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# An empirical relation between overburden pressure and firn density

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ABSTRACT. Two empirical equations for firn densification have been obtained, considering firn porosity as a function of overburden pressure. In the first equation, the reduction ratio of porosity in firn is assumed to be proportional to the increasing ratio of overburden pressure and the $m$th power of the porosity. The porosity exponent $m$ should be close to −2, so as to have a best-fit with 14 depth–density profiles from Greenland and Antarctica. In the second equation, the reduction ratio of porosity was assumed to increase proportionally to the increment of overburden pressure and the $n$th power of the porosity. The most satisfactory values of the exponent range from −1 to 1. It has been suggested that firn density, determined primarily by overburden pressure and firn temperature, contribute to a lesser degree.

## 1. INTRODUCTION

The transformation process of firn to ice is one of the most fundamental subjects in polar ice-sheet studies. A number of theories regarding the dry densification of firn have been proposed (Robin, 1958; Schytt, 1958; Bader, 1960, 1962; Benson, 1962; Anderson and Benson, 1963; Kojima, 1964; Gow, 1975). However, a comprehensive model including grain-settling, sublimation/condensation, volume/surface diffusion, ice deformation and recrystallization processes has not been firmly established. The densification processes have also been studied empirically (Robin, 1958; Schytt, 1958; Herron and Langway, 1980; Ling, 1985; Langway and others, 1993). Schytt (1958) found an empirical relation between firn density and the weight of overlying snow. Herron and Langway (1980) investigated the validity of Schytt's equation using 17 depth–density profiles from Greenland and Antarctica. Langway and others (1993) showed a simple relation between overburden pressure and firn density.

This paper focuses on obtaining a simple equation for a relationship between overburden pressure and firn density from the surface to the depth of pore close-off, considering the studies by Schytt (1958) and Langway and others (1993).

## 2. ANALYTICAL PROCEDURE

Instead of using the snow densification law (Bader, 1960, 1962), an assumption was made that snow behaves like a perfectly plastic material. That is, porosity, $s(= (\rho_i - \rho)/\rho_i)$ is determined only by overburden pressure, $P$, at each depth level. $\rho$ is the firn density and $\rho_i$ is the bubble-free ice density at −20°C (0.919 Mg m$^{-3}$). When the pressure $P$ increases to $P + dP$, $s$ decreases to $s - ds$. The reduction ratio of porosity, $-ds/s$ was assumed to increase proportionally to the increasing ratio of pressure, $dP/P$ and the $m$th of power of $s$ as follows:

$$-ds/s = Cs^m(dP/P) \tag{1}$$

where $C$ and $m$ are constants.

We obtained the following equations by integrating Equation (1):

$$\text{for } m \neq 0 \quad \ln(P) = C_1 s^{-m} + C_2 \tag{2}$$
$$\text{for } m = 0 \quad \ln(P) = C_3 \ln(s) + C_4 \tag{3}$$

where $C_1$, $C_2$, $C_3$ and $C_4$ are constants.



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Another equation was obtained by assuming that the reduction ratio of porosity, $-ds/s$ increases proportionally to the increment of pressure, $dP$, and the $n$th power of $s$ as follows:

$$-ds/s = Ds^n dP \tag{4}$$

where $D$ and $n$ are constants.

Thus, we obtained the following equations:

$$\text{for } n \neq 0 \quad P = D_1 s^{-n} + D_2 \tag{5}$$

$$\text{for } n = 0 \quad P = D_3 \ln(s) + D_4 \tag{6}$$

where $D_1$, $D_2$, $D_3$ and $D_4$ are constants.

Recently, Langway and others (1993) showed a linear relation between $(\rho_i - \rho)^2$ and $\ln(P)$ from the surface to the depth of pore close-off for three sites in Greenland and Antarctica. If we rewrite the expression $(\rho_i - \rho)^2$ of Langway and others (1993) to $((\rho_i - \rho)/\rho_i)^2$, the relation is expressed as follows:

$$\ln(P) = E_1((\rho_i - \rho)/\rho_i)^2 + E_2 \tag{7}$$

where $E_1$ and $E_2$ are constants.

This equation is identical with Equation (2) with $m = -2$.

On the other hand, Schytt (1958) proposed the following relation between firn density ($\rho$) and depth from the surface ($h$) for the Maudheim ice core, Antarctica:

$$d\rho/(\rho_i - \rho) = \text{const. } \rho dh\,. \tag{8}$$

This equation is identical with Equation (4) with $n = 0$.

