MODELLING OF UV RESONANCE LINES

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ABSTRACT

The envelopes of early type stars can be studied by comparing observed and theoretical P Cygni profiles of UV resonance lines. The lines are formed by scattering in the expanding envelope and complete frequency redistribution is a reasonable assumption. The Sobolev approximation can be used for the prediction of the profiles, except within a Doppler shift of $2xv_{thermal}$ from the line center. The profiles are sensitive to the optical depth $\tau_{rad}(v) \propto n_i(r)\ (dr/dv)$, but insensitive to the velocity law. We present a fairly simple method to calculate the effects of a photospheric profile or partly overlapping doublets.

I INTRODUCTION

The study of the ultraviolet P Cygni profiles is the major source of information about the mass loss rates of early type stars and the structure of their expanding envelopes. (The radio fluxes can be used to determine the mass loss rates accurately, but the observations are limited to only a very small number of the brightest stars. The infrared fluxes up to 22 μm also give information on the mass loss rate, but, since these fluxes arise from regions very close to the star with small velocities, they do not give information on the structure of the major part of the envelope.) Profiles of ultraviolet lines in about 60 early type stars observed by the Copernicus satellite have been published by Snow and Morton (1976) and Snow and Jenkins (1977); the forthcoming observations with the International Ultraviolet Explorer will extend this number dramatically. In order to derive information about the structure of the expanding envelopes, the observed profiles have to be compared with calculated profiles.

We will discuss the process of line formation, the Sobolev approximation for the calculation of the profiles and its accuracy and the accuracy of the physical quantities that can be derived from the profiles. We will show that the optical depth $\tau(v)$ can be derived fairly

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accurate, but that the profiles give little information on the velocity law.

II THE PROCESS OF LINE FORMATION

The P Cygni profiles of the ultraviolet resonance lines in the spectra of early type stars are formed by resonance scattering in the expanding envelopes. The probability of destruction of a photon and the corresponding rate of creation of photons are small, as we can see by computing the ratios.

$$\varepsilon = \frac{n_e C_{21}}{A_{21}} \left[1 - \exp\left(-\frac{\chi_{12}}{kT_e}\right)\right] \simeq 6.2\, T_e^{-1/2} n_e \lambda^3 \left[1 - \exp\left(-\frac{\chi_{12}}{kT_e}\right)\right] \quad (1)$$

and

$$\eta = \frac{R_{2k}}{A_{21}} \simeq 5.11\times10^{19}\ W\ a\ f_{21}^{-1}\ \lambda^2\ \chi_2^{\ 2}\ T_*\ \exp(-\chi_2/kT_*) \quad (2)$$

where $n_e C_{21}$ is the rate of collisional deexcitation and R_{2k} is the rate of photoionization, both per excited ion. We adopted Van Regemorter's (1962) approximation for C_{21} and a photoionizing radiation field of $J_\nu = W B_\nu(T_*)$. a is the photoionization cross section of level 2 (in cm^2), f_{21} is the emission oscillator strength, λ is the wavelength (in cm), χ_2 is the ionization potential of level 2 (in eV) and W is the dilution factor. The electron densities in the envelope range from about 10^8 to 10^{11} cm^{-3} with the result that ε ranges from less than 10^{-8} to 10^{-5}. The value of η is less than 3×10^{-5} if $T_* \gtrsim 4\times10^4$ K, $\chi_2 \gtrsim 40$ eV and $W \lesssim 0.5$. The effect of ε or η is negligible unless

$$\varepsilon\tau > \frac{WI_c}{B_\nu(T_e)} \quad \text{or} \quad \eta\tau > \frac{WI_c}{B_\nu(T_*)} \quad (3)$$

(see Castor 1970). In these expressions I_c is the intensity of the photosphere at the line frequency. For a line at 1000 Å, with $T_e \lesssim 2\times10^5$ K and $T_* \gtrsim 2.5\times10^4$ K, $I_c/B \gtrsim 10^{-2}$. We may suppose that $B^* \approx I_c$. Therefore, these effects are not important unless τ is very large, $\tau \gtrsim 10^3$.

