

WAVES IN MAGNETIC FLUX TUBES

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ABSTRACT. The basic aspects of wave propagation in a magnetic flux tube are reviewed, with particular emphasis on the types of flux tube that occur in the solar atmosphere. Two fundamental speeds arise naturally in a description of wave propagation in a flux tube: the slow magnetoacoustic (cusp) speed c_T , which is both subsonic and sub-Alfvénic, and a mean Alfvén speed c_k . Both surface and body modes are supported by a tube. It is stressed that a flux tube may act as a wave guide, similar to the guidance of light by a fibre optic, or sound in an ocean layer, or seismic waves in the Earth's crust.

1. INTRODUCTION

The evidence for the occurrence of magnetic flux tubes in astrophysical plasmas is increasing all the time. Flux tubes (or flux ropes) are believed to occur in extragalactic jets, in the magnetospheres of some planets, and in the Sun. Indeed, in the case of the Sun almost all of the magnetic flux emerging through the photospheric surface is found to occur in concentrated forms, ranging in scale from the visible sunspot to the sub-telescopic intense flux tubes. Thus, the source of coronal and solar wind magnetism is to be found in concentrated roots of magnetic field emerging from the deep interior. The field strengths in these sources, where they can be directly measured or indirectly inferred, are generally high: in a photospheric tube the field is as high as 0.2T (= 2kG), confined to a radius of about 100 km, whereas in a sunspot the field is typically 0.2-0.3T (= 2-3kG) and covers a wide area of the solar surface.

The concept of a magnetic flux tube goes back to the work of Michael Faraday (1791-1867) and James Clerk Maxwell (1831-1879). Faraday, the intuitive scientist with no formal training in mathematics, used the concept as a useful guide to a physical understanding of electrical and magnetic phenomena. Maxwell, with his more mathematical bent, gave the idea a more formal development and substance. Faraday was a little surprised, and no doubt pleasantly, to see the concept of a flux tube responding to Maxwell's mathematical overtures without exhibiting any undue signs of strain. It is amusing to note the manner in which Faraday, writing on the 4th May 1885 to the engineer Isambard Kingdom Brunel, puts his feelings: "I sometimes feel glad that I am not a mathematician for though Mathematical science is sure, I do not find that the conclusions of its professors can be trusted much more than those of other Professors" (Faraday 1885). Good advice indeed!

The mathematical substance that Maxwell gave to the concept of a magnetic flux tube rested on a very general definition. In his paper 'On Faraday's Lines of Force', Maxwell puts it thus: "If upon any surface which cuts the lines of fluid motion we draw a closed curve, and if from every point of this curve we draw a line of motion, these lines of motion will generate a tubular surface which we may call a *tube of fluid motion*" (Maxwell, 1855/6). By the term 'fluid' Maxwell means a purely imaginary substance and by 'lines of fluid motion' we would now say field lines.

The types of flux tubes that we are interested in astrophysical plasmas are more specific than those resting upon the rather general definition Maxwell gives us, for the interior and exterior of a tube are commonly distinct from one another by reason of physical differences additional to magnetism, though magnetism may well be the *cause* of such differences. For example, the field outside of sunspots and intense photospheric tubes is negligible in comparison with the interior field; so these objects are *isolated* flux tubes (see Roberts 1990a for a recent review). Such tubes are in magnetostatic pressure balance with their non-magnetic surroundings, so there is a deficit in gas pressure inside the tube: the tube is partially evacuated. For sunspots, the tube is also cooler than surroundings so there is a temperature deficit as well. The picture is more complicated for intense tubes, the temperature structure varying with height. (For recent reviews of solar magnetism and magnetic flux tubes see Schüssler (1987, 1990), Solanki (1987), Stenflo (1989) and Roberts (1990a).)

The tubes of the solar corona are different from those of the photosphere, for the corona is dominated by magnetic field and so there are no isolated tubes there. Instead, inhomogeneities in density and temperature give rise to magnetic flux tubes in which the magnetic field fills all of space; the interior of a coronal tube is generally denser than surroundings. Such *coronal loops* make up the coronal plasma in *active regions*.

