A FINITENESS CONDITION FOR LOCALLY COMPACT ABELIAN GROUPS

L. C. GROVE and L. J. LARDY

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1. Preliminaries

A map $f: A \to B$ in a category $\mathscr{C}$ is called *monic* if $fg = fh$ implies that $g = h$ for all maps $g, h : C \to A$; it is called *epic* if $gf = hf$ implies that $g = h$ for all maps $g, h : B \to C$. An object $A \in \mathscr{C}$ is called an *S-object* if every monic map $f: A \to A$ is also epic; it is called a *Q-object* if every epic map $f: A \to A$ is also monic. If A is both an S-object and a Q-object then A is called an *SQ-object*. In the category of sets the SQ-sets are the finite sets. In the category of vector spaces over a field F the SQ-spaces are precisely the finite dimensional spaces. In the light of these simple examples, it seems reasonable to view the SQ-objects of a category as being of 'finite type'. We shall be chiefly concerned with investigating the SQ-objects in certain subcategories of the category of locally compact abelian groups.

If $\mathscr{C}$ and $\mathscr{C}^*$ are dual categories then a map $f \in \mathscr{C}$ is epic if and only if $f^* \in \mathscr{C}^*$ is monic, and f is monic if and only if f^* is epic. As a result, we have

PROPOSITION 1. *If $\mathscr{C}$ and $\mathscr{C}^*$ are dual categories, then $A \in \mathscr{C}$ is an SQ-object if and only if $A^* \in \mathscr{C}^*$ is an SQ-object.*

PROPOSITION 2. *Suppose $\mathscr{C}$ is an additive category, $A_1, A_2 \in \mathscr{C}$, and $B = A_1 \oplus A_2$.* (a) *If B is an SQ-object then A_1 and A_2 are SQ-objects.* (b) *If A_1 and A_2 are SQ-objects,* Hom $(A_1, A_2) = 0$, *and* Hom $(A_2, A_1) = 0$, *then B is an SQ-object.*

PROOF. (a) If $f: A_1 \to A_1$ then there is [6, p. 251] a unique map $g : B \to B$ satisfying either, and hence both, of the dual conditions

(1) $\pi_1 g = f\pi_1$ and $\pi_2 g = \pi_2$, or

(2) $g\iota_1 = \iota_1 f$ and $g\iota_2 = \iota_2$

(the maps π_j and ι_j may be thought of as the canonical projections and injections associated with a direct sum). Suppose that f is monic and that $gh = gk$. Then $f\pi_1 h = \pi_1 gh = \pi_1 gk = f\pi_1 k$, so $\pi_1 h = \pi_1 k$. Also $\pi_2 h = \pi_2 gh = \pi_2 gk = \pi_2 k$. Thus $\iota_1 \pi_1 h = \iota_1 \pi_1 k, \iota_2 \pi_2 h = \iota_2 \pi_2 k$, so $h = \iota_1 \pi_1 h + \iota_2 \pi_2 h = \iota_1 \pi_1 k + \iota_2 \pi_2 k = k$, and g is monic. Since B is SQ, g is also epic. If $hf = kf$, then $h\pi_1 g = hf\pi_1 = kf\pi_1 = k\pi_1 g$, and so $h = k$, since $\pi_1 g$ is epic. Thus f is epic. A similar argument shows that if f is epic then f is monic. Thus A_1 is an SQ-object.

The proof of Proposition 4(b), below, is categorical in essence; it provides a proof of Proposition 2(b) as well.

2. Locally Compact Abelian Groups

The category of locally compact abelian groups with continuous homomorphisms will be denoted by $\mathscr{L}\mathscr{C}\mathscr{A}$. The subcategories of compact and discrete abelian groups will be denoted by $\mathscr{C}\mathscr{A}$ and $\mathscr{D}\mathscr{A}$, respectively. By the Pontryagin Duality Theorem the category $\mathscr{L}\mathscr{C}\mathscr{A}$ is self dual, and the categories $\mathscr{C}\mathscr{A}$ and $\mathscr{D}\mathscr{A}$ are dual to one another. Note that a map $f \in \mathscr{L}\mathscr{C}\mathscr{A}$ is monic if and only if it is one-to-one, and epic if and only if it has dense range.

