

# **AUTOMATIC PILOTS FOR HELICOPTERS**

## **Part I—Theoretical Considerations**

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## **Part II—Flight Development**

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DR G S HISLOP (*Chairman of the Executive Council*) in the Chair

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### INTRODUCTION BY THE CHAIRMAN

The CHAIRMAN, in introducing the Authors, expressed pleasure in having two lecturers together to talk upon a very important subject. Experiments with automatic pilots for helicopters had been started a considerable time ago, and they would have a very important application in the future.

Mr H COLLOMOSSE, B Sc, was a graduate of Leeds University, and during the war served for five years in the Signals Branch of the R A F. Prior to joining Louis Newmark Ltd in 1955 as Principal Design Engineer in charge of the development of auto pilots for helicopters, he was engaged for about seven years in the guided weapons field. For a number of years he was employed in the Research Department of Smiths Aircraft Instruments, Ltd, where he was the Project Engineer responsible for the control and navigation systems of a guided weapon.

Mr M C CURTIES, B Sc , A F R Ae S , had worked at the R A E since 1938, with the exception of the period 1940-1946, during which time he served as a pilot in the R A F and flew operationally with Bomber Command His flying experience, in some 50 types of aircraft, included about 1,000 hours as a pilot, and over 600 hours as a flight test observer, of which 200 had been in helicopters (seven types in all) Since 1947 he had been engaged mainly in the development and testing of automatic controls for both fixed wing and rotating wing aircraft He obtained his B Sc degree from the University of London in 1947 as an external student and became an Associate Fellow of the Royal Aeronautical Society in 1949

## MR H COLLOMOSSE

### PART I Theoretical Considerations

#### INTRODUCTION

In this part of the paper the theoretical considerations underlying the design of an autopilot system for helicopters are discussed , in particular, the system investigated has three modes of operation, viz —

(i) *Stabiliser*

In this mode, whilst the pilot still exercises overall control, the autopilot introduces artificial stabilisation so that the helicopter becomes more easy to fly

(ii) *Cruising*

Here the pilot establishes a desired attitude and heading in straight-line flight , on selection the autopilot maintains this mode of flight automatically

(iii) *Automatic Hover*

Assuming that signals are available defining the plan-position of the helicopter with respect to the point on the ground over which the pilot wishes to hover, on selection the autopilot maintains this hover condition automatically

The investigation is confined to the study of this system for a single rotor type of aircraft As may be expected from the geometry of the fuselage and the position of the tail rotor, the form of the control law required to stabilise a helicopter in yaw is similar to that used in fixed wing aircraft autostabilisers, *i e* , control is applied proportional to the angular rate of the fuselage , since this technique is well known it is not discussed further in the paper Also, since the longitudinal and lateral control of a helicopter are essentially similar, the discussion is confined to the problem of determining a suitable control law for one axis only, viz , the longitudinal axis

Initially an elementary theory involving simple aerodynamics is discussed This theory gives a reasonable approximation to hovering flight conditions and provides a simple stability model based on physical principles, with the advantage that the form of control law required can be easily recognised

Having determined the general form of the control law required for the stabilisation of a helicopter in hovering flight, the design of a "Stabiliser" system based on this law is discussed and approximate values of the control parameters are calculated. Further, it is shown how these values satisfy forward flight conditions, since the analysis becomes rather more complex, the problem is usually examined on a simulator and, as an example, typical responses for the Whirlwind helicopter are shown. No attempt is made to optimise the control parameters, only the general order of values is given.

Finally, the discussion is extended in a general manner to determine the control laws required for the cases of "cruising" and "automatic hover".

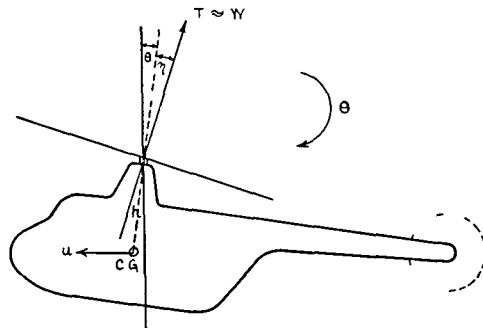
### ELEMENTARY STABILITY THEORY OF THE HELICOPTER IN HOVERING FLIGHT

In relation to a fixed wing aircraft certain peculiarities arise in the control of a helicopter due to the fact that both moments and forces are produced directly by tilting one control, viz, the rotor. Moments are produced by changes in rotor tilt relative to the fuselage, horizontal forces by changes in tilt relative to the vertical.

