$$
= \frac{\left|\begin{array}{cc} \left|\begin{array}{cc} 11 & 12 \\ 41 & 42 \end{array}\right| & \left|\begin{array}{cc} 13 & 14 \\ 43 & 44 \end{array}\right| \\ \left|\begin{array}{cc} 21 & 22 \\ 31 & 32 \end{array}\right| & \left|\begin{array}{cc} 23 & 24 \\ 33 & 34 \end{array}\right| \end{array}\right|}{\sqrt{(A_{14}{}^2+B_{14}{}^2+C_{14}{}^2)(A_{23}{}^2+B_{23}{}^2+C_{23}{}^2)-(A_{14}A_{23}+B_{14}B_{23}+C_{14}C_{23})^2}\sqrt{\&c.}}.
$$

Since $A_{14}{}^2+B_{14}{}^2+C_{14}{}^2=(l_1{}^2+m_1{}^2+n_1{}^2)\ (l_4{}^2+m_4{}^2+n_4{}^2)$
$-(l_1l_4+m_1m_4+n_1n_4)^2=1-14^2,$

and $A_{14}A_{23}+B_{14}B_{23}+C_{14}C_{23}=\left|\begin{array}{cc} 12 & 13 \\ 42 & 43 \end{array}\right|$;

we get

$$
57 = \frac{\left|\begin{array}{ccc} 1 & 34 & 12 \\ 14 & 13 & 24 \\ 23 & 24 & 13 \end{array}\right|}{\sqrt{\left\{(1-14^2)(1-23^2)-\left|\begin{array}{cc} 12 & 13 \\ 42 & 43 \end{array}\right|^2\right\}\left\{(1-12^2)(1-34^2)-\left|\begin{array}{cc} 13 & 14 \\ 23 & 24 \end{array}\right|^2\right\}}};
$$

In the particular case of the quadrantal quadrilateral this reduces to

$$
57 = \frac{13^2-24^2}{\sqrt{\left\{(1-14^2)(1-23^2)-13^2 24^2\right\}\left\{1-(13\cdot24-14\cdot23)^2\right\}}};
$$

from which 56 is obtained by interchanging 1 and 2.

On the Discrimination of Conics enveloped by the rays joining the corresponding points of two projective ranges.

By Professor CHRYSTAL.

It is evident in the first place as is pointed out by Steiner that the conic will be a parabola if, and cannot be a parabola unless the point at infinity on one range correspond to the point at infinity on the other, that is, the two ranges must be similar. This is the converse of the well-known proposition that a movable tangent to a parabola divides two fixed tangents similarly.

Steiner however does not take up the other cases, nor does Reye, or any other writer on the projective geometry of conics so far as I am aware.

We may however proceed in general as follows :

Join any two corresponding points P and P′. (Fig. 32.)

Project the range ω upon ω' by parallels to PP′.

We thus get a duplex range $\omega''\omega'$. Since this duplex range has already one double point P′P″, it must have another real double point, which can be easily constructed when k the power of the correspondence is given, if we observe that I projects into I″. To this second double point corresponds a tangent parallel to the tangent PP′.

From this it follows that the tangents to a curve of the second class occur in parallel pairs.

Remembering that points on the curve are the intersections of consecutive tangents we see that real points at infinity occur when the double points of the duplex range $\omega'\omega''$ are coincident.

Let $AI = i$, $B'J' = j'$, $B'I'' = x$; all with proper signs.

The condition for coincident double points is

$$I''J''^2 = -4k = -4\,\frac{B'I''}{AI}k\,;$$

$$(j'-x)^2 = -4\frac{x}{i}k\,;$$

$$x^2 - 2\left(j' - 2\frac{k}{i}\right)x + j'^2 = 0.$$

The roots of this equation must be real, that is, the condition for a hyperbola is

$$\left(j' - 2\frac{k}{i}\right)^2 - j'^2 > 0 \quad \text{or} - k(k - ij') > 0$$

There are two distinct cases. If $k = -p^2$, then $p^2 + ij'$ must be >0. This will be satisfied if i and j' have the same sign, or if they have opposite signs and ij' be numerically $<p^2$. If $k = +p^2$, then must $p^2 - ij' > 0$; which will be satisfied if i and j' have opposite signs, or if they have the same sign provided ij' be numerically $<p^2$.

In a letter which I received not long ago from Professor Cremona, he gave me, in answer to an enquiry what construction he used for asymptotes to a conic generated by means of its tangents, the following construction, which is more elegant than the above, although it proceeds on much the same lines. Regarding my own, I may observe that it was meant to come at the very beginning of a course on the projective theory of conics, and was not supposed to assume any proposition regarding conics except the fundamental fact of their projective generation by the lines joining the corresponding points of two projective ranges.

Construzione degli assintoti della conica inviluppata dalle rette AA′, BB′, ... congiungenti i punti corrispondenti di due punteggiate projettive $r \equiv \text{AB}\ldots\ldots, r' \equiv \text{A}'\text{B}'\ldots\ldots$

Le coppie di tangenti parallele determinano sopra una tangente fissa r una involuzione di punti AA_1, $\text{BB}_1,\ldots$ il cui punto centrale R è il punto in cui r tocca la conica. Percio, se, in r, si prende $\overline{\text{RP}}^2 = \overline{\text{RQ}}^2 = \text{RA.RA}_1$, saranno P,Q i punti d'intersezione di r cogli assintoti.

La conica sia adunque data mediante due rette punteggiate projettive, sia S il punto ad esse commune, R il punto di contatto della prima, e T il punto della stessa prima punteggiata che corrisponde all' infinito della seconda. Allora prendendo nella prima retta $\overline{\text{RP}}^2 = \overline{\text{RQ}}^2 = \text{RS.RT}$, i punti P,Q appartengono agli assintoti.

On a Problem in Partition of Numbers.

By Professor Chrystal.

At a recent meeting of the Royal Society of Edinburgh, Professor Tait proposed and solved the following problem:—

To calculate the number of Partitions of any number that can be made by taking any number from 2 up to another given number.

Let us denote by ${}_n\text{P}_r$ the number of partitions of r obtained by taking any of the numbers 2, 3, 4,......$(n-1)$, n. In the particular case $n=7$, $r=10$, the actual partitions are $3+7$, $4+6$, $5+5$; $2+2+6$, $2+3+5$, $2+4+4$, $3+3+4$; $2+2+2+4$, $2+2+3+3$; $2+2+2+2+2$; ten in all. Hence ${}_7\text{P}_{10} = 10$.

The object proposed here is not to find an analytical expression for ${}_n\text{P}_r$, but to give a process for quickly calculating a table of double entry for it. The following has some advantages over the method given by Professor Tait although the result is in reality much the same.

$$\text{Since } \frac{1}{(1-x^2)(1-x^3)\ldots\ldots(1-x^n)} = (1+x^2+x^{2.2}+x^{3.2}+\ldots\ldots)$$
$$\times(1+x^3+x^{2.3}+x^{3.3}+\ldots\ldots)$$
$$\times\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\times(1+x^n+x^{2n}+x^{3n}+\ldots\ldots);$$

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