The following groups, with their usual topologies, will enter into the discussion: Z will denote the additive group of integers, R the real numbers, $R_a \in \mathscr{D}\mathscr{A}$ the rationals, $S_{\boldsymbol{a}} \in \mathscr{C}\mathscr{A}$ the $\boldsymbol{a}$-adic solenoid (where $\boldsymbol{a} = (2, 3, 4, \cdots)$, see [5, p. 114]), $\Omega_p \in \mathscr{L}\mathscr{C}\mathscr{A}$ the p-adic numbers, and $\Delta_p \in \mathscr{C}\mathscr{A}$ the p-adic integers.

Proposition 3. *Suppose $G \in \mathscr{L}\mathscr{C}\mathscr{A}$ is the local direct product of groups $G_\alpha \in \mathscr{L}\mathscr{C}\mathscr{A}$ relative to open subgroups H_α* (see [5, p. 56]), *and let $\pi_\alpha : G \to G_\alpha$ and $\iota_\alpha : G_\alpha \to G$ denote the canonical projections and injections. Suppose $H \in \mathscr{L}\mathscr{C}\mathscr{A}$.* (a) *If $g, h \in \mathrm{Hom}\,(G, H)$ and $g\iota_\alpha = h\iota_\alpha$, all α, then $g = h$.* (b) *If $g, h \in \mathrm{Hom}\,(H, G)$ and $\pi_\alpha g = \pi_\alpha h$, all α, then $g = h$.*

The proof is elementary, and will be omitted.

Proposition 4. *Suppose $G \in \mathscr{L}\mathscr{C}\mathscr{A}$ is the local direct product of groups $G_\alpha \in \mathscr{L}\mathscr{C}\mathscr{A}$.* (a) *If G is SQ then each G_α is SQ.* (b) *If each G_α is SQ and if $\mathrm{Hom}\,(G_\alpha, G_\beta) = 0$ for $\alpha \neq \beta$ then G is SQ.*

Proof. (a) $\mathscr{L}\mathscr{C}\mathscr{A}$ is an additive category, and G is topologically isomorphic with $G_\alpha \oplus G'$, where G' is the local direct product of all G_β, $\beta \neq \alpha$. By Proposition 2(a), G_α is SQ. (b) If $f \in \mathrm{Hom}\,(G, G)$, define $f_\alpha = \pi_\alpha f \iota_\alpha \in \mathrm{Hom}\,(G_\alpha, G_\alpha)$ for each α. We show that f is monic (epic) if and only if every f_α is monic (epic). Observe that $\pi_\alpha \iota_\alpha \pi_\alpha = \pi_\alpha$, and hence that $\pi_\alpha f \iota_\alpha = \pi_\alpha \iota_\alpha \pi_\alpha f \iota_\alpha$. Also $\pi_\beta f \iota_\alpha = \pi_\beta \iota_\alpha \pi_\alpha f \iota_\alpha = 0$ if $\beta \neq \alpha$, and so, by Proposition 3(b), $f\iota_\alpha = \iota_\alpha \pi_\alpha f \iota_\alpha = \iota_\alpha f_\alpha$ for all α. A similar argument shows that $\pi_\alpha f = f_\alpha \pi_\alpha$ for all α.

Suppose then that $f \in \mathrm{Hom}\,(G, G)$ is monic and that $g, h \in \mathrm{Hom}\,(H, G_\alpha)$, with $f_\alpha g = f_\alpha h$. Then $f\iota_\alpha g = \iota_\alpha f_\alpha g = \iota_\alpha f_\alpha h = f\iota_\alpha h$. Since $f\iota_\alpha$ is monic we have $g = h$, and so f_α is monic.

Suppose every f_α is monic and that $g, h \in \mathrm{Hom}\,(H, G)$, with $fg = fh$. Then $f_\alpha \pi_\alpha g = \pi_\alpha fg = \pi_\alpha fh = f_\alpha \pi_\alpha h$, so $\pi_\alpha g = \pi_\alpha h$ for all α. Thus $g = h$ by Proposition 3(b), and so f is monic.