For small oscillations initiated from hovering flight the thrust of the rotor,  $T$ , may be assumed constant and equal to the weight of the helicopter,  $W$ . The oscillatory motion, therefore, has only two degrees of freedom, viz, the linear movements of the C of G along the horizontal axis, and the angular movements about the pitching axis through the C of G. Further, if we assume that the C of G is situated on the shaft of the rotor and that fuselage drag may be neglected, then an elementary stability theory of the helicopter in hovering flight may be developed.

Let us define the fore-and-aft tilt of the rotor thrust vector,  $T$ , relative to the rotor shaft as  $\eta$  measured positively in a clockwise direction (Fig 1),  $h$  as the distance between the rotor hub and the C of G,  $\theta$  as the angle of pitch of the fuselage measured positively in a clockwise direction from the vertical.

FIG 1 NOTATION



Let the helicopter be subject to a small disturbance, such that a small increment in velocity of the C of G in the horizontal direction,  $u$ , results. Due to the forward motion of the rotor hub through the air the velocity of the advancing blades is greater than that of the retreating blades. The lift on the advancing blades is greater, therefore, than that on the retreating

blades and, since the pitch of the blades is varying cyclically, this gives rise to a backward tilt of the tip-path plane until the redistribution of incidence cancels the difference in lift. The angle of tilt can be shown to be proportional to the forward speed of the rotor hub. Since the rotor hub is distant  $h$  from the C of G, angular motions about the C of G give translational movements of the hub, whence

$$\eta_u = a_u (u - h \theta)$$

Due to the inertia of the blades some delay exists between a tilt of the rotor shaft and the realignment of the rotor with the shaft. It may be shown that the angular displacement of the rotor plane with respect to the shaft is proportional to the angular velocity of the shaft, whence a contribution to  $\eta$  of magnitude  $-a_q \theta$

The general equation for  $\eta$  following a small disturbance from hovering flight is, therefore,

$$\eta = a_u u - (a_q + a_u h) \theta \quad \text{-----} \quad (1)$$

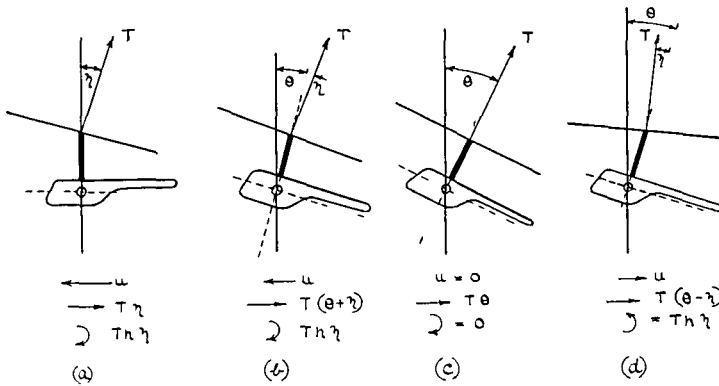


FIG 2 DIAGRAMMATIC REPRESENTATION OF THE UNCONTROLLED MOTION OF A HELICOPTER

A diagrammatic representation of the uncontrolled motion is shown in Fig 2. Due to the small increment of velocity,  $u$ , in the horizontal direction, the thrust vector,  $T$ , tilts through the angle  $\eta$ , in the opposite sense to the motion of the C of G, Fig 2(a). This produces a retarding force  $T\eta$  and a moment  $Th\eta$  about the C of G which in turn produces an angular acceleration of the fuselage. After a short interval of time the retarding force will be  $T(\theta + \eta)$ , Fig 2(b), and this force will eventually bring the C of G to rest,  $u = 0$ , however, the fuselage will still have a tilt,  $\theta$ , and the force  $T\theta$  will start accelerating the C of G in the opposite direction, Fig 2(c). When the C of G has acquired a velocity  $u$ ,  $T$  will tilt through an angle  $\eta$  in a direction opposing the motion, Fig 2(d). This motion may be examined as follows —