What can magnetic flux tubes do? Firstly, we note that a tube is a *wave guide*. It permits waves to propagate without spatial attenuation. Thus a tube is likely to be a *good communication channel* between one region of a plasma and another, perhaps providing a connection between an energy source and an energy sink. For example, photospheric flux tubes connect the convection zone - an ample source of energy, especially in granules - with the chromosphere and corona, and so are almost certainly central to the question of heating of those regions. Also, they guide spicular material that produces the spicule jets in the chromosphere. Secondly, we observe that a magnetic flux tube is an *elastic* object (and so an elastic, not rigid, wave guide) and as such is likely to respond to sudden changes by guiding *waves*. Sunspots, for example, are known to support a variety of wave phenomena (see Evans and Roberts (1990) for a recent discussion). Finally, we point out that flux tubes may both *leak* waves to and *absorb* waves from their environment (see Roberts and Webb 1979; Spruit 1982; Cally 1985, 1986; Davila 1985). Braun, Duvall and LaBonte (1987, 1989), for example, have discovered that sunspots may be absorbing over 50% of the sound waves (p-modes) in their vicinity.

2. OSCILLATIONS OF AN ISOLATED TUBE

2.1 Propagation Speeds

We consider the modes of oscillation of an isolated magnetic flux tube of strength B_0 embedded in a field-free gas of pressure p_e and density ρ_e . If such a tube is subjected to

a sudden twisting motion it will respond by producing a *torsional oscillation* of the tube; such a torsional motion may propagate as an *Alfvén wave* with speed $v_A = B_0/(\mu\rho_0)^{1/2}$, for a tube with gas density ρ_0 and magnetic permeability μ . On the other hand, a symmetric squeezing of the tube may produce the *sausage mode* (see Figure 1). Here both the gas and magnetic field within the tube are expanded and contracted in the motion. Compressions of the gas are characterized by its sound speed $c_s = (\gamma p_0/\rho_0)^{1/2}$, for a tube with gas pressure p_0 , whilst magnetic compressions are represented by the Alfvén speed v_A . The two speeds c_s and v_A combine in the form

$$\left(\frac{1}{c_s^2} + \frac{1}{v_A^2}\right)^{1/2} \equiv \frac{1}{c_T} \quad (1)$$

to produce the basic speed c_T . The tube speed c_T is both sub-sonic and sub-Alfvénic and therefore relates to the *slow* magnetoacoustic wave; see Roberts (1985a, 1990b) and Hollweg (1986) for a general discussion of magnetohydrodynamic waves.

It is interesting to note that any elastic tube will have a characteristic propagation speed of the form given by equation (1) for one dimensional waves (see Lighthill 1978). For example, a blood vessel is an elastic tube with a speed v_A arising from the elasticity of the vessel's membrane and c_s being the speed of sound in blood. In this example, the speed c_s is much greater than the 'elastic' speed v_A and so the propagation speed c_T is close to the elastic speed. Another example is provided by water in a hose-pipe and the phenomenon of 'water-hammer'. If the pipe is made of metal, then the elastic speed v_A is much greater than the sound speed in water, and so the propagation speed c_T is approximately the sound speed in water (about 1.4 km s^{-1}). On the other hand, for water in a plastic pipe the ordering in the speeds is reversed and now the propagation speed c_T is close to the elastic speed and so much lower (about 10 ms^{-1}). In the case of a magnetic flux tube under photospheric conditions, the sound and Alfvén speeds are roughly comparable at about 10 km s^{-1} and so the tube speed c_T is reduced to about 7 km s^{-1} .

Asymmetric disturbances of the tube produce *kink* (or serpentine) modes. This may be viewed as similar to the waves on an elastic string, producing a propagation speed of $(T/\rho)^{1/2}$. The tension T in the string is clearly here due to magnetic tension, so $T = B_0^2/\mu_0$. The density ρ is here taken to be the sum of the gas density ρ_0 within the tube and the gas density ρ_e in the environment, reflecting the fact that the vibrating tube displaces about the same amount of material in its surroundings as that in the tube itself. Thus, with $\rho = \rho_0 + \rho_e$ we obtain a characteristic kink speed c_k given by

$$c_k = \left(\frac{\rho_0}{\rho_0 + \rho_e}\right)^{1/2} v_A. \quad (2)$$

The kink speed c_k is sub-Alfvénic but not necessarily sub-sonic.