An analogous argument establishes that f is epic if and only if each f_α is epic, and the proposition follows immediately.

COROLLARY 1. *Suppose $G \in \mathscr{D}\mathscr{A}$ is a torsion group, with p-primary component G_p for each prime p. Then G is SQ if and only if each G_p is SQ.*

An example of an infinite primary SQ-group in $\mathscr{D}\mathscr{A}$ was given by Pierce [8, p. 302]. His construction was simplified by Megibben [7, p. 158]. It has been shown by Beaumont and Pierce [3, pp. 213 and 218] that any infinite primary S-group (hence any infinite primary SQ-group) must be uncountable but have cardinality less than or equal to that of the continuum.

We give another corollary to Proposition 4 that will prove useful later. If $n(p)$ is a cardinal number then $\Omega_p^{n(p)'}$ denotes the group consisting of all elements $x \in \Omega_p^{n(p)}$ for which the set of values of the p-adic valuation of the components of x is bounded. It is shown in [5, p. 420] that $\Omega_p^{n(p)'} \in \mathscr{L}\mathscr{C}\mathscr{A}$ is an injective envelope (minimal divisible extension) for $\Delta_p^{n(p)}$. Furthermore, the local direct product E of the groups $\Omega_p^{n(p)'}$ relative to the open subgroups $\Delta_p^{n(p)}$ is an injective envelope for $\prod_p \Delta_p^{n(p)}$.

COROLLARY 2. *The group E* (above) *is SQ if and only if each $n(p)$ is finite.*

PROOF. Observe that if $f \in \text{Hom}(\Omega_p, \Omega_q)$ then $f(r) = rf(1)$ for every $r \in R_a$ (viewing R_a as a subfield of both Ω_p and Ω_q). Since f is continuous and Ω_q is a topological field it follows that every p-Cauchy sequence in R_a is also a q-Cauchy sequence unless $f(1) = 0$, in which case $f = 0$. But if $p \neq q$ and $a_n = \sum_1^n p^k$ then it is easy to see that $\{a_n\}$ is p-Cauchy but not q-Cauchy. Thus $\text{Hom}(\Omega_p, \Omega_q) = 0$ if $p \neq q$.

Suppose E is SQ. If some $n(p)$ were infinite then clearly a shift map on $\Omega_p^{n(p)'}$ would be monic but not epic, contradicting Proposition 4(a). Suppose then that every $n(p)$ is finite. If $f \in \text{Hom}(\Omega_p, \Omega_p)$, then $f(r) = rf(1)$ for all $r \in R_a$, and hence $f(x) = xf(1)$ for all $x \in \Omega_p$, since R_a is dense in Ω_p and f is continuous. Thus f is either 0 or an isomorphism. If $\Omega_p^{n(p)'} = \Omega_p^{n(p)}$ is viewed as a vector space over Ω_p then clearly every $f \in \text{Hom}(\Omega_p^{n(p)}, \Omega_p^{n(p)})$ is a linear transformation. It follows that $\Omega_p^{n(p)}$ is an SQ-group, and hence that E is SQ, by Proposition 4(b).

THEOREM 1. *If $G \in \mathscr{D}\mathscr{A}$ is torsion free then G is SQ if and only if it is isomorphic with R_a^n for some non-negative $n \in Z$. Dually, if $G \in \mathscr{C}\mathscr{A}$ is connected then G is SQ if and only if it is* (*topologically*) *isomorphic with S_a^n for some non-negative $n \in Z$.*

PROOF. (see [2, p. 384]). If G is not divisible then nG is a proper subgroup of G for some positive integer n. Thus the map $x \to nx$ is monic but not epic. As a result G is divisible if it is SQ, hence it is a vector space over the field R_a. Every map $f: G \to G$ is a linear transformation so G is an SQ-group if and only if it is finite dimensional as a vector space over R_a. The dual statement follows from Proposition 1 since $G \in \mathscr{D}\mathscr{A}$ is torsion free if and only if $G^* \in \mathscr{C}\mathscr{A}$ is connected, the duality preserves direct sums, and $R_a^* = S_a$ (see [5, pp. 385 and 404]).

