available by the driving electric field $E_\perp$ by a factor $\tilde{\omega}_c/8\tilde{\omega}_g$ ($\gtrsim$ 10 in the case of the Crab pulsar). This would give the model more than enough energy to explain even the most energetic emissions known, even if the gap width is unchanged.

(2) Such "resistivity" can allow the electric field $E_\perp$ to penetrate deeper into the plasma, thus releasing still more of the energy tied up in the polar cap potential drop. These views of what might occur in the nonlinear state make clear that the boundary layer is a driven system; calling the plasma processes "instabilities" is a misnomer for microscopic dissipation processes which control the response of the plasma in the boundary layer to the electric field supported by the starvation zone in the neighboring gap at all altitudes above the star. The fact that this electric field is a strong function of P and a much weaker function of $\dot{P}$, as is clear by combining (13) and (4), indicates an explanation of the odd fact that pulsars exist with a wide range of $\dot{P}$ but a much smaller range of P. I suspect that the lack of long period pulsars is due to inability to form the boundary layer because of a lack of pairs, while the lack of short period objects is a consequence of $E_\perp$ being too weak to excite emission in the boundary layer. The determination of the relevant threshold for boundary layer excitation is one of the main tasks for work in progress.

V. CONCLUSIONS

I have reported on the current state of the slot gap model for pulsar emission both on aspects largely completed and on other calculations still in progress. It is possible that the new physics discussed here (slot gaps with acceleration along B, positron trapping in the pair plasma with polar cap heating and formation of current and torque fluctuations on the rotation time scale, formation of boundary layer

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sheats and their electromagnetic shear flow instabilities with consequent formation of radio and other emission and formation of subpulses in the boundary layer) will lead to a physical theory for pulsar emission. This is still quite a way off, even if I make the bold assumption that each of these separate pieces are not shot down by observational constraints. In addition, the model suggests some basic parameters to use in correlation studies, especially P/P_D which clearly measures the total energy efficiency and fraction of the polar cap potential which can appear in a form useful to excite emission. To make use of this parameter requires some understanding of the emission physics, since a group of pulsars can be classified as having the same P/P_D only if one knows P, $\dot{P}$, I, R_* and f_p. The last is particularly important, since it enters directly into the plasma density (see expression 9, for example) and momentum distribution which are crucial to all the rest. I and R_*, the other variables not directly observed, are much less important, and P and $\dot{P}$ are directly observable. Thus, one would like some way to relate the emitted spectra and pulse shapes to f_p and P/P_D separately and this requires more specific results on the emission (the simple case of thermal X-ray emission from the polar caps is a promising tool). So far, I can conclude only that this scheme is promising, but I have nothing to offer in the way of explicit tests, other than the predictions of thermal X-ray emission and fluctuations.

One can ask, finally, how general is this scheme? In particular, I assumed the transition field lines bounding the polar flux tube to be perfect conductors. Both the microscopic physics of plasma on these field lines, and/or the state of the rest of the magnetosphere, might make this a poor approximation in the emission region $R_* << r < c/\Omega_*$.

However, almost all the physics one can dream of (for example, high altitude vacuum regions, as described by Holloway 1973; Cheng, Ruderman and Sutherland 1976; Michel 1979 and Michel and Pellat 1980, or dissipation induced by the flow of the return current through precipitating plasma, as in the terrestrial auroral zone) would give rise to an increase of the energy released in the slot gap and to an increase of $E_\perp$ on the surface of the polar flux tube plasma, making the model more energetically favorable without changing the basic idea of boundary layer excitation as the seat of pulsar emission. Whether this idea works quantitatively, for any calculated or assumed $E_\perp$, remains to be seen.

Aspects of this work were begun in collaboration with E.T. Scharlemann. The work on dynamical fluctuations of polar cap current flow grew out of a conversation with W.M. Fawley. I have had interesting exchanges with E. Asseo on the subject of relativistic two-stream instabilities. My research on pulsars is supported by the U.S. National Science Foundation, by the John Simon Guggenheim Memorial Foundation, by the tax payers of California, and by the Commissariat à l'Energie Atomique de France.

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DISCUSSION

RICKETT: Do you think of the radio emission process as broad or narrow band?

ARONS: Possibly narrow band, if the instability stays at linear amplitudes (like a laboratory traveling wave amplifier) and if the growth is dominated by a sufficiently narrow range of density and particle energy in the boundary layer beam. However, if it saturates into a broader band "turbulence" spectrum and/or the gradients are sufficiently important, it can be broader band. In short, I do not yet know, but this whole class of effects <u>is</u> subject to quantitative investigation.

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