## 3. RESULTS

Depth–density information was obtained from 14 sites in Greenland and Antarctica as shown in Table 1. The 10 m depth firn temperatures of the borehole range from

*Table 1. Glaciological data and references for Greenland and Antarctic ice cores*

| *Site name* | *Location* Lat. | Long. | *Temperature at a depth of 10 m* | *Accumulation rate* | *References for $\rho$, $T$ and $A$ data* |
|---|---|---|---|---|---|
| | | | °C | m w.e. year$^{-1}$ | |
| *Greenland* | | | | | |
| Site J | 66°52′ N | 46°16′ W | −16.3 | 0.39 | This work; Shoji and others (1991) |
| Site 2 | 76°59′ N | 56°04′ W | −23.3 | 0.4 | Langway (1967) |
| Site A | 70°38′ N | 35°49′ W | −29.41 | 0.28 | Clausen and others (1988); personal communication from H. B. Clausen |
| Summit | 72°34′ N | 37°38′ W | −32 | 0.21 | This work; Johnsen and others (1992) |
| *Antarctica (east Dronning Maud Land)* | | | | | |
| S25 | 69°02′ S | 40°27′ E | −17.96 | – | Personal communication from K. Satow and O. Watanabe |
| H15 | 69°05′ S | 40°46′ E | −19.4 | – | Personal communication from Y. Fujii |
| G2 | 71°02′ S | 39°51′ E | −29.05 | 0.1 | Nishio (1984) |
| Mizuho Station | 70°42′ S | 44°22′ E | −33.55 | 0.09 | Fujii (1978); Narita and Maeno (1978); Nakawo and others (1989) |
| G15 | 71°11′ S | 45°58′ E | −37.5 | 0.1 | Moore and others (1991); personal communication from H. Narita |
| G6 | 73°07′ S | 39°42′ E | −43 | 0.08 | Shoji and Fujii (1991) |
| *Antarctica (other places)* | | | | | |
| Little America V | 78°10′ S | 162°13′ W | −24 | 0.22 | Gow (1968) |
| Old Byrd Station | 79°59′ S | 120°01′ W | −28 | 0.16 | Gow (1968) |
| Byrd surface camp | 80°00′ S | 120°00′ W | −28 | 0.11 | Langway and others (1993) |
| Dome C | 74°30′ S | 123°10′ E | −54.3 | 0.034 | Alley (1980) |



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−54.3° to −16.4°C and annual accumulation rates range from 0.034 to 0.39 m year$^{-1}$ in water equivalent. The data scatters in bulk-density values are generally ±0.005 Mg m$^{-3}$ for those from Summit, S25, H15, S25, G15, G6, Byrd surface camp and Dome C. On the other hand, the data scatters for site J, site 2, site A, G2, Mizuho, Little America V and Old Byrd Station are within ±0.003 Mg m$^{-3}$, since the densities were determined more precisely by a volumetric method. The reliability of 10 m depth temperatures is generally ±0.1°C or better.

A trial was made to find the most satisfactory values for $m$ and $n$ in Equations (1) and (4) using the above depth–density profiles. Depth–density values from the surface to the depth at which the density is 0.80 Mg m$^{-3}$ are used for the correlation-coefficient analysis.

Figure 1a shows the results of the linearity test for Equation (1) using different values of $m$. It is clear that correlation coefficients take the highest values around $m = -2$ ($r = 0.98$). Figure 2b shows the results for Equation (4) using different values of $n$. It was found that the correlation coefficients take the highest value from $n = -1$ to $+1$ ($r = 0.97$ at $n = 1$; $r = 0.98$ at $n = 0$; $r = 0.95$ at $n = 1$). Thus, we obtain the following equations, which are identical with Equations (7) and (8):

$$\ln(P) = C_1 s^2 + C_2 \tag{9}$$

$$P = D_3 \ln(s) + D_4 \tag{10}$$

where $C_1$, $C_2$, $D_3$ and $D_4$ are constants.

Figure 2 shows the relation between $\ln(P)$ and $s^2$ for the 14 ice cores from Greenland and Antarctica. It is seen that $\ln(P)$ and $s^2$ have excellent linear correlation. Correlation coefficients, linear slopes ($C_1$), $y$ intercepts ($C_2$) and degrees of freedom are summarized in Table 2. The lowest correlation coefficient at G6 is due to data variations in the surface-firn densities.

Figure 3 shows the relation between $P$ and $\ln(s)$ for the 14 ice cores. It seems that most of the overburden-pressure curves have a bend around a density of 0.50–0.60 Mg m$^{-3}$. The linear-correlation analysis is summarized in Table 3. The mean correlation coefficient for Equation (10), as shown in Figure 3 ($r = 0.98$), is the same as the mean correlation coefficients obtained for Equation (9) as shown in Figure 2 ($r = 0.98$).

