FORUM

Newtonian Inertial Navigation

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NEWTON's *Principia* contains a neglected passage, which is nevertheless the true source of inertial navigation, occurring in *De Motu Corporum Liber Primus*, toward the end of 'Sect. X. De Motu Corporum in Superficiebus datis, deq; Funipendulorum Motu reciproco' as 'Corol. 2' to Prop. LIII. Prob. XXXV. Concessis figurarum curvilinearum Quadraturis, invenire vires quibus corpora in datis curvis lineis Oscillationes semper Isochronas peragent'. The corollary reads in the first edition:

'Corol. 2. Igitur in Horologiis, si vires a Machina in Pendulum ad motum conservandum impressae ita cum vi gravitatis componi possint, ut vis tota deorsum semper sit ut linea quae oritur applicando rectangulum sub arcu TR & radio AR, ad sinum TN, Oscillationes omnes erunt Isochronae'.

This text shows only one or two very minor changes in the second and third editions, but the two extra drawings added in the third edition are helpful. Cajori, in a readily available publication quite adequate for reference here, gives these drawings, and his version of Motte's translation reads:

'Cor. II. And therefore in clocks, if forces are impressed by some machine upon the pendulum which continues the motion, and so compounded with the force of gravity that the whole force tending downwards will be always as a line which is obtained by dividing the product of the arc TR and the radius AR, by the sine TN, then all the oscillations will become isochronous.'

The class of machines so specified by Newton must include all inertial navigation systems which are *tautochronously* periodic in eighty-four minutes.

It would seem that if, as is probable, Newton did investigate the possibilities of his specified machine, these investigations would turn to the force of inertia and extensions of the third law of motion. Therefore, it seems unlikely that any record of such investigations has or will come to light, because the scholarly attention which has been devoted to this question in connection with d'Alembert's *Traité de dynamique* would have discovered it if it were extant. It is the application of d'Alembert's principle and his fundamental equation of dynamics that further characterizes both the Newtonian tautochronous, and the d'Alembertian compound pendulum, systems of inertial navigation (the latter not so far having been reduced to practice).

There are many possibilities of carrying out in detail, as a reader's exercise, the problems raised in Book One, Section X, of *Principia*. One simple case in brief outline will suffice here. In the second extra drawing, let us take SRQ as the figure of Newton's model of Earth, A the centre, AR the radius, and let T be a navigated craft restrained to the surface of Earth's model. Then TZ can be the direction of the true vertical at R, and TY a horizontal direction of known apparent azimuth, both physically available on board at T. This is the general situation in, e.g., the navigation system of the U.S. weapon 'Snark'. By Newton's prescription, the exterior force of inertia developed at the craft must have always identically the component *V* arc TR in magnitude, but oppositely directed along if the path of the craft is to be a tautochrone relative to the path of R. Therefore, the path of the craft must have the appropriate curvature to balance this much force of inertia, correct in sense and direction. This curvature can be applied by using the physically available directions on board, even though they are not locally true, as steering references. The sensors of error from the desired balance of exterior forces at the craft will often be a set of bubbles, pendulums, or accelerometers, appropriately fixed in the reference directions physically available on board. The path of R is in the 'Snark' system prescribed by functions of real time carried on board (or more commonly in other systems by a process equivalent to dead reckoning, in which the path of T as indicated on board is assumed to be that of R). In plain English, Newton's prescription is that the craft must be so steered (or the 'machine' so arranged) that the error of position plus the error of motion, as an algebraic sum, vanishes identically. These errors are *apparent* at the craft, and both have the dimension either of force or of acceleration.

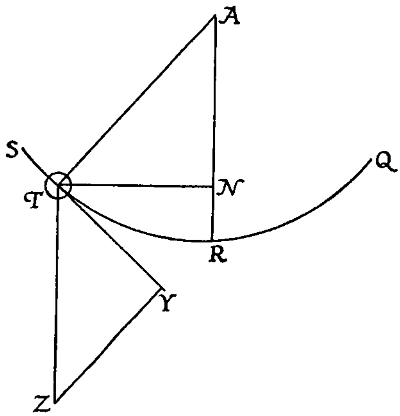

FIG. 1.

For the purposes to which inertial navigation is intended, it does not appear that any exterior force but the force of inertia could be used in Newton's machine, but a Newtonian inertial navigation system is as much a gravity system as it is an inertial system. Not so the d'Alembertian compound pendulum of swing length equal to Earth's radius, which is independent of gravity, and therefore might well be called 'purely inertial'. Such a system has been proposed, but never yet reduced to practice—the popular notion, that working inertial navigation systems are of this class, to the contrary notwithstanding. One gets the strange impression from this 'Corol. 2' that here Newton inadvertently reveals and uses a certain mastery of d'Alembert's principle which he ought not to possess, because d'Alembert has yet to be born. We speak here, of course, only of the exterior forces acting on the system, and do not consider the interior stresses, strains, isolations, &c., however important these latter may be in the actual design and realization of *the* system by means of practical *sub*systems. Almost by statement of the problem, the exterior forces are limited to three in practical 'self-contained' navigation on Earth, *viz.* that of attraction, that of inertia, and the pushes and pulls of the air, water, &c. in which the craft moves. Newtonian inertial navigation uses all three, whereas d'Alembertian inertial navigation suppresses attraction. There can be no other inertial navigation system than these two, and the latter is a degenerate case of the former.

REFERENCE

Cajori, Florian (1934). *Sir Isaac Newton's Mathematical Principles of Natural Philosophy and his System of the World, translated into English by Andrew Motte in 1729. The*

translations revised, and supplied with an historical and explanatory appendix. Cambridge University Press, London, and University of California Press, Berkeley, California, pp. 156–160 *et passim.*

Collision and Stranding at Sea

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HAVING studied marine statistics for a number of years, especially those concerning casualties at sea and in particular collisions between ships, the opening paragraph of the Traffic Separation Working Group's report (this *Journal*, 19, 4) is, to say the least, lacking confirmatory evidence on two counts.

I refer to the general statements which allege that:

(i) 'The main risk at sea today arises not as it used to from stranding and shipwreck, but from collision'.

(ii) 'In the open sea, where nearly 30 per cent of all collisions occur . . .'

As both include the words 'at sea' or 'open sea' it is not unreasonable to suppose that they imply freedom to manœuvre under own power unhampered by depth of water or similar navigational restriction.

This being so, and not an irrational definition of the area of concern to the Working Party, what are the facts and how exactly do casualties at sea compare with those in other areas of marine interest throughout the world.

To find the right answer to the global problem is not easy and there are no short cuts, such as for example making a sample analysis in a limited area where traffic density is high, in order to arrive at fairly large numbers which are then used to deduce the statistical probability applicable to all areas.

Not only is this misleading but it denigrates the value of any study it is meant to support, primarily because it is restrictive in two dimensions; size of sample and location.

Many authorities produce statistics of one kind or another though, as they are all tailored to meet a particular need, no two can in fact be directly compared one with the other without some re-arrangement.

In short, there is at present no universality applied to the compilation of statistics relating to marine casualties and, unless there is loss of life or injury to persons, there is also doubt in some quarters that all casualties do in fact come to the notice of responsible authorities.

The most satisfactory solution therefore seems to be to mean the available information. This overcomes any tendency towards bias or personal inhibitions in the choice of facts presented in the individual tables.

As an example the following table has been prepared.