2.2 Modes of Oscillation

To describe the modes of oscillation of an isolated tube in greater detail, we consider the usual equations of ideal magnetohydrodynamics as applied to a tube. The tube is taken to have a field strength B_0 , gas pressure p_0 , density ρ_0 , and sound speed c_0 ($\equiv (\gamma p_0/\rho_0)^{1/2}$), confined to a radius a by a field-free environment of gas pressure p_e , density ρ_e , and sound speed c_e ($\equiv (\gamma p_e/\rho_e)^{1/2}$). Then pressure balance across the boundary of the tube implies that

$$p_e = p_0 + \frac{B_0^2}{2\mu}, \quad (3)$$

which combined with the ideal gas law implies that

$$\frac{\rho_e}{\rho_0} = \frac{c_0^2 + \frac{1}{2} \gamma v_A^2}{c_e^2}. \quad (4)$$

Modes of oscillation of this equilibrium configuration may be sought by requiring in cylindrical coordinates (r, θ, z) that perturbations in flow, pressure and field be of the form

$$f(\mathbf{r}, t) = f(r) e^{i(\omega t + n\theta - kz)}, \quad (5)$$

representing a wave of frequency ω , longitudinal wave number k , azimuthal wave number n , and amplitude $f(r)$. The geometry of the modes is then described in terms of the integer n : sausage modes correspond to $n = 0$; kink modes to $n = 1$; and fluting modes to $n \geq 2$. Additionally, the radial behaviour of the amplitude $f(r)$ within $r \leq a$ may be either oscillatory or decaying. Oscillating modes are classified as *body* waves; decaying or evanescent modes as *surface* waves (Roberts 1981 a,b). See Figure 1.

It proves convenient to work in terms of the total pressure perturbation p_T , made up of the sum of the gas pressure perturbation and the magnetic pressure perturbation. For p_T is readily shown (see Edwin and Roberts 1983) to satisfy Bessel's equation:

$$\frac{d^2 p_T}{dr^2} + \frac{1}{r} \frac{dp_T}{dr} - \left(m_0^2 + \frac{n^2}{r^2} \right) p_T = 0, \quad r < a, \quad (6)$$

where the effective transverse wavenumber (squared) is given by

$$m_0^2 = \frac{(k^2 c_0^2 - \omega^2)(k^2 v_A^2 - \omega^2)}{(c_0^2 + v_A^2)(k^2 c_T^2 - \omega^2)}. \quad (7)$$

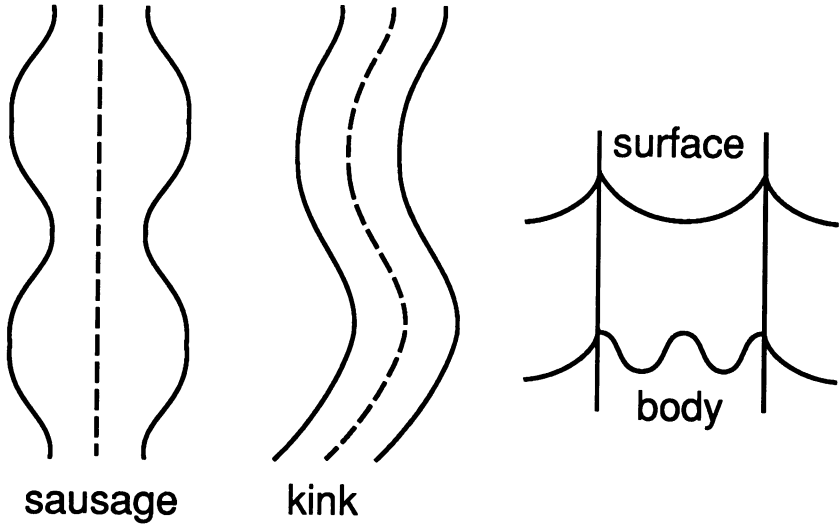


Fig. 1. Surface and body waves in a flux tube, showing the sausage ($n=0$) and kink ($n=1$) modes.