An investigation of (discrete) S-groups and Q-groups was conducted by R. Baer in [1]. Special cases of two theorems from [1, pp. 268 and 274] can be combined to give a set of sufficient conditions in order that G should be an SQ-group.

BAER'S THEOREM. *Suppose $G \in \mathscr{D}\mathscr{A}$ has a finite chain of subgroups $0 = H_0 \subseteq H_1 \subseteq \cdots \subseteq H_n = G$, satisfying*

(1) *if $f: G \to G$ is monic, then $fH_i \subseteq H_i$,*

(2) *if $f: G \to G$ is epic, then $fH_i = H_i$, and*

(3) *H_{i+1}/H_i is an SQ-group, $i = 0, 1, \cdots, n-1$.*

Then G is an SQ-group.

The proof, which is valid even for G nonabelian, will not be reproduced here. The essential steps may be seen, however, in the proof of Theorem 3, below, if the topological details are ignored.

THEOREM 2. *Suppose $G \in \mathscr{D}\mathscr{A}$ has torsion subgroup T. If T and G/T are SQ-groups then G is an SQ-group. Dually, suppose $G \in \mathscr{C}\mathscr{A}$ has connected component of the identity C. If C and G/C are SQ-groups then G is an SQ-group.*

PROOF. We prove only the first statement; the dual statement follows as in the proof of Theorem 1.

Set $H_0 = 0$, $H_1 = T$, and $H_2 = G$. If $f: G \to G$ is epic, set $g(x+T) = fx+T$. Then g is clearly well defined and epic in Hom $(G/T, G/T)$. Since G/T is SQ, g is also monic, so $fx+T = T$ if and only if $x \in T$, i.e. $fx \in T$ if and only if $x \in T$. Thus $f|T$ is epic in Hom (T, T), and condition (2) of Baer's Theorem holds. Conditions (1) and (3) obviously hold, so G is an SQ-group.

COROLLARY. *If $G \in \mathscr{D}\mathscr{A}$ splits, i.e. $G \cong T \oplus G/T$, then G is SQ if and only if both T and G/T are SQ.*

It is reasonable to ask whether the splitting hypothesis in the corollary is necessary. The answer, unfortunately, is yes, and as a result the torsion free and primary cases are not independent of one another in $\mathscr{D}\mathscr{A}$. For example, let G_p be the integers mod p for each prime p and set $G = \prod_p G_p$. Then $T = \Sigma_p \oplus G_p$, and it can be shown that Hom $(G, G) = \prod_p$ Hom (G_p, G_p) in the obvious fashion. It follows easily that G is SQ. Also, T is SQ by Proposition 4, Corollary 1. However, G/T is divisible and infinite dimensional as a vector space over R_a. Thus, by Theorem 1, G/T is not an SQ-group.

The next theorem is a topological version of Baer's Theorem.

THEOREM 3. *Suppose $G \in \mathscr{L}\mathscr{C}\mathscr{A}$ has a finite chain of subgroups $0 = H_0 \subseteq H_1 \subseteq \cdots \subseteq H_n = G$, satisfying, for $i = 0, 1, \cdots, n-1$,*

(1) *if $f: G \to G$ is monic, then $fH_i \subseteq H_i$,*

(2) *if $f: G \to G$ is epic, then $fH_i = H_i$,*

(3) $H_i \in \mathscr{C}\mathscr{A}$, *and*

(4) H_{i+1}/H_i *is SQ*.

Then G is an SQ-group.

PROOF. We prove the theorem with $n = 2$, writing $H_1 = H$. An easy induction completes the proof.