## 4. DISCUSSION

### 4.1. The relation between $\ln(P)$ and $s^2$

The slope and $y$ intercept for each site were investigated to find the dependence on 10 m depth firn temperature and/or annual accumulation rate. It was found that the $y$ intercepts, $C_2$, are well correlated with 10 m firn temperatures ($r = 0.69$) as shown in Figure 4. This is the only meaningful correlation. It is clear that the $y$ intercepts of site J (SJ) and Mizuho (MZ) are shifted from other data. The correlation coefficient without the above two sites is as high as 0.88. With this temperature-dependence of $y$ intercepts for 12 data points, we obtain the following equation:

$$\ln(P) = -12.9 s^2 - 0.0251 T + 7.60 \tag{11}$$

where $T$ is the 10 m depth firn temperature in K, $P$ is an overburden pressure in bar, $\rho$ is the firn density in Mg m$^{-3}$ and $\rho_i$ is the bubble-free ice density at −20°C (0.919 Mg m$^{-3}$).

Comparisons of measured and calculated density profiles with Equation (11) are shown in Figure 5. It is seen that the site J and Mizuho ice-core data are shifted significantly from the curves calculated from Equation (11). Site J is located in a percolation zone of the

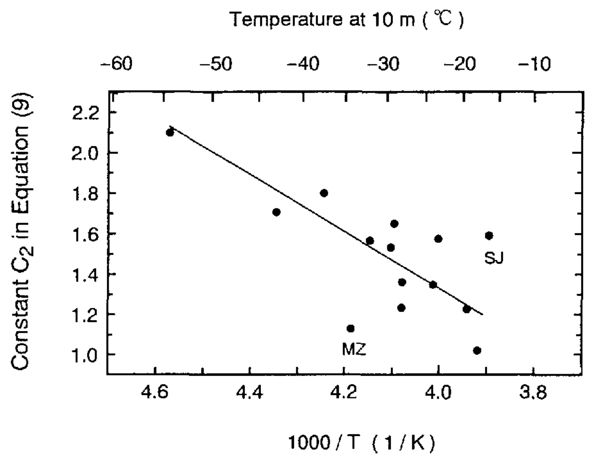


*Fig. 4. Temperature-dependence of the y intercepts in Equation (9). SJ and MZ refer to ice cores from site J and Mizuho, respectively.*

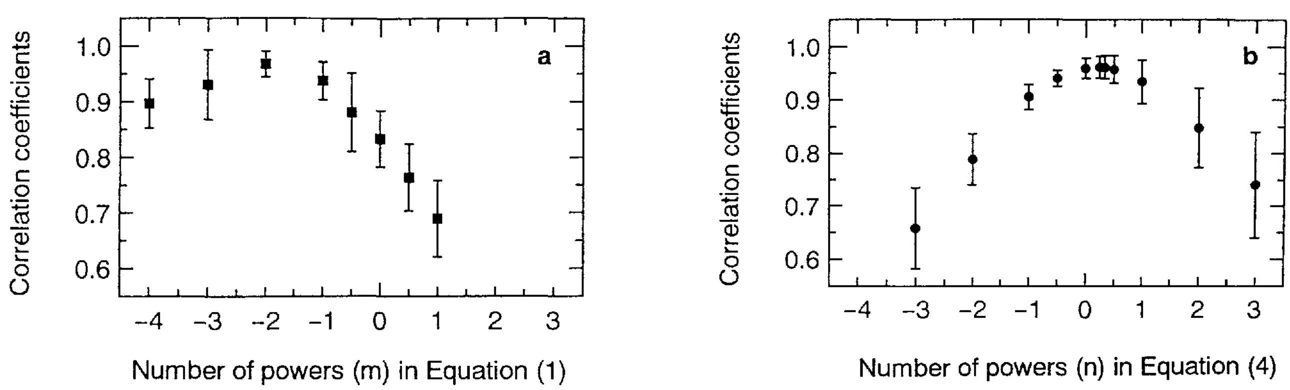


*Fig. 1. The relation between values of powers and correlation coefficients with their standard deviations. Figure 1a shows the results for m in Equation (1) and Figure 1b for n in Equation (4).*