One can assume somplete frequency redistribution for the scattering process. Mihalas *et al.* (1976) showed that the assumption of complete frequency redistribution gives an accurate approximation to the source function of the lines in a moving atmosphere. They found that even if the lines have a coherent scattering wing, the assumption of complete redistribution is still to be preferred to coherent scattering for the line as a whole provided that the optical thickness is not too large.

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III THE SOBOLEV APPROXIMATION

The transfer equation for a spherically symmetric expanding envelope can be solved if the Sobolev approximation is adopted. This approximation is valid if the velocity gradient is so large that the scale length for the density and temperature variations in the envelope is much larger than the length in which the expansion velocity increases by the thermal velocity. The intrinsic line width can then be neglected relative to the Doppler displacements. There are two methods for calculating the profiles of resonance lines in a spherically symmetric expanding envelope with the Sobolev approximation. In Lucy's (1971) method the angular dependence of the intensity is approximated by two streams with a properly chosen boundary between them, whereas in Castor's (1970) method the angular dependence is treated exactly using escape probabilities. For the same model, the profiles calculated with the two methods agree within one per cent.

In order to estimate the effect of the Sobolev approximation on the profile we compared the profiles for a rapidly expanding plane-parallel envelope calculated by Noerdlinger and Rybicki (1974) using the modified Feautrier method including thermal broadening with the profiles for the same envelope calculated without thermal broadening using Lucy's method. This comparison is shown in Figure 1, where the profiles computed by Noerdlinger and Rybicki with the parameters (ZM = 1, VM = 10) and (ZM = 5, VM = 20) are compared with our profiles for the same case.

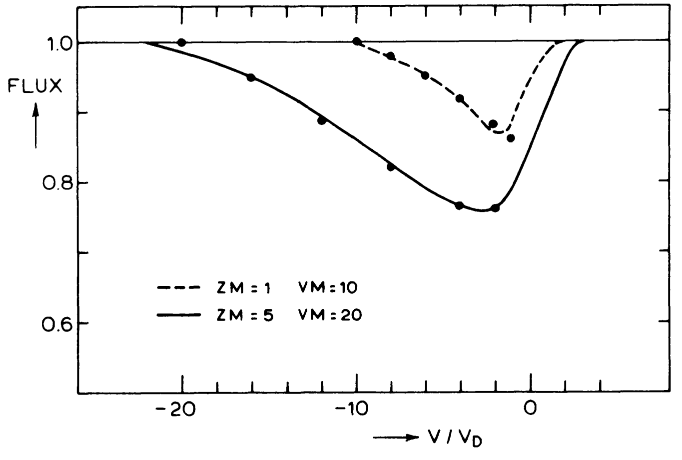

Figure 1. A comparison between theoretical profiles of scattering lines in a plane parallel expanding atmosphere, calculated with the Sobolev approximation (dots) and without the Sobolev approximation (full and dashed lines; from Noerdlinger and Rybicki, 1974). The agreement is very good in the region where $|V| > 2xV_{Doppler}$.

We note that the agreement is better than 0.5% throughout the violet absorption wing, except within two Doppler widths of the line center. At such small velocities the thermal motions of the ions cannot be neglected and so the Sobolev approximation breaks down. This implies that for O stars, where the photospheric turbulent velocity can amount to about 30 km/s (Ebbets, 1978), profiles calculated with the Sobolev approximation can not be trusted within 60 km/s from the line center.

IV THE ION DENSITY AS A FUNCTION OF DISTANCE: $n_i(r)$

The information that one wants to derive from the study of the P Cygni profiles of UV resonance lines is: the velocity law, v(r), and the density of the absorbing ions, $n_i(r)$. These two quantities enter the calculations of the profiles in a coupled way via the radial optical depth.

The radial optical depth in an expanding envelope is given by

$$\tau_{rad}(r) = \frac{\pi e^2}{mc} f \lambda_o n_i \left(\frac{dr}{dv}\right) , \tag{4}$$

where f is the absorption oscillator strength, λ_o (in cm) is the rest wavelength of the transition, n_i (in cm^{-3}) is the number density of the absorbing ions at a distance r, and dr/dv (in s) is the inverse velocity gradient. The quantities n_i and dr/dv are evaluated at the radial distance r. We assume that the velocity increases monotonically with radius, so that dr/dv is uniquely defined. This τ_{rad} is the total optical depth presented by the envelope to a photon traveling radially at that frequency which would be resonantly absorbed at radius r. Most of this optical depth is contributed by a thin shell across which $\Delta v = v_{thermal}$.