Suppose first that $f : G \to G$ is monic. Then $f|H$ is epic, since H is SQ. If $q : G \to G/H$ is the quotient map then qf induces a homomorphism $g : G/H \to G/H$ via $g(x+H) = fx+H$, i.e. $gq = qf$. If U is an open neighborhood of 0 in G/H, set $W = qf^{-1}(q^{-1}(U))$. Then W is open in G/H since f is continuous and q is both open and continuous. But $gW = qff^{-1}(q^{-1}(U)) \subseteq U$, so g is continuous. Also, $\ker qf = \{x : fx \in H\} = H$ since f is monic and $f|H$ is epic, and so g is monic. But then g is epic since G/H is SQ. Since q is an open map, $q(G \setminus f(G)^-)$ is an open subset of G/H. If it were not disjoint from $qfG = \text{range } g$, then there would exist $x \in G \setminus f(G)^-$ and $y \in G$ such that $qx = qfy$, i.e. $x - fy \in H$. But $fH = H$, so then $x - fy = fz$ for some $z \in H$. This is impossible because $x \notin fG$. Since g is epic we conclude that $q(G \setminus f(G)^-)$ is empty, hence that $f(G)^- = G$, i.e. that f is epic.

Suppose next that $f : G \to G$ is epic, and denote $\ker f$ by N. As above, qf induces $g \in \text{Hom}(G/H, G/H)$, with $gq = qf$. Since H is compact q is a closed map [5, p. 37]. Since f is epic we have $G/H = qG = q(f(G)^-) = (qf(G))^- = (gq(G))^- = (g(G/H))^-$, i.e. g is epic. Thus g is monic since G/H is SQ, and so $fx+H = g(x+H) = H$ if and only if $x \in H$. In particular, if $x \in N$ then $fx = 0$, so $x \in H$, and $N \subseteq H$. But $f|H$ is epic and H is SQ, so $f|H$ is monic and G is an SQ-group.

THEOREM 4. *If* $H \in \mathscr{C}\mathscr{A}$ *is SQ and* $0 \leqq n \in Z$ *then* $G = R^n \oplus H$ *is SQ in* $\mathscr{L}\mathscr{C}\mathscr{A}$.

PROOF. Since maps are continuous in $\mathscr{L}\mathscr{C}\mathscr{A}$ every $f \in \text{Hom}(R^n, R^n)$ is a linear transformation if R^n is considered as a vector space over R, and so R^n is SQ. If $f \in \text{Hom}(G, G)$ then, since $\text{Hom}(H, R^n) = 0$, we have $f(x, y) = (gx, hx+ky)$, where $g \in \text{Hom}(R^n, R^n)$, $h \in \text{Hom}(R^n, H)$, and $k \in \text{Hom}(H, H)$.

If f is epic then g must clearly be surjective since it is a linear transformation and since the range of f is a subset of $(\text{range } g) \times H$. But then g is also monic, and in fact a homeomorphism. Let us show that $kH = H$. If not, $H \setminus kH$ is an open subset of H. Choose $y \in H \setminus kH$ and an open neighborhood N of 0 in H such that $(y+N) \cap (kH+N)$ is empty. Next choose a neighborhood W of 0 in R^n such that $hx \in N$ if $x \in W$, and finally choose an open neighborhood U of 0 in R^n such that $x \in W$ if $gx \in U$ (g^{-1} is continuous). Then $U \times (y+N)$ is an open set in G; we show it to be disjoint from the range of f. Suppose $(u, v) \in U \times (y+N)$. If also $(u, v) \in \text{range } f$, then there exists $(r, s) \in G$ such that $gr = u$ and $hr+ks = v$. But then $r \in W$ and so $hr \in N$. Thus $v = ks+hr \in kH+\text{N}$, contradicting $v \in y+N$.

If we consider H as a subgroup of G then we have just shown that $fH = H$ for every epic f in $\text{Hom}(G, G)$. The theorem now follows from Theorem 3.

COROLLARY 1. *Suppose $G \in \mathscr{LCA}$ is compactly generated. Then G is SQ if and only if it is topologically isomorphic with $R^n \oplus H$, where $H \in \mathscr{CA}$ is SQ and $0 \leqq n \in Z$.*

PROOF. Since G is compactly generated it is topologically isomorphic with $R^n \oplus Z^m \oplus H$ [5, p. 90]. If G is SQ then it follows from Proposition 2 that $m = 0$ and H is SQ.

Corollary 1, together with Proposition 1, shows that the problem of determining all compactly generated SQ-groups in $\mathscr{LCA}$ is equivalent with that of determining all SQ-groups in the category $\mathscr{DA}$. An application of Theorem 1 determines all compactly generated connected SQ-groups in $\mathscr{LCA}$.