Since  $T = W = mg$  the equations of motion are

$$u = -g(\theta + \eta) \quad \text{-----} \quad (2)$$

$$\theta = M_\eta \eta \quad \text{-----} \quad (3)$$

where  $M_\eta = \frac{gh}{k_y^2}$  and  $k_y$  is the radius of gyration about the pitch axis

Equations (1), (2) and (3) lead to the characteristic equation of the uncontrolled motion

$$p^3 + [a_u g + (a_q + a_u h) M_\eta] p^2 + a_u g M_\eta = 0 \quad \text{-----} \quad (4)$$

(where  $p \equiv \frac{d}{dt}$ ) which is clearly unstable since the coefficients are essentially positive and there is no term in  $p$

Typical values of  $a_u$ ,  $a_q$ ,  $h$  and  $k_y^2$  for a lightweight type of helicopter are

$$a_u = 0.607 \times 10^{-3}, \quad a_q = 0.0766, \quad h = 4 \text{ ft}, \quad k_y^2 = 14.6 \text{ ft}^2$$

Assuming  $g = 32.2 \text{ ft/sec}^2$ ,  $M_\eta = 8.82$  and eqn (4) becomes

$$p^3 + 7164 p^2 + 1724 = 0$$

$$\text{i.e. } (p + 92)(p^2 + 2036 p + 1873) = 0$$

The uncontrolled motion consists, therefore, of a subsidence and a divergent oscillation of period equal to 15 sec, whose amplitude is doubled in 6.8 sec

#### GENERAL FORM OF CONTROL LAW REQUIRED TO STABILISE A HELICOPTER IN HOVERING FLIGHT

When a cyclic control input,  $\eta_s$ , is applied the general equation for  $\eta$  becomes

$$\eta = \eta_s + a_u u - (a_q + a_u h) \theta \quad \text{-----} \quad (5)$$

and the problem is, therefore, to find a suitable control law

$$\eta_s = f(\theta)$$

such that the motion of the helicopter is stable

From equations (2), (3) and (5) we have

$$\theta + [a_u g + (a_q + a_u h) M_\eta] \theta + a_u g M_\eta \int \theta dt = M_\eta \eta_s$$

which suggests that a control law

$$\eta_s = -K_\theta \theta$$

will be suitable. The resulting characteristic equation is

$$p^3 + [a_u g + (a_q + a_u h) M_\eta] p^2 + K_\theta M_\eta p + a_u g M_\eta = 0 \quad (6)$$

and the motion is stable, since  $K_\theta$  may be chosen such that

$$K_{\theta} [a_u g + (a_q + a_u h) M_{\eta}] > a_u g$$

We have established, therefore, the fundamental law that the motion of a helicopter is stable if cyclic control displacements are applied, in the correct phase, proportional to changes in attitude of the fuselage

However, a closer examination of equation (6) reveals that an improvement to this fundamental law may be made. Since the sum of the roots is equal to  $- [a_u g + (a_q + a_u h) M_{\eta}] / e$ ,  $- a_q M_{\eta}$  approximately, the damping of the oscillatory motion can only be increased at the expense of the subsidence, and the inherent damping,  $a_q$ , of the helicopter is insufficient to provide adequately for both. This difficulty may be overcome by applying cyclic control proportional to  $\theta$  as well as  $\dot{\theta}$

The general control law now becomes

$$\eta_s = - K_{\theta} \theta - K_q \dot{\theta} \quad \text{-----} \quad (7)$$

resulting in the characteristic equation

$$p^3 + [a_u g + (K_q + a_q + a_u h) M_{\eta}] p^2 + K_{\theta} M_{\eta} p + a_u g M_{\eta} = 0 \quad (8)$$