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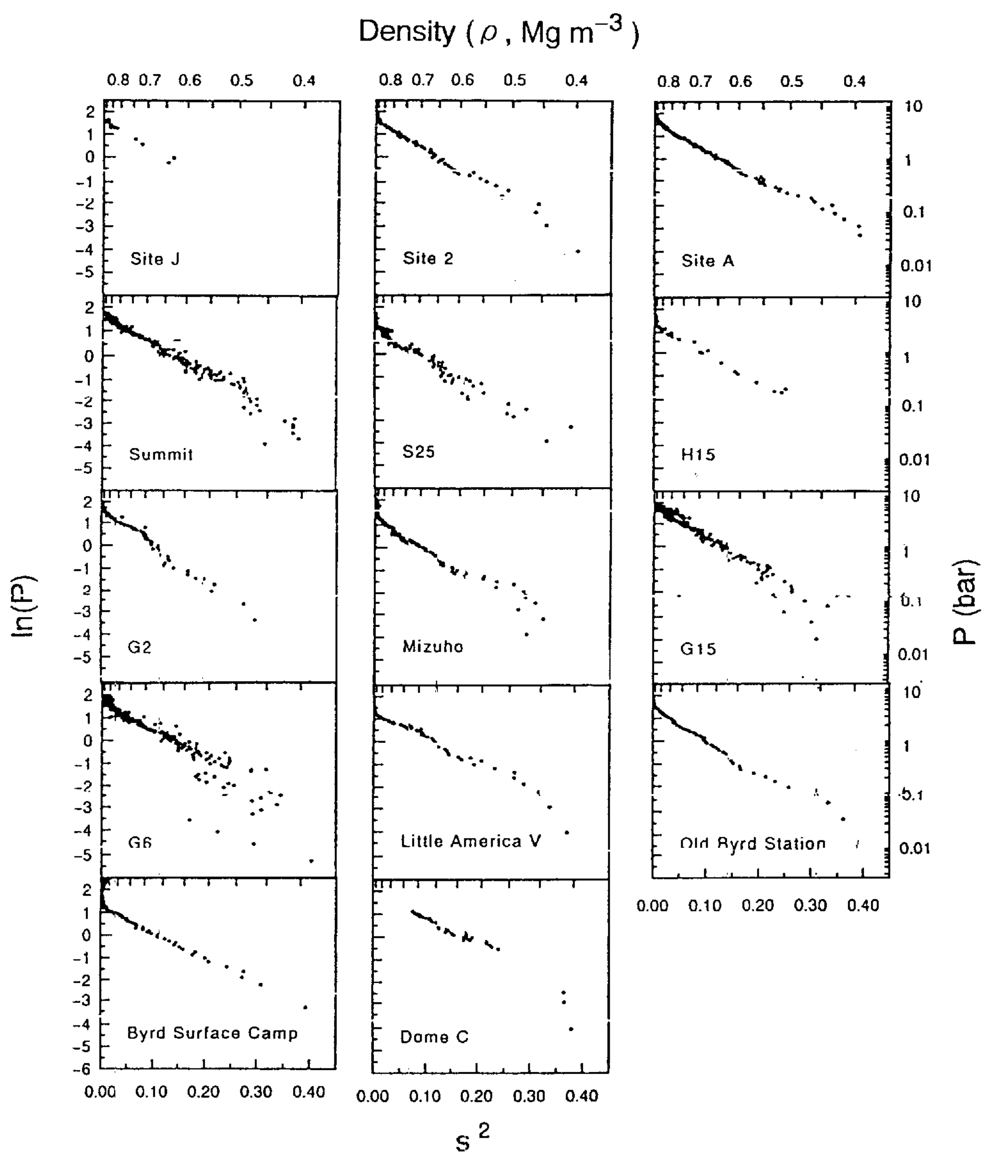


*Fig. 2. Relations between* $\ln(P)$ *and* $s^2$ *for 14 sites in Greenland and Antarctica.* $P$ *is an overburden pressure and* $s$ *is the porosity of firn, where* $s = (\rho_i - \rho)/\rho_i$. $\rho_i$ *is the bubble-free ice density and* $\rho$ *is the firn density.*