COROLLARY 2. *If G in $\mathscr{LCA}$ is compactly generated and connected, then G is SQ if and only if it is topologically isomorphic with $R^n \oplus S_a^m$, $0 \leqq n, m \in Z$.*

Finally, we determine all divisible torsion free SQ-groups in $\mathscr{LCA}$.

THEOREM 5. *Suppose G in $\mathscr{LCA}$ is torsion free and divisible. Then G is SQ if and only if it is topologically isomorphic with $R^j \oplus R_a^k \oplus S_a^m \oplus E$, where j, k, and m are non-negative integers and E is the injective envelope of $\prod_p \Delta_p^{n(p)}$ with $0 \leqq n(p) \in Z$ for each prime p.*

PROOF. By [5, p. 421] G is topologically isomorphic with $R^j \oplus (\Sigma \oplus R_a) \oplus S_a^m \oplus E$, where $0 \leqq j \in Z$, k and m are cardinals, there are k copies of R_a in the (discrete) direct sum, and E is the injective envelope of $\prod_p \Delta_p^{n(p)}$, with $n(p)$ a cardinal for each prime p. By Proposition 1, Theorem 1, and Corollary 2 of Proposition 4 we see that m, k, and all $n(p)$ are finite.

For the converse we first observe that E is self dual. This follows from remarks on page 422 of [5] and the fact that $\Delta_p^\perp \cong (\Omega_p/\Delta_p)^* \cong (Z(p^\infty))^* = \Delta_p$. For each prime p let π_p be the projection on E to $\Omega_p^{n(p)}$. If $f \in \mathrm{Hom}\,(R^j, E)$ then $\pi_p f \in \mathrm{Hom}\,(R^j, \Omega_p^{n(p)})$ so $\pi_p f = 0$ since R^j is connected and $\Omega_p^{n(p)}$ is totally disconnected. Thus $f = 0$ by Proposition 3, and $\mathrm{Hom}\,(R^j, E) = 0$. It follows, since both R^j and E are self dual, that $\mathrm{Hom}\,(E, R^j) = 0$. By Proposition 2(b), $R^j \oplus E$ is SQ. Since S_a^m is compact and connected we have $\mathrm{Hom}\,(S_a^m, R^j \oplus E) \cong \mathrm{Hom}\,(S_a^m, R^j) \oplus \mathrm{Hom}\,(S_a^m, E) = 0$. Viewing S_a^m as a subgroup of $R^j \oplus E \oplus S_a^m$ we see that $R^j \oplus E \oplus S_a^m$ is SQ, by Theorem 3, and hence that $R^j \oplus R_a^k \oplus E$ is SQ, substituting k for m and applying Proposition 1. Finally, $\mathrm{Hom}\,(S_a^m, R_a^k) = 0$ since S_a is connected and R_a is discrete, and so $\mathrm{Hom}\,(S_a^m, R^j \oplus R_a^k \oplus E) = 0$. Another application of Theorem 3 yields the fact that $R^j \oplus R_a^k \oplus S_a^m \oplus E$ is SQ, proving the theorem.

The divisibility hypothesis in Theorem 5 may not be necessary. The argument used in the proof of Theorem 1 yields only denseness of nG in G, but not divisibility.

3. Remarks

It might be of interest to study the SQ-objects in other specific categories. To mention one example, suppose $\mathscr{B}$ is the category of commutative B^*-algebras with identity and symmetric algebra homomorphisms. Then $\mathscr{B}$ is dual to the category $\mathscr{T}$ of compact Hausdorff spaces. It can be checked that epic means onto and monic means one-to-one in both categories. Thus if $A \in \mathscr{B}$, $X \in \mathscr{T}$, and $A = X^* = C(X)$, then A is an SQ-algebra if and only if X is an SQ-space.

An example has been constructed (see [4]) of an infinite compact connected Hausdorff space X for which the only continuous maps $f : X \to X$ are the identity and constant maps onto single points. Thus X is SQ in $\mathscr{T}$ and $A = C(X)$ is SQ in $\mathscr{B}$.

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The University of Oregon
Syracuse University