Bearing in mind the orders of value of  $a_u$  and  $a_q$ , for sufficiently large autopilot gearings, *i.e.*, large value of  $K_{\theta}$  and  $K_q$ , this equation splits into the approximate factors,

$$\left( p + \frac{a_u g}{K_{\theta}} \right) (p^2 + K_q M_{\eta} p + K_{\theta} M_{\eta}) = 0$$

Since  $a_u g$  is small the first factor implies a slow subsidence, the larger  $K_{\theta}$  the smaller the damping implied by this root. The second factor in the equation, the short period oscillation, has an undamped natural frequency  $\frac{(M_{\eta} K_{\theta})^{\frac{1}{2}}}{2 \pi}$  determined by the choice of  $K_{\theta}$ , whilst the damping is determined

by the choice of  $K_q$ , the damping ratio being equal to  $\pi \frac{K_q}{K_{\theta}}$  times the undamped natural frequency. (As far as the controlled motion is concerned, we shall refer to oscillations of the order of 5 secs period as short period oscillations and oscillations of the order of 70 secs period as long period oscillations)

This slow subsidence is not of great importance during pilot monitored flight, *i.e.*, on the "stabiliser" condition, the short period stability being the essential gain. For the "cruise" and "automatic hover" conditions, however, a slow subsidence may prove to be unsatisfactory, and it is interesting to note here that some improvement may be expected from the introduction of a term in our general control law proportional to  $u$  or  $\int \theta dt$

#### A "STABILISER" SYSTEM BASED ON THE GENERAL CONTROL LAW

Having established the general form of the control law required to stabilise a helicopter we must now turn our attention to the problem of designing a "Stabiliser" system based on this law

Due to the requirement for an angular position signal, difficulties arise in the design of such a system when manoeuvre cases are considered. Clearly it is desirable that the system should confer stability on the helicopter during all possible modes of flight.

This requirement can be met by a system in which  $\theta$  is measured by a rate gyro and a quasi-position signal,  $\theta$ , is derived by means of a leaky-integrator network of suitable time constant. This "short memory"  $\theta$  signal is effective for the purposes of stabilisation, but does not imply a long term datum such as that which would be present if  $\theta$  were measured by a position gyro. The control law now becomes

$$\eta_s = - \left[ K_q + \frac{K_\theta n}{1 + np} \right] \theta \quad \text{-----} \quad (9)$$

where  $n$  is the time constant of the leaky-integrator network in seconds.

In order that the analysis may be simplified, let us assume for the moment that our elementary theory, which provides a reasonable approximation to hovering flight only, will give values of the control parameters which will prove satisfactory for other conditions of flight. The more complex analysis which must be used to study other conditions of flight is discussed in para 5.

The characteristic equation now becomes

$$\begin{aligned} p^4 + \left[ (a_u g + (a_q + a_u h + K_q) M_\eta) + \frac{1}{n} \right] p^3 \\ + \left[ K_\theta M_\eta + \frac{(a_u g + (a_q + a_u h + K_q) M_\eta)}{n} \right] p^2 \\ + a_u g M_\eta p + \frac{a_u g M_\eta}{n} = 0 \end{aligned}$$

It will be observed that this quartic equation reduces to the cubic, equation (8), as  $n \rightarrow \infty$ . Choosing the ratio  $\frac{K_q}{K_\theta} = \frac{1}{2}$  as being a reasonable practical value and taking the typical values of  $a_u, a_q$ , etc., given in para 2, the characteristic modes of oscillation are plotted in Fig. 3 for values of autopilot gearing,  $i.e.$ ,  $K_q$  equal to 0.1, 0.2, 0.3 and values of  $n$  equal to 5, 10,  $\infty$ . Here the time taken to halve the amplitude is plotted rather than the damping ratio since the latter can be somewhat misleading when long period oscillations are under discussion.

Three main points emerge from an examination of Fig. 3, *viz.*,

- (1) The system is not critical, the motion being stable over a wide range of values of the control parameters,  $K_q$  and  $n$ . The motion consists of a short period oscillation and a long period oscillation, except in the case  $n \rightarrow \infty$  when the latter degenerates into a subsidence.