One solution of equation (6) which we should not overlook is the apparently uninteresting case of $p_T = 0$. In fact, with $p_T = 0$ the motions are torsional (i.e. $v_\theta \neq 0$, $v_r = v_z = 0$) and incompressible (i.e. $\text{div } \mathbf{v} = 0$); these are the torsional Alfvén waves satisfying the wave equation

$$\frac{\partial^2 v_\theta}{\partial t^2} = v_A^2 \frac{\partial^2 v_\theta}{\partial z^2}. \quad (8)$$

Returning to equation (6) we may construct its general solution in terms of Bessel functions. Requiring p_T to be bounded at the centre of the tube gives

$$p_T = \begin{cases} A_0 I_n(m_0 r), & m_0^2 > 0, \\ A_0 J_n(n_0 r), & m_0^2 < 0, \end{cases} \quad (r < a), \quad (9)$$

where I_n is the modified Bessel function (of order n) and J_n is the Bessel function. We use the form $I_n(m_0 r)$ for *surface* waves ($m_0^2 > 0$), giving exponential decay from the boundary at $r = a$, and the form $J_n(n_0 r)$, giving oscillatory behaviour, for *body* waves ($m_0^2 < 0$). Here $n_0^2 = -m_0^2$.

The region outside the tube may be treated in a similar fashion. Pressure perturbations in $r > a$ are governed by an equation of the form (6), with m_0^2 replaced by

m_e^2 (the equivalent form of the squared transverse wave number for the environment). In the absence of an external magnetic field, m_e^2 is given by

$$m_e^2 = \frac{k^2 c_e^2 - \omega^2}{c_e^2} \tag{10}$$

We consider pressure variations in the environment that are evanescent, declining to zero as we move away from the tube (i.e. $p_T(r > a) \rightarrow 0$ as $r \rightarrow \infty$). The vibrations of the tube are then confined to the region within and close to the tube. Such solutions require $m_e^2 > 0$, so that $\omega^2 < k^2 c_e^2$, and give an external pressure perturbation of the form

$$p_T = A_e K_n(m_e r), \quad r > a. \tag{11}$$

Matching of the interior and exterior of the tube at $r = a$ by requiring continuity of the radial velocity and the total pressure perturbation then yields the required dispersion relation, with the result (McKenzie 1970; Roberts and Webb 1978; Wilson 1980; Spruit 1982; Edwin and Roberts 1983; Abdelatif 1988; Evans and Roberts 1990)

$$\rho_e \omega^2 m_0 \frac{I_n'(m_0 a)}{I_n(m_0 a)} + \rho_0 (k^2 v_A^2 - \omega^2) m_e \frac{K_n'(m_e a)}{K_n(m_e a)} = 0, \tag{12}$$

where the dash denotes the derivative of the Bessel function. Equation (12) is valid for $m_e^2 > 0$. The derivation of the dispersion relation (12) given in Roberts and Webb (1978) was for $n = 0$ (sausage) modes only. It is interesting to note that a structurally similar relation was obtained by Tayler (1957) in a stability analysis of a twisted tube with a vacuum environment.

The dispersion relation (12) is transcendental and so obtaining its solutions is a complicated and largely numerical process, though certain cases (e.g. thin tubes, $kl_a \ll 1$) may be investigated analytically. In the special case of an *incompressible* medium ($c_0 \rightarrow \infty$, $c_e \rightarrow \infty$), for which $m_0^2 \rightarrow k^2$, $m_e^2 \rightarrow k^2$ and $c_T \rightarrow v_A$, the transcendental nature of equation (12) is removed and the solutions become transparent (Dungey and Loughhead 1954; Roberts and Webb 1978; Uberoi and Somasundaram 1980, 1982; Edwin and Roberts 1983)

$$\frac{k^2 v_A^2}{\omega^2} = 1 - \left(\frac{\rho_e}{\rho_0} \right) \frac{I_n'(kl_a) K_n(kl_a)}{I_n(kl_a) K_n'(kl_a)} \tag{13}$$

The modes described by dispersion relation (13) are *surface* waves (since $m_0^2 = k^2 > 0$); they propagate dispersively, a reflection of the fact that the tube provides a lengthscale, namely its radius, against which the waves may be measured. In the *thin*

tube limit ($kl_a \ll 1$), the sausage mode ($n=0$) propagates with a speed close to v_A ($\equiv c_T$, since the medium is incompressible) and the kink ($n=1$) mode with a speed close to c_k .

