THE MOTION OF HYPERION

ON THE ACCURACY OF THE OBSERVATIONS

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Abstract.
93 observations of the seventh satellite of Saturn, Hyperion, with 473 observations of Titan—Saturn's sixth and the most massive satellite and the one nearest to Hyperion—are considered. The observations were made in 1967–1981 at several observatories. The values of (O–C) across and along the orbits are obtained. The normality of the distributions of (O–C) is studied.

1. Introduction

Since its discovery, Saturn's seventh satellite Hyperion has attracted the attention of scientists working in the field of celestial mechanics by peculiarities of its motion, which are related to the strong influence of Titan [2, 5]. Being rather dark (magnitude near 14^m) Hyperion is a difficult object for observations. In this sense, it is very interesting to investigate the accuracy of existing photographic astrometric observations comparing them with a numerical theory of motion.

2. Observations

I used photographic astrometric observations (α, δ) made at different observatories between 1967 and 1981 (a total of 473 observations for Titan and 93 observations of Hyperion). The observations were used also in [4] in order to improve the nonsingular orbital elements. The observations are made at the following observatories: Pulkovo, Table Mountain, McDonald, McCormick, Greenwich, US Naval Observatory, Bordeaux, European Southern Observatory.

S. Ferraz-Mello et al. (eds.), Dynamics, Ephemerides and Astrometry of the Solar System, 137–140.

3. Theory of Motion

I compared the observed equatorial coordinates (reduced to the geocentric ones) with those given by my numerical theory [4]. In this theory the equations of motion for Titan and Hyperion were integrated jointly by the method of extrapolation by the rational functions [1] in order to avoid any loss of accuracy modified by Encke transformations. The program provides an accuracy better than $0''\!.001$ in the satellite's geocentric position over an interval of no less than 10 years. The theory allows for perturbations caused by the Sun, Jupiter, zonal harmonics J_2 and J_4 of Saturn's gravitational field, and other satellites (from Mimas to Phoebe).

4. (O–C)

4.1. PROJECTION OF THE ORBIT

I obtained the values of (O–C) across and along the projection of the orbits of the satellites on the celestial sphere. In order to project the (O–C) onto the visible orbits I used the following suggestions. Let

$$\Delta\alpha = O_\alpha - C_\alpha,$$

$$\Delta\delta = O_\delta - C_\delta.$$

Then, define the vector (O–C) by the following:

$$\vec{oc} = (\Delta\alpha \cos\delta, \Delta\delta)$$

(we can suppose that (O–C) is rather small to allow us to manipulate with this part of the celestial sphere as if it were planar).

Define the tangent to the visible orbit by the following:

$$\vec{tng} = (\dot{\alpha}\cos\delta, \dot{\delta}).$$

Then let γ be angle between the α axis and $\vec{oc}$, ϵ be angle between the α axis and $\vec{tng}$, ϕ be angle between the $\vec{tng}$ and $\vec{oc}$. Then the (O–C) across and along the visible orbit may be written by the following:

$$(\mathrm{O}-\mathrm{C})_{across} = |\vec{oc}|\sin\phi,$$

$$(\mathrm{O}-\mathrm{C})_{along} = |\vec{oc}|\cos\phi,$$

where

$$\phi = \gamma - \epsilon,$$

$$\cos\gamma = \frac{\Delta\alpha\cos\delta}{|\vec{oc}|}, \qquad \sin\gamma = \frac{\Delta\delta}{|\vec{oc}|},$$

https://doi.org/10.1017/S0074180900127238 Published online by Cambridge University Press

