CORRESPONDENCE

The Joint Editors
The Journal of the Institute of Actuaries Students' Society

12 *December* 1950

Spot the prior reference

Sirs,

A game which is fast become a favourite relaxation of the more priggish type of mathematician is one which might be called: Spot the prior reference. The equipment is elementary—a good memory or an extensive system of card records with appropriate cross-references. The object of the game is simple—the infliction of a blow to the self-esteem of a colleague while retaining an appearance of scientific detachment.

The first move is made by an author who inadvertently omits that thorough search through the numerous volumes of *Mathematical Reviews* and the *Zentralblatt für Mathematik* which nowadays occupies as much of a mathematician's time as the preparation of a supposedly original article. The second move falls to the editor whose referees fail to notice that the work submitted has already appeared in print in a substantially similar form ten, twenty or even a hundred years earlier—and the game is on. The reviewer now appears on the scene and scores one or more points according to the number of years he can span and the amount of scorn he can convey in a politely worded account of the author's limitations. The game continues as a third and fourth writer show that even the reviewer himself has not found the site of original publication of the material presented. Final honours go to the player who has revealed the greatest number of missing references in the previous writers' articles.

An amusing example of the game in progress is to be found in the 1944, 1945 and 1947 volumes of the *Philosophical Magazine* and concerns a subject which has recently been discussed in your pages, namely, the probability distribution of the sum of n continuous or discrete rectangular variates.

Silberstein started the ball rolling with a derivation of the continuous distribution for $n=2, 3, 4$ and 5 by Laplace's iterative method but failed to generalize his result, although he was able to throw it into an (exact) integral form also due originally to Laplace. Haldane quickly pointed out that explicit general expressions were available in Irwin's and Hall's developments, both written under the impression of prior authorship. Shortly after, Grimsey stated that Silberstein's integral had been evaluated in terms of a sum by Edwards in his text on calculus (1921–2), apparently overlooking that it had been included in Bierens de Haan's standard table of definite integrals since the first edition of 1858. Goddard followed this with a derivation of the distribution due to Fränz (1940) which has certain similarities with that provided by Mr Packer in this *Journal*. Finally, Parker used moment generating functions to derive the integral form of Laplace's result, apparently ignorant that such was Laplace's own method of arriving at this integral preparatory to investigating its asymptotic properties.

Extraordinarily, neither Simpson's (1757), Lagrange's (1773) nor Laplace's (1776, 1781, 1810) names are mentioned in any of these five notes, though a reviewer referred to the latter. In fact, the references uncovered by these writers and by Mr Packer only represent a small part of the numerous independent derivations of the distributions, continuous and discrete, under consideration. As a demonstration of the efficiency of my own card index and to provide your readers with a quiver of weapons with which to participate in the game in future years, I append a list of the post-classical derivations I have encountered, most of which have not, to my knowledge, been collected together previously. In each case I indicate whether the derivation relates to the continuous or the discrete case (c and/or d), whether or not the author refers to any earlier solutions (e or $\bar{e}$), and whether or not the method used was sufficiently different to be considered (by me) original at the time it was written (o or $\bar{o}$).

LOBATSCHEWSKY (1842). Probabilité des résultats moyens tirés d'observations répetées. *J. reine angew. Math.* XXIV, 164.

[c, d, $\bar{e}$, o]

https://doi.org/10.1017/S0020269X00004606 Published online by Cambridge University Press

JULLIEN, M. (1858). Sur la probabilité des erreurs dans la somme ou dans la moyenne de plusieurs observations. *Ann. math.* I, 76, 149 and 227. [*c*, *e*, *o*]

KUMMELL, C. H. (1882). On the composition of errors from single causes of error. *Astron. Nachr.* CIII, 177.

[*c*, *ē*, *ō*]

SCHOLS, CH. M. (1887). La loi de l'erreur résultante. *Ann. l'École Polytech. Delft*, III, 140.

[*c*, *ē*, *o*]

HAUSDORFF, F. (1901). Beiträge zur Wahrscheinlichkeitsrechnung. *Ber. Verh. könig. sächs. Ges. Wiss. Leipzig*, Math.-Phys. Cl., LIII, 152. [*c*, *ē*, *ō*]

SOMMERFELD, A. (1904). Eine besondere anschauliche Ableitung des Gaussischen Fehlergesetzes. *Festschrift Ludwig Boltzmann.* Leipzig. [*c*, *e*, *o*]

RISSER, R. (1924). Recherche de la loi de dispersion de la somme des erreurs $(x_1+x_2+\dots x_n)$, lorsque la dispersion de chacune de ces erreurs est définie par une loi simple. *Bull. trim. inst. actu. franç.* XXXV, 57. [*c*, *ē*, *ō*]

RIETZ, H. L. (1924). On a certain law of probability of Laplace. *Proc. Int. Math. Congr. Toronto*, II, 795.

[*c*, *e*, *ō*]

TRICOMI, F. (1928). Su di una questione di probabilità. *Atti* 1° *Cong. Naz. Sci. Assic.* (Turin).

[*c*, *ē*, ?]

TRICOMI, F. (1931). Su di una variabile casuale connessa con un notevole tipo di partizioni di un numero intero. *Gior. ist. ital. attuar.* II, 455. [*d*, *ē*, *ō*]

TRICOMI, F. (1933). Über die Summe mehrerer zufälliger Veränderlichen mit konstanten Verteilungsgesetzen. *Jber. Dtsch. Math. Verein.* XLII, 174. [*c*, *ē*, *ō*]

BRUN, V. (1932). Gauss' fordelingslov. *Norsk Mat. Tidsskr.* XIV, 81.

[*c*, *d*, *e*, *ō*]

AUERBACH, H. (1933). Über die Fehlerwahrscheinlichkeit einer Summe von Dezimalzahlen. *Z. angew. Math. Mech.* XIII, 386.

[*c*, *e*, *ō*]

Two comments may be made on the above list. It will be noticed that I disagree with Mr Packer that Reitz's proof was any simpler than Laplace's. In fact it was Laplace's own derivation (for which, in modern notation, see the Appendix to my paper in the 1949 Swiss *Bulletin*) with only formal differences. Secondly, it may be mentioned that a 3-decimal table similar to Mr Packer's Table 3 is provided by Auerbach in the paper cited.

I hasten to assure you that the provision of this list, which contains four original proofs between Laplace and Rietz, is not intended as a criticism of Mr Packer's excellent note. The modest title of your *Journal* would, in any case, forbid the scoring of points on the part of your averagely priggish correspondent who signs himself

Yours faithfully,

H. L. SEAL

295 Madison Avenue
New York 17

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The variance-ratio distribution

Sirs,

The formula given by Bizley (*J.S.S.* X, 62) for the probability distribution of the variance-ratio distribution is certainly useful when the smaller of n_1 or n_2 is not too large to make the computation tedious. As a matter of historical record it should perhaps, however, be pointed out that the result, which is essentially obtained by integration by parts, has been known for a long time. As long ago as 1924 Karl Pearson (*Biometrika*, XVI, 202) showed that the ratio