- (ii) The period and time to half-amplitude of the short period oscillation are virtually independent of  $n$ , but both are decreased as the autopilot gearing is increased.
- (iii) For the long period oscillation, as  $n$  is increased the period is increased and the time to half-amplitude is decreased, but both are increased as the autopilot gearing is increased.

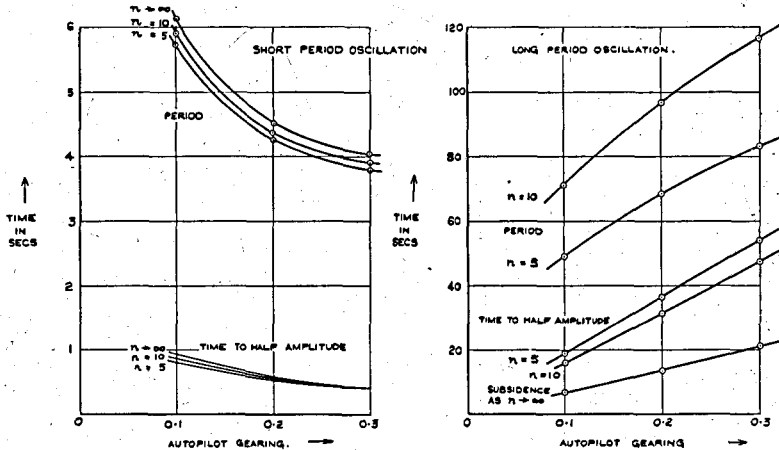


FIG. 3. PERIODS AND TIMES TO HALF AMPLITUDE FOR VARIATIONS IN AUTOPILOT GEARING AND LEAKY INTEGRATOR TIME CONSTANT. (ELEMENTARY THEORY.)

In a "Stabiliser" system, since the pilot exercises overall control, the long period oscillation is not of great importance, the short period stability being the essential gain. The autopilot gearing may therefore be chosen so that the period and damping of the short period oscillation give the "feel" desired by the pilot, whilst  $n$  may be chosen so that the performance of the system is satisfactory under manoeuvre conditions.

Let us examine these manoeuvre conditions more closely ; there are two cases to consider :

- (a) when the pilot demands a change of attitude of the helicopter,
- (b) when the pilot puts the helicopter into a steady turn.

Clearly the system must be designed so that the servo does not saturate or stabilisation of the aircraft will be lost ; further, during the initiation of a manoeuvre it is desirable that the servo does not move so as to oppose the pilot's demand.

These problems may be solved by the use of a parallel servo system as shown in Fig. 6. When the pilot wishes to manoeuvre, a signal from his controller precesses the rate gyro which causes the servo to move and put on cyclic control. The response of the aircraft causes the rate gyro to precess in the opposite direction and cancel the demand signal to the servo. Thus the rate gyro is maintained substantially at neutral and the servo system does not saturate ; only a small error signal is required from the gyro pick-off to maintain the servo in a desired position since the steady state gain of the system is high [ $K_q (1 + 2n)$  when  $K_\theta = 2 K_q$ ]. Transient disturbances are therefore measured by the rate gyro and the cyclic controls moved so as to stabilise the aircraft. Whilst the autopilot gearing is determined by



stability considerations, the pilot's gearing may be determined by the choice of the signal levels fed from his controller to the rate gyro

In the above discussion we have assumed that forces and moments are applied instantaneously by the cyclic controls according to the control law. Due to lags in the system, *e.g.*, the servo lag, it is necessary to phase-advance the signals from the rate gyro and its associated circuitry so that cyclic control is applied in the correct phase. If necessary the phase-advance circuitry may be designed to improve the damping of the short period oscillation, its effect is to decrease the short period frequency, since phase-advance of the rate component of the gyro signal is equivalent to an added inertia term, and phase-advance of the quasi position signal gives added damping.

#### EXTENSION OF THE THEORY TO OTHER CONDITIONS OF FLIGHT

In the above discussion we have used an elementary theory which gives a reasonable approximation to hovering flight only, it remains to be seen whether our control law, equation (9), proves satisfactory for other conditions of flight and the theory will now be extended to cover these cases.