The modes of the *compressible* case (equation (12)) are more complicated to describe, their character depending upon the relative orderings between the speeds c_e , c_0 , and v_A (Edwin and Roberts 1983; Cally 1985, 1986; Davila 1985; Abdelatif 1988; Evans and Roberts 1990). For example, a magnetic flux tube with $c_e > c_0 > v_A$, typical of conditions below the surface layers of a sunspot, supports a slow surface wave, with speed close to c_T for $kl_a \ll 1$, as well as the harmonics of slow and fast body waves; a fast surface is unable to propagate under these conditions. The slow body waves have phase speeds between c_T and v_A . The fast body waves have phase speeds between the two sound speeds and are particularly interesting in that, with the exception of the fundamental sausage mode, they possess propagation cutoffs, corresponding to total internal reflection of the fast modes on the boundary of the tube.

By contrast, for a flux tube with speeds ordered as $v_A > c_e > c_0$, typical of conditions in an intense flux tube or in the upper layers of a spot, the fast body waves no longer arise and instead fast surface waves may propagate; the slow body waves remain much as before and there is a slow sausage surface mode. See diagrams in Edwin and Roberts (1983) and Evans and Roberts (1990).

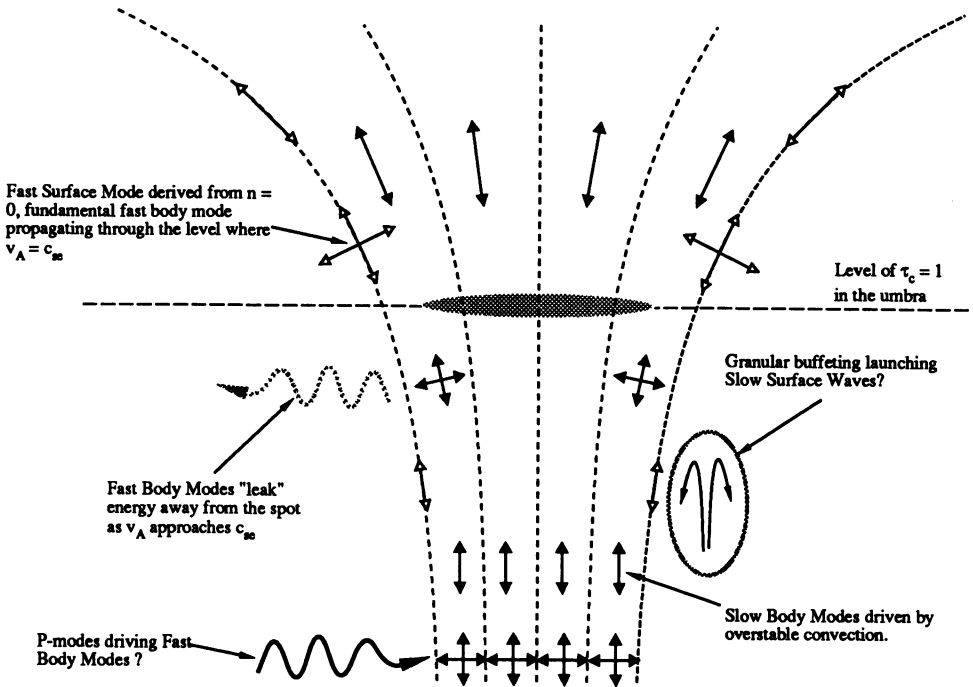


Fig. 2. A sketch of the various waves possible in a sunspot on the basis of a simple flux tube model. After Evans and Roberts 1990.

