In Fig. 2(*b*), which is the case of the hyperbola,

$$\angle \text{ACD is supplementary to } \angle \text{RP}_1\text{P}_2,$$

and $$\angle \text{BCD} = \angle \text{RP}_2\text{P}_1 = \angle \text{RP}_1\text{P}_2.$$

Therefore angles ACD and BCD are supplementary.

G. PHILIP

Note on Geometric Series.—The formula for the sum of n terms of the geometric series

$$a + ar + ar^2 + \ldots + ar^{n-1},$$

viz. $$s = \frac{a(r^n - 1)}{r - 1}, \text{ may be written } s = \frac{\text{T}_{n+1} - \text{T}_1}{r - 1},$$

where T_1 is the first term and T_{n+1} the term immediately succeeding the last to be summed This form is useful for finding the sum of a closed geometric series without first finding the number of terms. Thus

$$\tfrac{1}{8} + \tfrac{1}{4} + \tfrac{1}{2} + \ldots\ldots\ldots + 32 = \frac{64 - {}^1/_8}{2 - 1},$$

and $$a^{-5} + a^{-3} + a^{-1} \ldots\ldots + a^{19} = \frac{a^{21} - a^{-5}}{a^2 - 1},$$

since the terms of the series which succeed 32 and a^{19} are respectively 64 and a^{21}.

R. J. T. BELL

Distance of the Horizon.—The approximate rules given in this note may perhaps be new to some readers.

If an observer be at a height h above the Earth's surface, the distance of the horizon as seen by him is given by the formula

$$d = \sqrt{2rh},$$

where r is the radius of the Earth.

If h is given in feet, then

$$\sqrt{\left(\frac{h}{5280} \times 8000\right)}$$

will give d in miles.

Now $$\tfrac{8000}{5280} = \tfrac{100}{66} = \tfrac{3}{2}, \text{ nearly.}$$

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Hence the following convenient rule: Multiply the given height in feet by $1\frac{1}{2}$ and extract the square root. The result gives the approximate distance of the horizon, in miles.

Example: What distance along the surface of the Earth can be seen from a height of 80 feet?

The working may be set down thus:

$$\begin{array}{r} 80 \\ 40 \\ \hline 120 \end{array}$$

$$\text{Distance} = \sqrt{120} = 11 \text{ miles, nearly.}$$

The converse problem is to find the height of an observer to whom an object on the Earth's surface at a given distance is just visible. This is easily solved by the following rule: Multiply the square of the distance in miles by $\frac{2}{3}$; the result is the required height in feet.

Thus, if the object is at a distance of 30 miles, the required height $= \frac{2}{3} \times 900$ feet

$= 600$ feet.

PETER RAMSAY

The Asymptotes of the Hyperbola.—The equation of the hyperbola usually presents itself to the student in the form $y = \dfrac{a}{x}$, in the elements of graphs, before the canonical form is reached in Analytical Geometry. Some of the properties of the curve are easily obtained from the above equation by the methods of elementary geometry; the object of this note is to illustrate the process. We shall use the equation in the form $xy = c^2$, and it will be convenient to arrange the discussion in numbered Theorems. The axes may be rectangular or oblique.

Theorem 1.

If PM, PN and $P'M'$, $P'N'$ are drawn parallel to the asymptotes Cy, Cx of a hypherbola from points P and P' on the curve, to meet Cx and Cy in M, M' and N, N' respectively, then MN', $M'N$ and PP' are parallel.