Using the standard fixed-wing aircraft notation the longitudinal equations of motion of the helicopter in wind-body axes are —

$$u = x_u u + x_w w - \left(k - \frac{x_q}{\mu_2 p}\right) \theta + x_{\eta_s} \eta_s$$

$$w = z_u u + z_w w + \left(\mu + \frac{z_q}{\mu_2}\right) \theta + z_{\eta_s} \eta_s$$

$$\theta = \frac{\mu_2 m_u}{I_B} u + \frac{\mu_2 m_w}{I_B} w + \frac{m_q}{I_B} \theta + \frac{\mu_2 m_{\eta_s}}{I_B} \eta_s$$

In non-dimensional form our control law becomes

$$\eta_s = - \left[ A + \frac{BN}{1 + Np} \right] \theta$$

where  $N = \frac{n}{\hat{t}}$ ,  $A = \frac{K_q}{\hat{t}}$ ,  $B = \frac{K_\theta}{\hat{t}}$

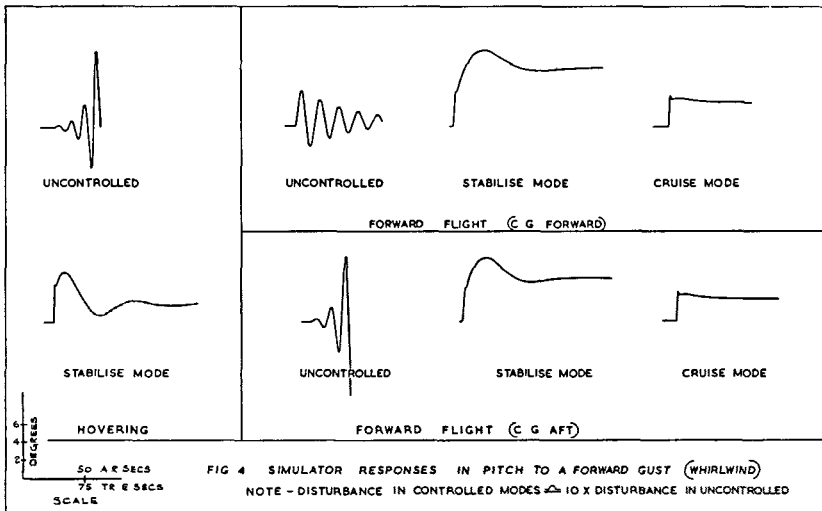
$\hat{t}$  being the unit of time in arseconds

The eliminant gives the characteristic equation

$$\begin{vmatrix} p - x_u & -x_w & \left(k - \frac{x_q}{\mu_2 p}\right) & -x_{\eta_s} \\ -z_u & p - z_w & -\left(\mu + \frac{z_q}{\mu_2}\right) & -z_{\eta_s} \\ -\frac{\mu_2 m_u}{I_B} & -\frac{\mu_2 m_w}{I_B} & p^2 - \frac{m_q}{I_B} p & -\frac{\mu_2 m_{\eta_s}}{I_B} \\ 0 & 0 & (A + BN) + ANp & (1 + Np) \end{vmatrix} = 0$$

which is a quintic. In practice it is usual to examine an equation of this order on a simulator since analytical methods are laborious, especially when one wishes to optimise the control parameters for the various conditions of flight.

As an example, typical responses of the Whirlwind to a forward gust are shown in Fig. 4, the aerodynamic derivatives having been provided by Messrs Westland Aircraft Ltd. Three conditions of flight are shown, viz,  $\mu = 0$  (hover) and  $\mu = 2$  in level flight (*i.e.* a forward speed of 118 ft/sec) with the C.G. forward and the C.G. aft. The values of the control parameters are  $K_q = 25$ ,  $\frac{K_\theta}{K_q} = 2.7$ ,  $n = 7.5$ ,  $\hat{t} = 1.525$  secs.