The fact that fast body waves in the deep layers of a spot, regarded as a simple flux tube, give way to fast surface waves in its upper layers has led Evans and Roberts (1990) to suggest that p-modes in the environment of a spot will generate fast body modes within the spot, and these in turn will be ducted up or down the spot. Consequently, sunspots will act as sinks for p-modes. This is consistent with the observations by Braun *et al.* (1987, 1988), of sunspots absorbing p-modes, though a detailed theory of the effect has yet to be worked out. The fast body modes thus generated in the spot will then convert to fast surface waves in the upper layers of the spot. Such surface modes may in fact be the *running penumbral wave* (see Small and Roberts 1984; Miles and Roberts 1989). A schematic of the complex structure of modes possible in a sunspot flux tube is given in Figure 2.

3. CORONAL LOOPS AS COMMUNICATION CHANNELS

The isolated magnetic flux tubes of the photosphere act as communication channels between the dense energy reservoir of the convection zone, with its granules, supergranules and p-modes, and the relatively tenuous atmosphere of the chromosphere and corona. In much the same way the loops of magnetic field in the coronal atmosphere may act as communication channels there, covering distances of the order of 10^5 km. Of course, such flux tubes are quite different from photospheric tubes, since the magnetic field of the corona fills all of the coronal gas and in general dominates the plasma both mechanically (the plasma beta is low) and thermally (heat conduction across the field is severely limited, allowing regions of different temperature to exist in close proximity to one another).

The magnetoacoustic modes of such a situation can be modelled in terms of a magnetic cylinder of radius a surrounded by a uniform magnetic field. For a low beta plasma, the Alfvén speeds within and external to the cylinder will be much larger than the corresponding sound speeds c_0 and c_e . The dispersion relation for such circumstances (see Edwin and Roberts 1983) can be obtained much as for the isolated tube discussed above. There are basically two distinct cases, depending upon whether the Alfvén speed v_A within the tube is greater than or less than the Alfvén speed v_{Ae} in the tube's environment. If the tube has a gas density that is *higher* than that in the surroundings, then $v_A < v_{Ae}$, provided the magnetic field is not too different from a uniform one; this is the "top-hat" profile of Alfvén speed. The opposite case, corresponding to a region of low density, gives $v_A > v_{Ae}$ and looks like an "inverted top hat"; see Roberts (1990b).

The coronal plasma provides examples of both structures, with coronal loops matching the top hat profile (in Alfvén speed) and coronal holes the inverted hat. Coronal loops with $v_A < v_{Ae}$ act as wave guides, ducting fast magnetoacoustic waves along the structure, much the same as light is guided along an optical fibre (Edwin and Roberts 1983; Roberts, Edwin and Benz 1984). Coronal holes or indeed any regions of the atmosphere with high Alfvén speed, $v_A > v_{Ae}$, are *leaky* wave guides (Spruit 1982; Edwin and Roberts 1983; Cally 1985, 1986; Davila 1985) for fast modes. The slow mode is ducted irrespective of whether $v_A > v_{Ae}$ or $v_A < v_{Ae}$.

The guided fast body modes are analogous not only to light in a fibre optic but also to Love waves in the Earth's crust and Pekeris waves in an internal ocean layer (Edwin and Roberts 1983; Roberts *et al.* 1984). This rather pleasing mathematical analogy implies that coronal loops are likely to produce seismic signatures when fast waves are

impulsively generated within them (Roberts *et al.* 1984). Such signatures could provide a diagnostic means of determining magnetic conditions in the corona (Roberts 1986b; Edwin and Roberts 1986). Also, because the waves propagate with high phase speeds, between v_A and v_{Ae} , the pulsations they produce are likely to be rapid, of the order of the crossing time for the diameter of the structure. For example, a loop with diameter 10^3 km and Alfvén speed of order 10^3 km s⁻¹ will sustain fast waves with timescales of the order of a second. Such rapid oscillations may in fact be the long-observed radio pulsations (see Aschwanden 1987 for a review).

