

HALO MASS IN SPIRAL GALAXIES

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I. INTRODUCTION

We have solved the dispersion relation given by the theory of Lynden Bell and Kalnajs (1972) using models with spherical halos under the assumption that the halo population does not participate in the spiral arms. Especially we have studied the conditions under which the solutions of this dispersion relation reach corotation so that the excitation mechanism of the density waves due to the theory of Lynden Bell-Kalnajs is applicable. We have used as basic axisymmetric models the model of Schmidt (1965) (S-model) and the model proposed by Miyamoto and Nagai (1975) (M-model). On these models we have superimposed spherical halos so that in a sphere of radius 20 kpc there exists a given amount of mass with two different density laws: ρ as r^{-1} and ρ as r^{-2} .

The details of this work will be presented in a paper in the near future.

II. DESCRIPTION OF THE DISPERSION RELATION

The Lynden Bell-Kalnajs dispersion relation in the case of a two armed spiral galaxy can be written in the form:

$$(Q_a x + Q_a^2 D)^{1/2} = 2e^{-x} \sum_{n=1}^{\infty} \frac{I_n(x)}{1 - \nu^2/n^2} \quad (1)$$

where the variable x in the Bessel function I_n is given by:

$$x = \left[k^2 + \left(\frac{4\Omega}{\kappa r} \right)^2 \right] \frac{\langle r^2 \rangle}{2} \quad (2)$$

where k is the wave number, Ω the angular velocity of the stars, κ their epicyclic frequency, r the galactocentric distance and $\langle r^2 \rangle^{1/2}$ the dispersion of radial velocities. The parameter D is given by:

$$D = \frac{1}{k_T^2 r^2} \left[4 - \left(\frac{4\Omega}{\kappa} \right)^2 \right] \quad (3)$$

where k_T is the basic wave number connected with the critical value of the dispersion of radial velocities found by Toomre (1964), in order to have a disk which is stable against axisymmetric collapse. Q_a is the product: $Q_a = 0.2857Q^2$, where Q , the stability parameter, is the ratio between the real dispersion of radial velocities to the critical value.

It is worth noting that the parameter D which we call model parameter to distinguish it from the stability parameter is essentially the same as a parameter found lately by Lau and Bertin which characterizes the growth of spiral waves. On the other hand this parameter remains negative throughout a galaxy if the angular velocity decreases outwards and this is important since a set of solutions of the dispersion relation reach corotation for Q larger than 1 if the model parameter is negative.

III. CONCLUDING REMARKS

We consider as acceptable those solutions of the dispersion relation which have one minimum in k larger than zero and with no angular points in the resultant shape of the spiral arms. Examining the relations between the stability and the model parameters for several values of the variable x and the relative frequency $|v|$ and under the restriction that the values of the variable x must be larger than a critical value, we have found the ranges of the stability parameter Q and the halo mass for which the solutions of the dispersion relation are acceptable. These ranges are given in Fig.1. We distinguish two cases: Q between 1 and 1.1, and Q larger than 1.1.

In the first case (Fig.1a), the various curves give for

each value of Q the maximum value of the halo mass for which there are acceptable solutions up to corotation. The curve $S(r^{-1})$ refers to the S-models where the density law of the halo is as r^{-1} , whereas for the curve $S(r^{-2})$ the density law of the halo mass is as r^{-2} . The meaning of the curves $M(r^{-1})$ and $M(r^{-2})$ is similar but they refer to the M-models. The pattern speed for these curves is $12.5 \text{ kmsec}^{-1} \text{ kpc}^{-1}$, whereas for the curve marked with I_a (which refers to the S-models with density law of the halo mass as r^{-2}), the pattern speed is $17 \text{ kmsec}^{-1} \text{ kpc}^{-1}$. In this range of Q the solutions which have no angular points reach corotation with values of the wave number larger than zero. Therefore the acceptable solutions in this range of Q are defined uniquely by these curves.

On the contrary if Q is larger than 1.1 the solutions of the dispersion relation have no angular points in the range of Q and the halo mass above the solid curves $S(r^{-1})$ and $S(r^{-2})$ (Fig.1b), which have the same meaning as previously. But here the solutions reach corotation with values of the wave number larger than zero only for the ranges of Q and the halo mass below the dashed curves $S_{cr}(r^{-1})$, $S_{cr}(r^{-2})$. Therefore we have acceptable solutions of the dispersion relation in the ranges of Q and the halo mass given in the hatched regions of the figure. The curves Ib and IIb correspond to the set of the parameters given below figure 1b. This figure refers to the S-models whereas figure 1c gives the corresponding regions for the M-models.

By studying these figures we conclude that if Q is small (very near unity) we have acceptable solutions of the dispersion relation even for large values of the halo mass. On the contrary if Q is large (larger than 1.1), the halo mass is limited to about $3 \times 10_{\odot}^{10}$.

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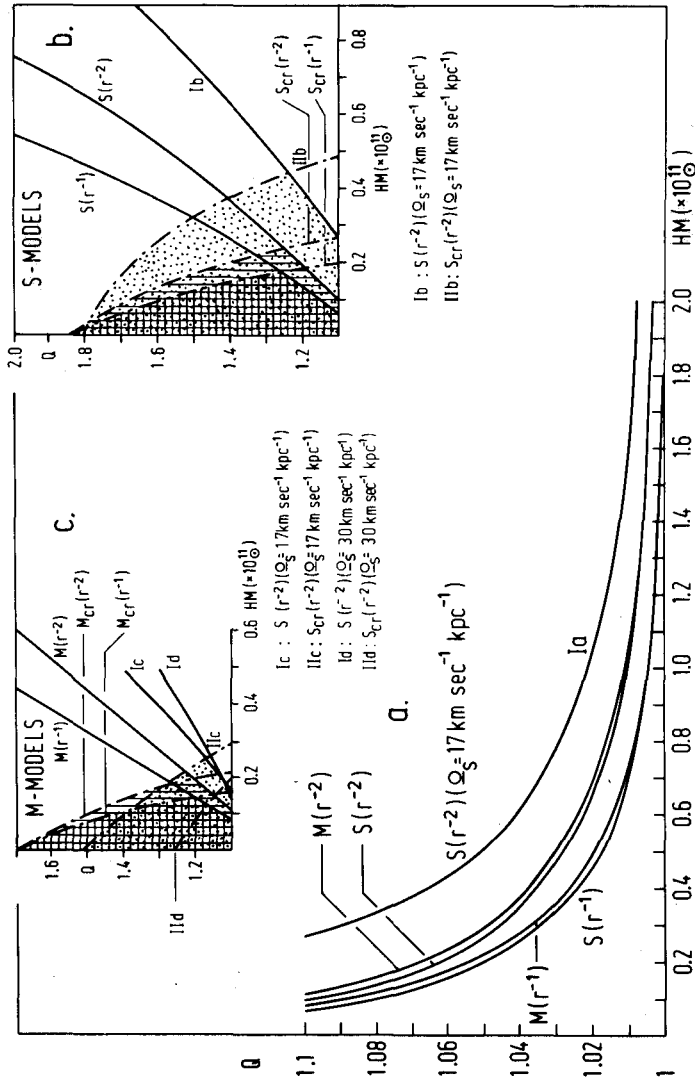


Fig. 1

The ranges of values of the stability parameter Q and the halo mass, so that the solutions of the Lynden Bell-Kalnajs dispersion relation be "acceptable".