In the uncontrolled motion the short period instability is apparent for the hover and forward flight (C.G. aft) cases, but it is interesting to note that the motion is just stable for the same forward speed when the C.G. is forward, this is due to the stabilising effect of the fins mounted on the tail of the aircraft.

The controlled motion is seen to be stable in all three cases, the well damped short period oscillation being just visible as a kink at the beginning of each trace. The period of the long period oscillation is about 50 airsecs, *i.e.*, 76 secs, in hover and about 70 airsecs, *i.e.*, 106 secs, in forward flight.

Other conditions of flight may be examined in this manner, giving similar results, and it may be concluded that our control law is satisfactory for a "Stabiliser" system.

#### A "CRUISING" AUTOPILOT SYSTEM

The quasi-position signal,  $\theta$ , derived by integrating the rate gyro signal, gives "hands-off" stability for short intervals of time, but such a signal is not suitable as a long term datum. In a "Cruising" autopilot, therefore, a more suitable  $\theta$  signal must be used, such as that which may be obtained from the position gyro of an artificial horizon.

If a  $\theta$  datum signal is used to monitor our "Stabiliser" system and added to the  $\theta$  signal from the rate gyro before integration, the control law becomes of the form

$$\eta_s = - \left[ K_q + \frac{K_\theta n}{1 + np} \right] (\theta + K\theta) \quad (10)$$

where  $K$  is a constant

Using the control parameters given in para 5, typical responses are shown in Fig 4

It is observed that the long period oscillation which was present in the "Stabiliser" system has degenerated into a subsidence of reasonable damping. This improvement was predicted on page 50 by means of the elementary theory. On page 49, it was shown that a  $\theta$  signal (case  $n \rightarrow \infty$ ) caused the long period oscillation to degenerate into a subsidence with a resulting decrease in the time to half-amplitude. On page 50 it was pointed out that the damping of this subsidence could be improved by using a term in our control law proportional to  $\int \theta dt$ , in equation (10) a quasi-integral of  $\theta$  is introduced by the leaky-integrator network

#### THE "AUTOMATIC HOVER" AUTOPILOT

Since ability to hover is one of the important roles of the helicopter, a desirable feature of an autopilot would be to control the helicopter automatically in this mode of flight. Clearly in order to achieve this, signals giving the plan-position of the helicopter relative to the ground are required, for instance if suitable detecting equipment were installed in the aircraft such signals could be obtained from a radio or radar beacon situated on the ground. Assuming this intelligence to be available, a system may be designed so that cyclic control is applied proportional to the plan-position signals in such a sense so as to return the helicopter to the point of hover when disturbances take place in longitudinal and lateral directions. Vertical control may be achieved by applying collective pitch control proportional to changes in height, measured by a sensitive barometric instrument.

On page 50 we established that cyclic control inputs proportional to  $\theta$  and  $\dot{\theta}$  would stabilise a helicopter in hovering flight when the aircraft is under the control of the pilot. If we define the linear displacement of the helicopter from the point of hover as  $x$ , we wish to examine the stability of the system when cyclic control is applied proportional to  $\theta$ ,  $\dot{\theta}$  and  $x$ . Although tilting of the thrust vector,  $T$ , in the correct sense will tend to decrease  $x$ , this will be accompanied by a moment which will cause a change in attitude,  $\theta$ , and it does not follow, therefore, that the motion will necessarily be stable.

Writing  $x = \frac{u}{p}$  the control law becomes

$$\eta_s = - K_\theta \theta - K_q \dot{\theta} + K_x \frac{u}{p}$$

and the equation for  $\eta$

$$\eta = (a_u + \frac{K_x}{p}) u - K_\theta \theta - (a_q + a_u h + K_q) \dot{\theta}$$

The resulting characteristic equation is

$$p^4 + [a_u g + (a_q + u_u h + K_q) M_\eta] p^3 + (g K_x + K_\theta M_\eta) p^2 + a_u g M_\eta p + g M_\eta K_x = 0$$

Whence, for stability we must have

$$[a_u K_\theta - K_x (a_q + a_u h + K_q)] > \frac{g a_u^2}{a_u g + (a_q + a_u h + K_q) M_\eta}$$