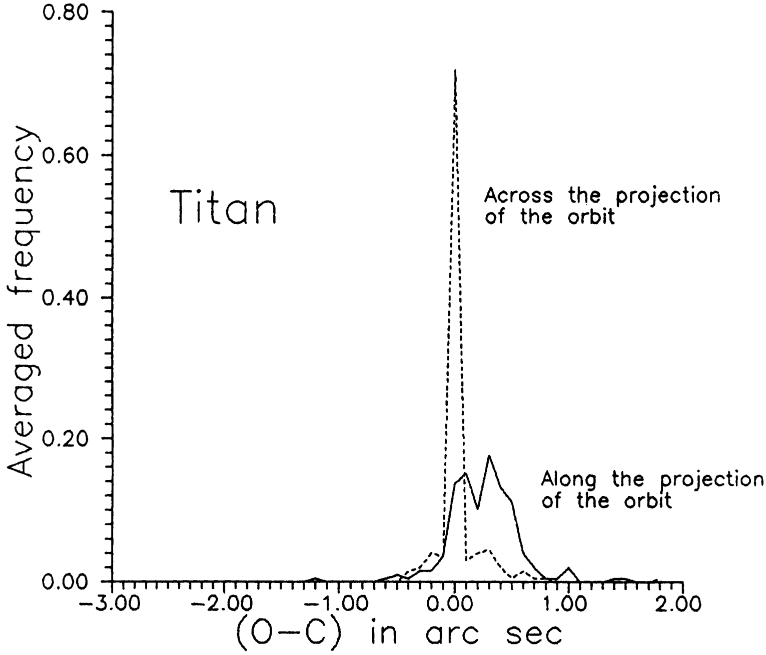

Figure 1. **Distribution of (O–C) for Titan**

$$\sin \epsilon = \frac{\dot{\delta}}{|\vec{tng}|}.$$

For the purpose of my investigation, I supposed that I can replace $\dot{\alpha}$ and $\dot{\delta}$ by $\dfrac{\alpha_{i+1} - \alpha_i}{t_{i+1} - t_i}$ and $\dfrac{\delta_{i+1} - \delta_i}{t_{i+1} - t_i}$—we only need the rather rough approximation to the real tangent if we wish estimate the distributions.

4.2. THE DISTRIBUTIONS

Then, I studied the normality of the distribution of the (O–C). The figures give us the distributions of (O–C) (in given intervals in the arc sec) along and across the projection of the orbit on the celestial sphere for Hyperion and for Titan. The y-axes give the averaged frequencies of occurrence of the (O–C) defined by the values of the x-axes.

One may see that the distributions along the orbits deviate slightly from the normal ones. The most probable values of the (O–C) along the projection of the orbit on the celestial sphere are near $0\rlap{.}''3$ for Titan and near $0\rlap{.}''4$ for Hyperion that corresponds with the accuracies of [4].

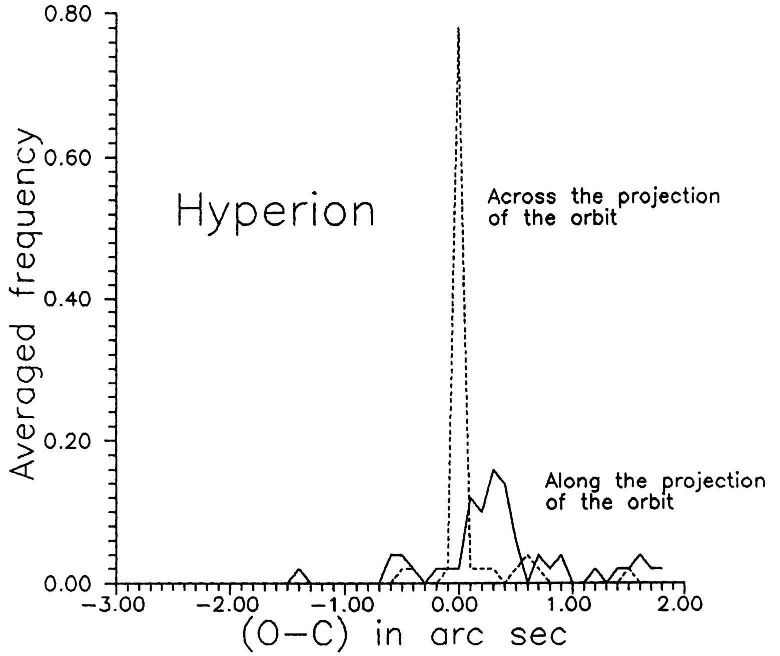

Figure 2. Distribution of (O–C) for Hyperion

Using the known formula for dispersion, I obtained the values of the root mean square errors of distributions of (O–C). They are: for Hyperion, 0.0255 across the orbit and (formally) along, 0.855; for Titan, 0.0236 across the orbit and (formally) 0.571 along the orbit.

The numbers above mean that there is a small systematical error which is to be eliminated by taking into account, in the numerical theory, the omitted factors, e. g. the influence of the errors in the ephemerides of Saturn.

References

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