A BOUSSINESQ MODEL FOR THE CONVECTION ZONE AND THE SOLAR ANGULAR VELOCITY

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Abstract. In this paper we study the dependence of the solar angular velocity on depth and latitude as produced by a meridional circulation in the convection zone.

We assume that the main mechanism responsible for setting up and driving the circulation is the interaction of rotation with convection. We solve the first-order equations (perturbation of the spherically symmetric state) for the motion and energy (diffusion equation) in the Boussinesq approximation and in the steady state for the axisymmetric case. The interaction of convection with rotation is considered through the convective transport coefficient $k(r) = k_0 + \varepsilon k_2(r) P_2(\cos\theta)$, where $P_2(\cos\theta)$ is the 2nd Legendre Polynomial which allows for the latitudinal perturbation and $k_2(r) \sim T_a(r)(F_c(r)/F_T)$, where T_a is the local Taylor number, F_c, F_T are convective and total fluxes, while ε is an expansion parameter. Here T_a takes into account the interaction of convection with rotation and F_c/F_T the energy transport.

The equations are numerically solved in order to determine the stream function ψ, with the boundary conditions that the fluid must be confined in a spherical shell, that there are no stresses at the boundaries and that the temperature and flux are spherically symmetric on the inner boundary. Once determined ψ we solve for the angular velocity and then determine ε by a fit with the observed angular velocity.

We obtained the following results for a Rayleigh number $R_a = 10^3$:

(1) A single cell circulation extending from poles to the equator and with the circulation directed toward the equator at the surface. Radial velocities are of the order of 10 cm s^{-1} and meridional ones of the order of 150 cm s^{-1}.

(2) A flux difference between poles and equator $\Delta F \approx 5 \times 10^{-2}$ at the surface, the poles being hotter.

(3) A negligible temperature difference between poles and equator at the surface.

(4) An angular velocity increasing inward.

(5) Angular velocity constant surfaces of spheroidal shape.

The model is consistent with the fact that the interaction of convection with rotation sets up the circulation (driven by the temperature gradient) which carries angular momentum toward the equator against the viscous friction. Unfortunately also a large ΔF is obtained. Nevertheless it seems that the model has the basic requisites for a correct dynamo action.

DISCUSSION

Paternò: I would like to add one more point to the model of Belvedere and myself. When we made the calculations for a Prandtl number equal to unity we obtained a flux difference between poles and equator $\Delta F = 5 \times 10^{-2}$. But using a Prandtl number $\sigma = 10^{-2}$ we obtained ΔF's as small as a few parts in 10^{-4}. In

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this case the same circulation carries a smaller amount of flux towards the surface. However a more appealing idea suggested by Prof. Roxburgh looks to be promising to solve the ΔF dilemma. We also made calculations for a convection zone twice as deep. The preliminary results seem to indicate that in this case the model generates larger differential rotation than flux differences and ΔF's of the order of 10^{-3} can be obtained. On the other hand, the depth of the convection zone, and the extent of convective overshooting into the radiative zone, is still a matter of discussion.