Taking typical values as given in para 2, the R H S of this inequality is extremely small ( $a_u = 607 \times 10^{-3}$ ), and since  $K_q$  is large compared with  $(a_q + a_u h)$  we have approximately

$$K_x < a_u \frac{K_\theta}{K_q}$$

for stability This condition implies that  $K_x$  must be very small and the control law is unsatisfactory, therefore, since only a very weak  $x$  control can be applied

Clearly, in order to overcome this difficulty it is necessary to increase the effective value of  $a_u$  and this may be achieved by applying cyclic control proportional to the helicopter velocity,  $u$ , in addition to  $\theta$ ,  $\dot{\theta}$  and  $x$ , i.e.,

$$\eta_s = -K_\theta \theta - K_q \dot{\theta} + K_x \frac{u}{p} + K_u u$$

$$\text{giving } \eta = (a_u + K_u + \frac{K_x}{p}) u - K_\theta \theta - (a_q + a_u h + K_q) \dot{\theta}$$

With this control law the criterion for stability becomes

$$[(a_u + K_u) K_\theta - K_x (a_q + a_u h + K_q)] > \frac{g (a_u + K_u)^2}{(a_u + K_u) g + (a_q + a_u h + K_q) M_\eta}$$

or for  $K_u$  sufficiently large we have approximately

$$(K_u K_\theta - K_x K_q) > \frac{g K_u^2}{(g K_u + K_q M_\eta)}$$

This condition may be satisfied by choosing  $K_\theta$  and  $K_u$  sufficiently large Reasonable values of  $K_x$  may now be chosen to give adequate control of  $x$

In conclusion, therefore, control terms in  $\theta$ ,  $\dot{\theta}$ ,  $x$  and  $u$  are necessary to achieve satisfactory automatic hovering of a helicopter It is interesting to observe that this is only possible when autopilot gearings are chosen so that the aerodynamic derivatives themselves become insignificant

## NOTATION

$T$	rotor thrust vector
$W = mg$	weight of helicopter
$\eta$	fore-and-aft tilt of rotor thrust vector relative to line joining rotor hub to C of G
$h$	distance from rotor hub to C of G
$\theta$	angle of pitch of fuselage
$u$	increment in forward velocity of C of G
$\eta_u$	fore-and-aft tilt of rotor thrust vector due to increment in velocity $u$
$a_u$	derivative of rotor tilt with respect to $u$
$a_q$	derivative of rotor tilt with respect to angular velocity of pitch of fuselage
$M_\eta$	derivative of angular acceleration in pitch with respect to $\eta$
$k_y$	radius of gyration about the pitch axis
$\eta_s$	fore-and-aft cyclic control input
$K_\theta$	coefficient of $\theta$ in control law
$K_q$	coefficient of $\dot{\theta}$ in control law
$K$	coefficient of $\theta$ datum in control law for "Cruising" autopilot
$K_x$	coefficient of $x$ in control law
$K_u$	coefficient of $u$ in control law
$n$	time constant of leaky-integrator network in seconds
$x$	linear displacement of helicopter from point of hover in fore-and-aft direction
$\dot{x}_u, \dot{x}_w, \text{ etc}$	non-dimensional derivatives as in standard fixed-wing notation
$\hat{t}$	unit of time in airseconds
$p \equiv \frac{d}{dt}$	

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## PART II Flight Development

### INTRODUCTION TO THE FLIGHT EXPERIMENTS

In the first part of this paper the theoretical considerations underlying the design of an automatic control system for helicopters have been discussed. This second part describes some flight experiments carried out at R A E, Farnborough. In addition to demonstrating the validity of the theory presented, there has emerged from the flying a clear conception of the design features needed in a practical system for every day use by the average helicopter pilot.

One of the more interesting aspects of the experimental programme has been in the fact that the results were achieved almost entirely from flight tests. At the time when this work was commenced, aerodynamic information of the test aircraft was scanty. Thus whilst the theoretical form of the control law was known, the optimum values of the control parameters could only be roughly forecasted and the actual values were obtained in flight. Similarly the desirable operational features of a system were also appreciated.