The radial optical depth is expressed in equation (4) as a function of distance from the star. If the velocity increases monotonically, τ_{rad} can also be expressed as a function of the velocity in the envelope: $\tau_{rad}(v)$. Calculations of line profiles show, as expected, that the absorption part of the P Cygni profiles, expressed in relative flux versus velocity-shift, depends very strongly on $\tau_{rad}(v)$ and very little on the velocity law v(r). On the other hand, the emission component depends on v(r) but not on its detailed structure. The total amount of emission is a measure of the extent of the envelope: a soft velocity law, (i.e. v/v_∞ increases slowly outwards) gives a stronger emission component than a fast velocity law.

If the line is strong and saturated, the profiles yield little information on $\tau_{rad}(v)$. In contrast to the case for static atmospheres, for which the profiles continue to change if τ increases, the P Cygni profiles become completely independent τ when $\tau_{rad}(v)$ is larger than about 3 at all velocities (but smaller than 10^3, see section II). Such saturated profiles depend on the velocity law only, but the dependence is weak except near the line center.

In conclusion: we find that for lines which are not saturated the function $\tau_{rad}(v) \propto n_i(r).(dr/dv)$ can be derived from the profiles, except at small velocities $v < 2v_{thermal}$. For the determination of the ion density or the ionization balance as a function of distance one has to know the correct velocity law. As the P Cygni profiles are not very sensitive to the velocity law, this law may have to be derived by other methods and from other observations.

V THE VELOCITY LAW: v(r)

The strongest P Cygni profiles of the UV resonance lines of luminous O and B stars have a steep violet edge at the short wavelength side of the absorption component and this edge velocity is about the same for all strong lines in the spectrum of a star. (see e.g. Snow and Morton, 1976). The emission components are strong at small velocities and decrease towards large velocities. These two facts suggest that the expansion velocity v(r) increases outwards and asymptotically approaches a terminal velocity, v_∞, at large distances. Abbott(1978) found that the terminal velocity of the most luminous stars is about 3.5 times the effective escape velocity at the stellar surface.

For those stars which have weak P Cygni profiles or extended violet absorption wings at the UV resonance lines, like early-B mainsequence stars and giants, the terminal velocity cannot be determined and may in fact be considerably larger than the extent of the violet wings.

In Figure 2 we show several velocity laws. The broken lines are theoretical laws for the radiation driven wind models by Castor, Abbott and Klein (1975) and by Abbott (1977).

The steep law (CAK) includes no effects of lineblending and the slow law (A) includes a maximum of blending. These two laws can be taken as the upper and lower limit for the steepness of the velocity law predicted for the radiation driven winds. The full lines are the laws derived from the observations by Lamers and Morton (1975) from the UV lines, using the observed ionization balance, and by Barlow and Cohen (1977) from the radio and infrared flux distribution of P Cygni. We can only conclude that either there is no unique velocity law for all early type stars or the true velocity law is still unknown.

VI THE EFFECT OF A PHOTOSPHERIC ABSORPTION LINE

There are two additional effects which have to be taken into account for an accurate analysis of the P Cygni profiles of the UV resonance lines:

a. the presence of an underlying photospheric absorption component and

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b. the overlap of doublet lines.

In the literature one finds examples where these effects are allowed for by simple addition or superposition of the profiles. However, calculations of profiles in which both effects are properly taken into account show that these simple corrections can give a large error in the profiles. Therefore we have developed a simple method, based on the separate superposition and addition of flux contributions, arising from different parts of the envelope or photosphere. This simple method gives profiles which are fairly accurate approximations of the exact calculations.

