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CORRECTION

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Proposition 2.1 of the above paper is not correct. We thank Johan Segers (Université catholique de Louvain) for making us aware of this fact. The proof is correct except for the last line. Let us denote the mean value operator by E. The question is, what can we say about the limit behaviour of

$$\mathrm{E}\,M_k := \mathrm{E}\max(1, A_1^\alpha, A_1^\alpha A_2^\alpha, \dots, A_1^\alpha A_2^\alpha \cdots A_k^\alpha)$$

as $k \to \infty$? We can prove that the function $\mathrm{E}\,M_{\lfloor x \rfloor}$ is regularly varying with index 1 but the limit of $\mathrm{E}\,M_k/k$ (which is interesting in view of the examples in the above paper) is as yet unclear. Here is a sketch of the proof of regular variation.

Spitzer's identity (see Chung (1968, Theorem 8.5.1)) tells us that

$$\sum_{r=0}^{\infty} r^k\,\mathrm{E}\,M_k^\tau = \exp\biggl\{\sum_{k=1}^{\infty} \frac{r^k}{k}\,\mathrm{E}\,P_k^\tau\biggr\}, \tag{1}$$

with $0 < r < 1$, $\operatorname{Re}\tau \le 0$, and $P_k := \max(1, A_1^\alpha A_2^\alpha \cdots A_k^\alpha)$.

It is not difficult to see that

$$\mathrm{E}\,P_k = 2 - \int_0^1 \mathrm{P}\{A_1^\alpha A_2^\alpha \cdots A_k^\alpha > t\}\,\mathrm{d}t$$

and, in fact, $\lim_{k\to\infty} \mathrm{E}\,P_k = 2$. It follows that we can continue the right-hand side of (1) analytically to the half-plane $\operatorname{Re}\tau < 1$ and, by continuity, we get (1) with $\tau = 1$.

Next, we consider

$$Q(\lambda) := \sum_{k=1}^{\infty} \frac{\mathrm{e}^{-k\lambda}}{k}\,\mathrm{E}\,P_k, \qquad \lambda > 0,$$

as a Laplace transform as in Feller (1971, Chapter XIII.5, Theorem 5). By an Abelian theorem for Π-functions (De Haan (1976)) we find, for $a > 0$, that

$$\lim_{\lambda \downarrow 0} Q\biggl(\frac{\lambda}{a}\biggr) - Q(\lambda) = 2\log a.$$

Then (1), with $\tau = 1$, tells us that the function $L(\lambda) := \sum_{k=0}^{\infty} \mathrm{e}^{-\lambda k}\,\mathrm{E}\,M_k$ satisfies

$$\lim_{\lambda\downarrow 0} \frac{L(\lambda/a)}{L(\lambda)} = a^2, \quad \text{for some } a > 0.$$

Next, Karamata's Tauberian theorem (see Feller (1971, Chapter XIII.5, Theorem 5)) implies that the function $\sum_{k=1}^{\lfloor x \rfloor} \mathrm{E}\,M_k$ is regularly varying with index 2. Then the nondecreasing function $\mathrm{E}\,M_{\lfloor x \rfloor}$ must be regularly varying with index 1.

References

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