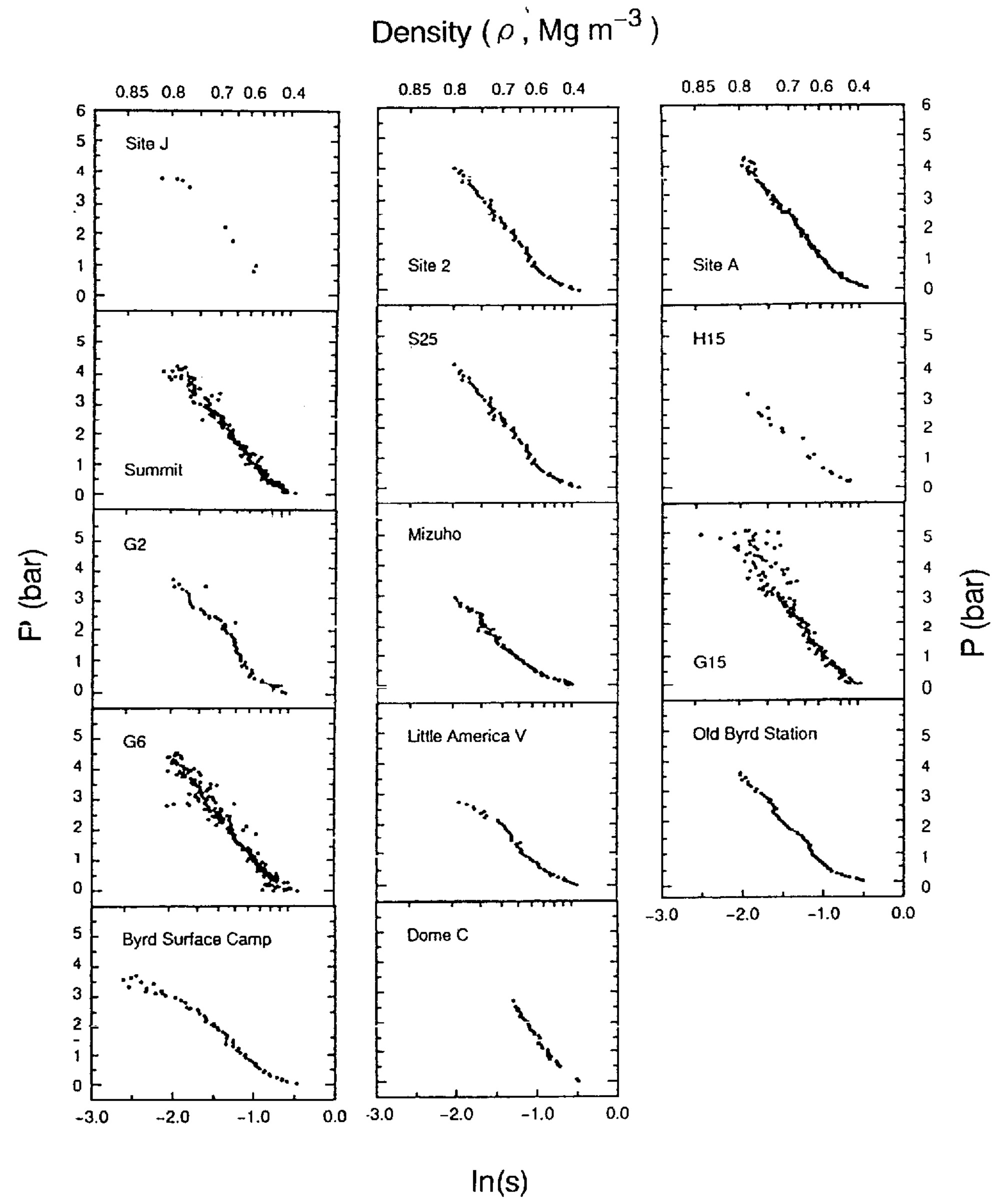


*Fig. 3. Relations between* $P$ *and* $\ln(s)$ *for 14 sites in Greenland and Antarctica.*

*Table 2. Slopes, y intercepts, correlation coefficients and degrees of freedom for the relation in Equation (9)*

| *Site* | *Slope* $C_1$ | *y intercept* $C_2$ | *Correlation coefficient* $r$ | *Degrees of freedom* |
|---|---|---|---|---|
| Site J | −13.3 | 1.59 | 0.98 | 18 |
| Site 2 | −13.4 | 1.58 | 0.99 | 62 |
| Site A | −11.5 | 1.54 | 0.99 | 121 |
| Summit | 11.8 | 1.57 | 0.99 | 266 |
| S25 | −13.5 | 1.02 | 0.98 | 85 |
| H15 | −11.7 | 1.23 | 0.99 | 19 |
| G2 | −15.9 | 1.65 | 0.99 | 54 |
| Mizuho Station | −13.3 | 1.13 | 0.98 | 85 |
| G15 | −14.0 | 1.80 | 0.97 | 146 |
| G6 | −13.4 | 1.71 | 0.95 | 227 |
| Little America V | −12.3 | 1.35 | 0.99 | 44 |
| Old Byrd Station | −12.8 | 1.36 | 0.99 | 41 |
| Byrd surface camp | 11.1 | 1.23 | 0.99 | 55 |
| Dome C | −13.2 | 2.10 | 0.97 | 37 |
| Average | −12.9 | 1.49 | 0.98 | |
| Standard deviation ($\sigma$) | 1.2 | 0.28 | 0.01 | |