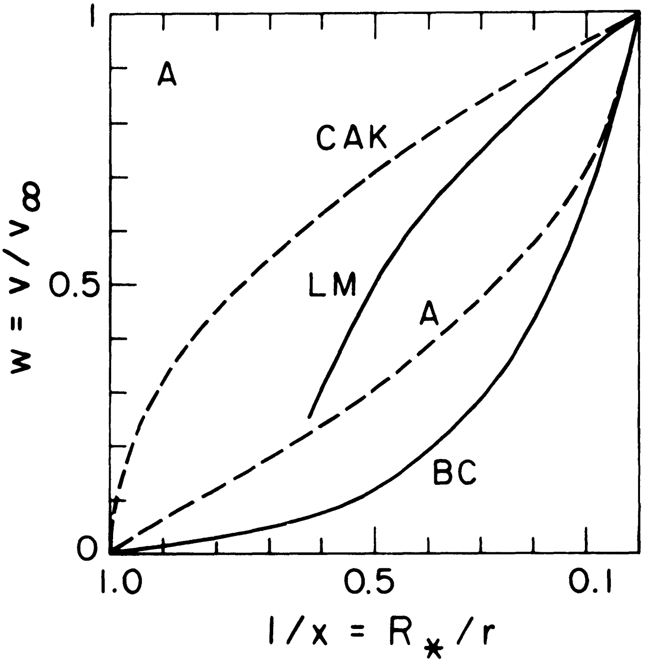

Figure 2. Theoretical (dashed lines) and observed (full lines) velocity laws normalized to $v_\infty = 1$ and $R_* = 1$. The theoretical laws are for radiation driven wind models without line blending (Castor, Abbott and Klein, 1975; CAK) and with maximum line blending (Abbott, 1977; A). The observed laws are derived from the infrared and radiofluxes of P Cygni (Barlow and Cohen, 1977; BC) and from the ultraviolet lines of ζ Puppis (Lamers and Morton, 1976; LM).

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Let $F_{phot}(v)$ be the flux of the photospheric profile relative to the continuum, where v is the Doppler shift from the line center, $v = c (\lambda-\lambda_o)/\lambda_o$. Let $F_{env}(v)$ be the flux of the envelope P Cygni profile calculated for a continuous photospheric spectrum. The profile which results from the interaction of the two profiles can be approximated with reasonable accuracy by the following expressions:

$$F(v) = F_{phot}(v) + q[F_{env}(v) - 1] \tag{5a}$$

for the long wavelength side of the profile (v>o), and

$$F(v) = F_{phot}(v) \; [F_{env}(v) - \{F_{env}(-v) - 1\}] + q\,[F_{env}(-v) - 1] \tag{5b}$$

for the short wavelength side of the profile ($v \leqslant o$),

where

$$q = \int_{v=-v_\infty}^{v=o} F_{phot}(v) \; d(v/v_\infty) \tag{5c}$$

The approximation can be understood as follows: The P Cygni profile consists of the sum of a shortward shifted absorption, produced in the part of the envelope that is seen projected against the stellar disc, and an emission that is symmetrical around v = 0 produced by photons that are scattered by the envelope into the line of sight. If the photospheric spectrum contains an absorption profile, the amount of photospheric radiation that can be scattered into the line of sight is reduced by a factor q, the mean photospheric flux in the wavelength interval corresponding to $v = -v_\infty$ to $v = o$; thus the long-wavelength emission is reduced by a factor q (second term of eqs 5a and 5b). On the short-wavelength side, the reduction factor $F_{env}(v) - F_{env}(-v) + 1$ that accounts for envelope absorption is impressed on the photospheric profile before the reduced emission is added. Figure 3 shows an example of this simple correction.

VII PARTLY OVERLAPPING DOUBLETS

Many of the ultraviolet resonance lines in the spectra of early type stars are doublets with a separation smaller than their widths, which results in partly overlapping lines. One of the authors (J.I.C.) has adapted the escape probability method to solve the transfer equations for doublets. The calculations show that the doublet profile differs strongly from the profile obtained by a simple addition or superposition of two separately calculated components. This is due to the fact that radiation scattered by the blue component in the direction of the observer can be scattered again by the red component. So the radiation scattered by both components appears as an enhancement of the emission on the long wavelength side of the red component.