*Table 3. Slopes, y intercepts, correlation coefficients and degrees of freedom for the relation in Equation (10)*

| *Site* | *Slope* $D_3$ | *y intercept* $D_4$ | *Correlation coefficient* $r$ | *Degrees of freedom* |
|---|---|---|---|---|
| Site J | 1.73 | 1.20 | 0.97 | 18 |
| Site 2 | −2.58 | −3.13 | 0.99 | 62 |
| Site A | −3.00 | −1.91 | 0.99 | 121 |
| Summit | −3.01 | −1.90 | 0.99 | 266 |
| S25 | −2.22 | −1.73 | 0.97 | 85 |
| H15 | −2.25 | 1.50 | 0.99 | 19 |
| G2 | −2.91 | −2.12 | 0.97 | 54 |
| Mizuho Station | −2.12 | −1.55 | 0.97 | 85 |
| G15 | −3.46 | −2.31 | 0.96 | 146 |
| G6 | −3.06 | −1.90 | 0.96 | 227 |
| Little America V | −2.19 | −1.32 | 0.99 | 44 |
| Old Byrd Station | −2.61 | −1.84 | 0.99 | 41 |
| Byrd surface camp | −2.23 | −1.39 | 0.99 | 55 |
| Dome C | −3.19 | −1.75 | 0.99 | 37 |
| Average | −2.61 | −1.82 | 0.98 | |
| Standard deviation ($\sigma$) | 0.48 | 0.47 | 0.01 | |

Greenland ice sheet (Watanabe and Fujii, 1989) and the core contains 16.1% by volume of melt features (paper in preparation by Kameda and others). Thus, refrozen water around the grains can strengthen the grain bonding which may result in a delay in grain settling in the upper firn. On the other hand, Mizuho Station is located in a characteristic area where an hiatus in snow deposition has been studied (Watanabe and others, 1978). Katabatic winds produce denser snow/firn layers at the surface around this area. This may cause a higher density value for the Mizuho ice core.

Standard deviations of measured densities from the calculated densities in 14 ice cores are 0.02 $\mathrm{Mg\,m^{-3}}$ on average (i.e. 3.2% accuracy). The standard deviations in the site J and Mizuho ice cores are 0.05 $\mathrm{Mg\,m^{-3}}$ and 0.06 $\mathrm{Mg\,m^{-3}}$, respectively. These disagreements are mainly from $C_2$ values at site J and Mizuho. That is, the agreement could be improved considerably, if the initial density value is adopted instead of $T$ in Equation (11) for each site.

### 4.2. The relation between $P$ and $\ln(s)$

The slopes and $y$ intercepts were investigated to find the dependencies on the 10 m depth firn temperatures and/or annual accumulation rates. It was found that the slopes, $D_3$, are well correlated with 10 m firn temperatures ($r = 0.72$) as shown in Figure 6. For the same reasons as above, the values at site J and Mizuho could be deleted from Figure 6. The correlation coefficient between the slopes and the 10 m depth firn temperatures has a slightly higher value of $r = 0.76$ if site J and Mizuho values are excluded. Using this temperature-dependence, we obtain the following equation:

$$P = (0.0326T - 10.6)\ln(s) - 1.82 \tag{12}$$

where $T$, $P$, $\rho$ and $\rho_i$ are the same as in Equation (11).

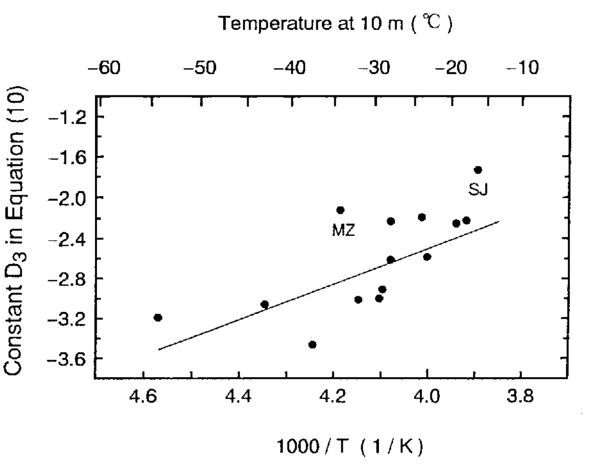


*Fig. 6. Temperature-dependence of the slopes in Equation (10). SJ and MZ refer to ice cores from site J and Mizuho, respectively.*



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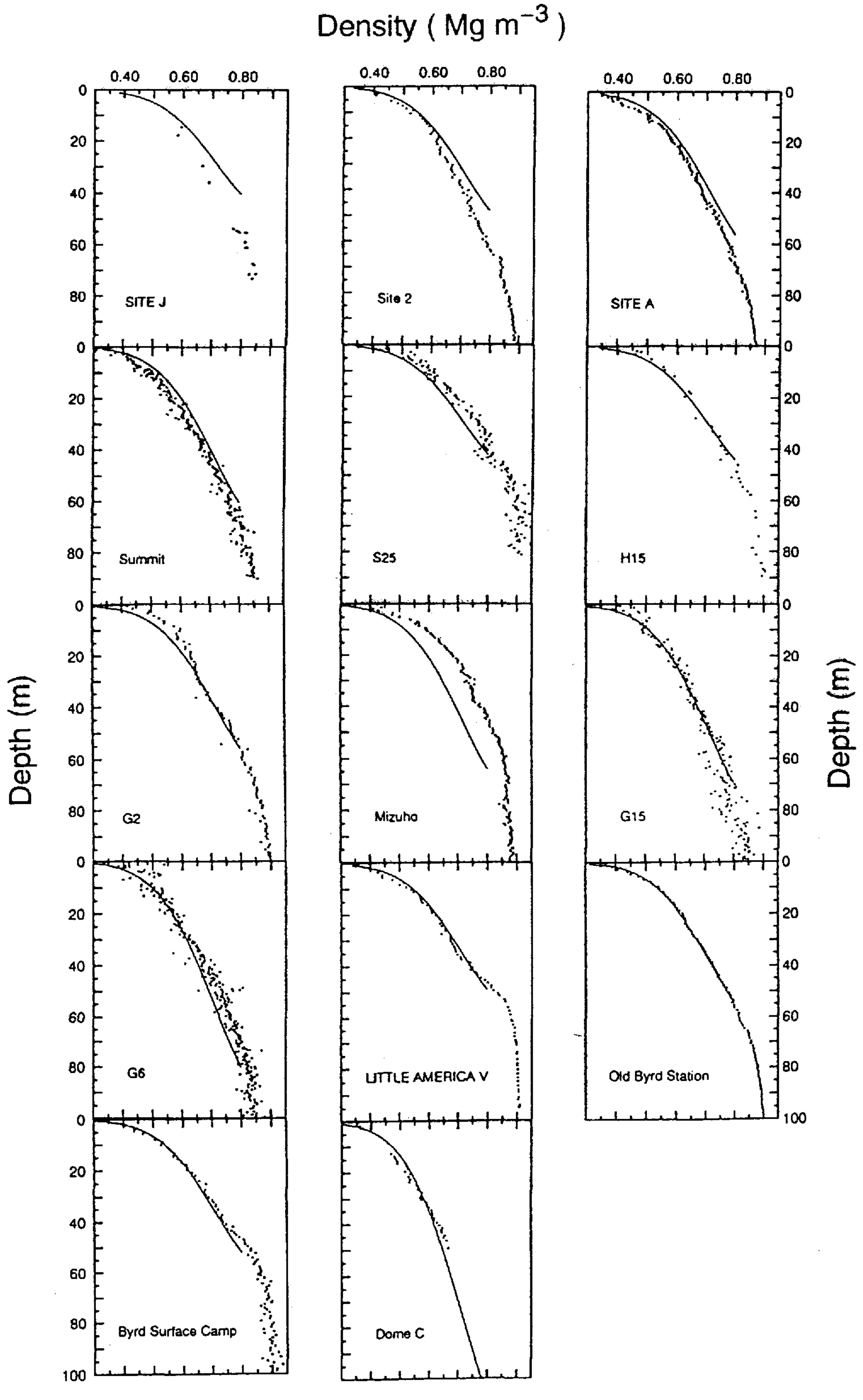


*Fig. 5. Comparisons between measured density values (dots) and calculated profiles (lines). The calculated profiles are obtained from Equation (11).*

Comparisons of measured and calculated density values using Equation (12) are shown in Figure 7. Again, it is seen that site J and Mizuho ice-core data have significant shifts from the curves calculated from Equation (12). Average differences between the measured and calculated densities in 14 ice cores are 0.03 Mg m$^{-3}$ (i.e. 4.7% accuracy). The average differences between measured and calculated values for site J and Mizuho ice cores are both 0.06 Mg m$^{-3}$, which is approximately the same magnitude as for the case using Equation (11).