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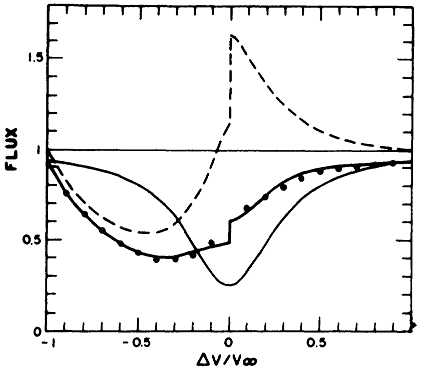

Figure 3

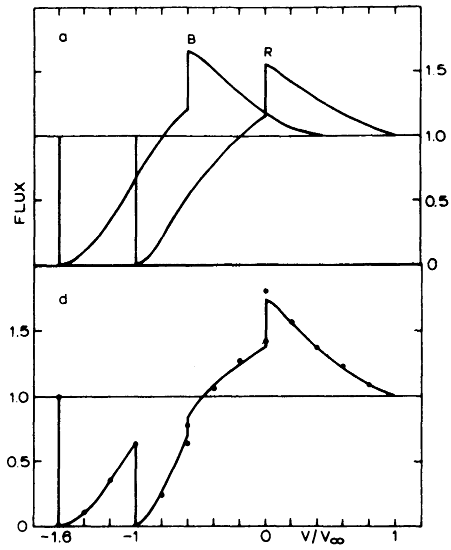

Figure 4

Figure 3. The effect of a photospheric absorption line. The dashed line is the profile if the stellar spectrum were continuous. The thin solid line is the photospheric absorption line. The thick solid line is the profile calculated with the photospheric absorption properly taken into account as a boundary condition to the equation of transfer in the envelope. The dots show the approximate profile calculated with the simple method of section VI. The approximation fits the exact profile very nicely.

Figure 4. The effect of partly overlapping doublet lines. The top part shows the profiles of the two components, if calculated separately. The blue (B) and red (R) components are calculated for the model with $v = v_\infty(1-R_*/r)^{\frac{1}{2}}$ and $\tau^B_{rad} = 1.5/1-(v/v_\infty)^2$ and $\tau^R_{rad} = 0.75/1-(v/v_\infty)^2$. The lower part shows the exact profile with the overlap properly taken into account in the equation of transfer (solid line) and the approximate profile calculated with the method of Section VII (dots). The approximation fits the exact profile nicely. Note the increased emission of the blue component due to the effect of overlap.

Let $F^B(v)$ and $F^R(v)$ be the flux of the blue and red components relative to the continuum if calculated separately and let $F^D(v)$ be the flux profile of the doublet. The velocity v is the Doppler velocity corresponding to the shift relative to the wavelength $\lambda_o{}^R$ of the undisplaced red component, $v = c(\lambda - \lambda_o{}^R)/\lambda_o{}^R$. Let $\tau^R(v)$ be the radial optical depth of the red component. The profile of the doublet can be approximated by

$$F^D(v) \simeq F^R(v) + [F^B(v)-1]\ e^{-\tau^R(|v|)} + Q\cdot R(v) \tag{6a}$$

with

$$Q = \int_{-v_\infty}^{v_\infty} \left[F^B(v)-1\right] . \left[1-e^{-\tau^R(|v|)}\right] . d(v/v_\infty) \tag{6b}$$

and

$$R(v) = \left[F^R(|v|)-1\right] \Big/ 2 \int_0^{v_\infty} \left[F^R(|v|)-1\right] . d(v/v_\infty) . \tag{6c}$$

The first term of equation (6a) is the profile of the red component, which is not affected by the presence of a blue component. The second term is the contribution to the blue component reduced by a factor $e^{-\tau R}$ due to scattering in the red component. The third term is the radiation from the blue component which is scattered and redistributed over frequency by the red component. The factor Q is the amount of scattered radiation, i.e., the complement of the second term integrated over the velocities in the region where the lines overlap. The function R(v) describes the redistribution of the radiation Q over frequency or velocity in the profile. We assume that this redistribution is the same as the distribution of the emission, $F^R(|v|)-1$, of the red component. In Figure 4 we show an example of this approximation.