## CONCLUSION

These findings suggest that firn density can be determined primarily by overburden pressure; firn temperature



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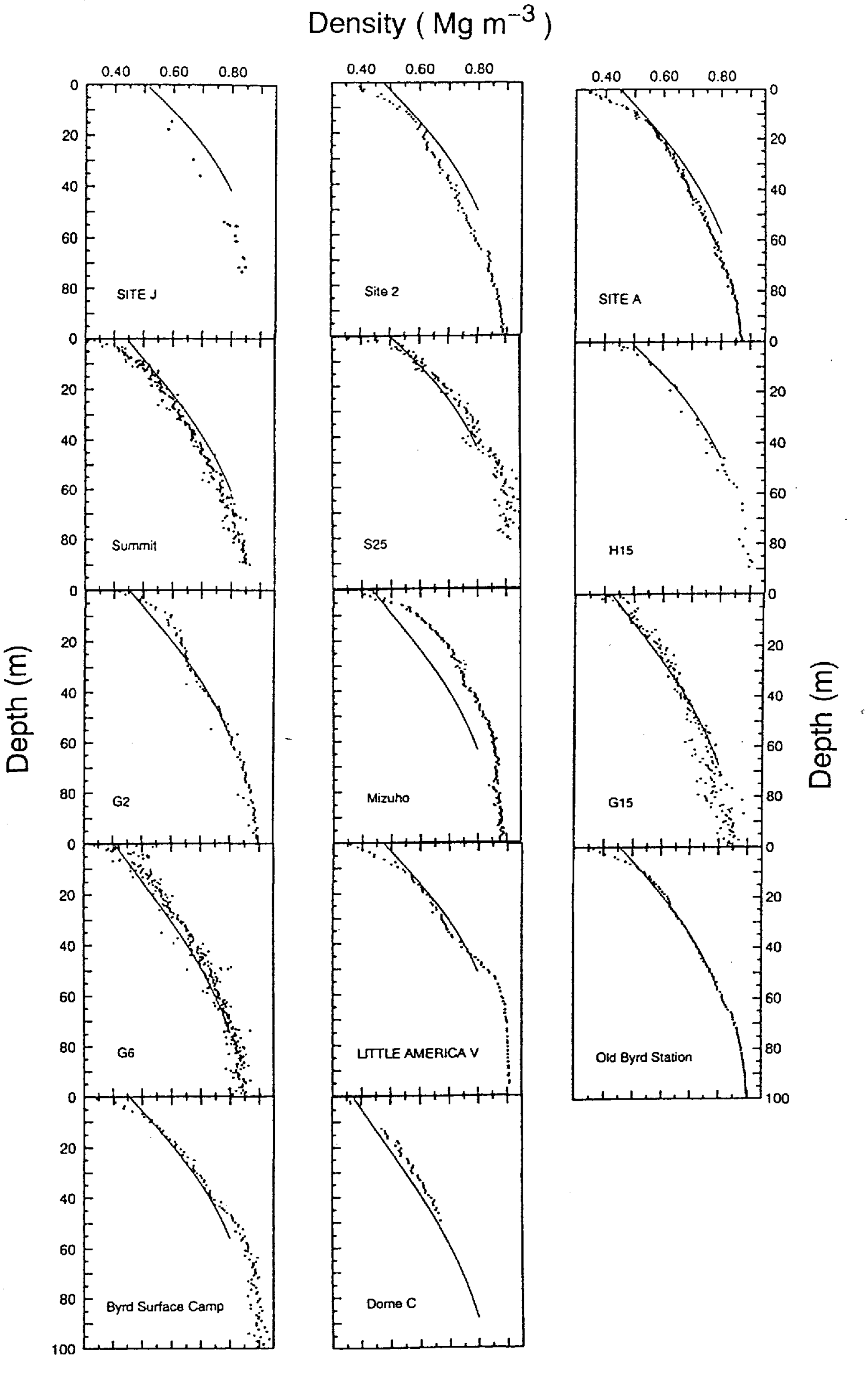


*Fig. 7. Comparisons between measured density values (dots) and calculated profiles (lines). The calculated profiles are obtained from Equation (12).*

contributes to a lesser degree, as expressed by Equations (11) and (12). That is, the validity of the equation proposed suggests that overburden pressure is the most dominant factor in the densification process. Apparently, age-dependency is not as strong as expected previously. For sites in a percolation zone or hiatus-observed area, each initial density at the surface should be taken into account for a better fit with the measured density data.

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from GRIP members at Summit, Greenland. Ice cores from east Dronning Maud Land, Antarctica, were recovered by the 23rd, 24th, 25th, 26th, 27th and 32nd Japanese Antarctic Research Expeditions.

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