VIII CONCLUSIONS

Accurate observations of a P Cygni type profile supply two functions of the velocity, v, namely the fluxes at the displacement v in either direction from line center. Therefore in principle it should be possible to derive both the velocity law and the optical depth function from a single line profile. In practice, the computed profiles have shown us that the short wavelength wing of the line is primarily sensitive to $\tau_{rad}(v)$, and not sensitive at all to v(r), while the long wavelength wing is sensitive to both. Furthermore, the sensitivity of the long wavelength wing to the velocity law is large only near the center of the line, and vanishes toward the extreme. The central part of the profile is afflicted with uncertainties due to the underlying photospheric profile and the error of the Sobolev approximation. The Sobolev approximation is valid, in the case of a rapidly rising velocity law, where $v \gtrsim 2v_{thermal}$. Thus observations readily yield only the optical depth law $\tau_{rad}(v)$. If the central part of the profile can be corrected for the effect of a photospheric component, then the total amount of emission on the long wavelength side of the profile can be used to distinguish between a slowly rising v(r) and a steeply rising one. Finer discrimination of the velocity law than this may not be possible.

We have presented a fairly simple method for estimating the correction due to a photospheric absorption line or the overlap of doublets.

One has to keep in mind that one of the basic assumptions used in the analysis of UV resonance lines -- spherically symmetric outflow -- may be incorrect. Aspect effects in an asymmetric flow can cause systematic changes in the absorption and emission strengths (Cassinelli and Rumpl 1978). Turbulence is a definite possibility, and its existence would vitiate the Sobolev approximation if $v_{turb} \approx v$. (A small amount of turbulence would have no effect.) The effect of stellar rotation may also be important. The profiles for stellar envelopes with a combination of expansion and rotation have a bewildering variety (Magnan 1970) and such cases are better treated individually.

(This paper contains some of the results of a study of theoretical P Cygni profiles by the authors. The details of this study and an atlas of theoretical profiles will be published in the Astrophysical Journal Supplement Series).

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DISCUSSION FOLLOWING LAMERS AND CASTOR

Underhill: Your way of estimating when it is valid to use the Sobolev approximation seems to miss the essential character of the physical assumptions lying behind this drastic simplification of radiative transfer in the presence of a velocity field. What is involved has been clearly expressed by Rybicki (1970, NBS Special Publ. 332) and by Rybicki and Hummer (1978, Ap. J.). If one follows this discussion for the case of a spherically symmetric atmosphere in which a velocity field v(r) exists, then one may make the Sobolev approximation if the following inequality is valid everywhere in the atmosphere:

$$\frac{v_{th}}{v(r)} << \frac{\ell_o}{r}\left(1 - \frac{(r^2 - p^2)}{r^2}\left\{1 - \frac{d\,\ell n\, v(r)}{d\,\ell n\, r}\right\}\right) \quad .$$

Here v_{th} is the thermal velocity at the radius r in the atmosphere, that is the velocity which describes the width of the line absorption coefficient, ℓ_o is the characteristic length in the atmosphere over which the state properties of the gas are constant, r is the radius to the point under consideration and p is the perpendicular distance to the line of sight being considered. The emergent flux spectrum is obtained by integrating the specific intensity I_ν emerging along each line of sight for the range in p from 0 to R_{shell}. Usually r runs from the radius of the photosphere, R_*, to the outer radius of the atmosphere R_{shell}, which lies in the range 5 to 10 R_* for early-type stars. From a simple diagnosis of observed line widths and displacements and guesses at the electron temperature or microturbulence in the flowing atmosphere, one finds that the left side of the inequality lies in the range 0.01 to 0.3. If microturbulence is significant it lies in the range 0.1 to 0.3. The quantity ℓ_o is never defined by authors desiring to use the Sobolev approximation, but it seems unlikely that it would exceed R_*. Typically r may lie in the range 3 to 5 R_*. If v(r) increases outwards, the term in braces is less than unity. It is multiplied by a factor in the range 0 to 1; typically this factor may be 0.5.

Consequently for the Sobolev approximation to be valid one must have $v_{th}/v(r) << 0.1$ to 0.2. Such conditions are only met if v_{th} is small, i.e., in a cool outer atmosphere in which there is little or no microturbulence. This is not the case for the layers in which the strong resonance lines from high ions are formed. It certainly is not so at deep layers where v(r) is just beginning to increase from its small photospheric value.

Olson: In response to A. Underhill's remarks on the accuracy of the Sobolev approximation, I think that the worst case of low velocity near the stellar surface is being used as an example to claim that the approximation is poor is true everywhere. The low velocity results are inaccurate as Lamers has said, but this does

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not mean that an inaccuracy at low velocity affects the high velocity value of Sobolev-type calculations.