4. THIN FLUX TUBE THEORY

The thin flux tubes of the photosphere provide important communication channels between the convection zone, photosphere, and chromosphere. This region of the solar atmosphere is the most strongly stratified part of the Sun, for here the pressure scale height $\Lambda(z)$ of the atmosphere is smallest: Λ is about 100 km at the temperature minimum, compared with some 10^5 km in the high temperature corona. Additionally, the upper layers of the convection zone are strongly super-adiabatic. These effects of stratification have important consequences for the modes of oscillation of a tube.

Analyzing the modes of oscillation of a magnetic tube in a stratified atmosphere is fraught with difficulties, not least because the very equilibrium profile of the tube is complicated and has largely to be determined numerically (e.g., Pizzo 1986; Pneuman *et al.* 1986; Fiedler and Cally 1990). Analytical progress has, however, proved possible for thin tubes. The idea has been to utilize the fact that physical variables such as pressure, density and longitudinal velocity are not likely to vary strongly *across* the tube, their main variations being along the tube. Other variables, such as the radial component of velocity, may vary linearly about the axis of the tube. A systematic expansion (in a Taylor series) of the magnetohydrodynamic equations about the axis of a tube is possible and provides the thin tube equations for symmetric modes of the tube (Roberts and Webb 1978):

$$\frac{\partial}{\partial t} \rho A + \frac{\partial}{\partial z} \rho v A = 0, \quad (14)$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} - g, \quad (15)$$

$$\frac{\partial p}{\partial t} + v \frac{\partial p}{\partial z} = \frac{\gamma p}{\rho} \left(\frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial z} \right), \quad (16)$$

$$p + \frac{B^2}{2\mu} = p_e, \quad (17)$$

$$BA = \text{constant}. \quad (18)$$

Here $B(z,t)$ is the longitudinal field strength, $A(z,t)$ the cross-sectional area of the tube, $v(z,t)$ the longitudinal flow speed, and $p(z,t)$ and $\rho(z,t)$ the gas pressure and density

within the tube. The external gas pressure $p_e(z,t)$ is calculated on the boundary of the tube.

Equations (14)-(18) are the *thin tube equations* for the sausage ($n=0$) mode of oscillation. The equilibrium of the tube is one of hydrostatic stratification, with gas pressure $p_0(z)$ and density $\rho_0(z)$ satisfying

$$p'_0(z) = -g\rho_0(z) \quad (19)$$

for gravity g ($= 274 \text{ m s}^{-2}$). The temperature structure may be arbitrary.

In general, the isolated tube expands with height z as it maintains pressure balance with its surroundings (eqn (17)). Specifically, the equilibrium magnetic pressure $p_m(z) \equiv B_0^2(z)/2\mu$, for a tube with longitudinal magnetic field strength $B_0(z)$, satisfies

$$p'_m = g(\rho_0(z) - \rho_e(z)). \quad (20)$$

Introducing the ideal gas law, viz.

$$p = \frac{k_B}{\hat{m}} \rho T, \quad (21)$$

for Boltzmann's constant k_B , mean particle mass \hat{m} , and temperature T , applied both inside and outside the tube, gives

$$p'_m = \frac{1}{\Lambda_0} \left[\left(1 - \frac{T_0}{T_e} \right) p_0 - \frac{T_0}{T_e} p_m \right], \quad (22)$$

where $\Lambda_0 \equiv p_0/g\rho_0$ is the pressure scaleheight inside the tube. (We have assumed that the levels of ionization, represented by \hat{m} , are the same inside and outside the tube.) If the temperatures inside and outside the tube are the same ($T_0(z) = T_e(z)$), then

$$p'_m = -\frac{1}{\Lambda_0} p_m. \quad (23)$$

Thus, the magnetic pressure declines at the same rate as the hydrostatic pressures p_0 and p_e , and so the magnetic field strength $B_0(z)$ declines at *half* the gas pressure rate. Consequently, the tube expands in height.

It should be noticed that the nonlinear set of equations (14)-(18) may be grouped into two sets, equations (14)-(16) describing longitudinal motions in *any* elastic tube and equations (17) and (18) defining that elasticity as specifically magnetic. This property affords a convenient way of identifying a variety of effects and attributing those effects appropriately to the tube's elasticity, shape or magnetism.

4.1 Linear Disturbances

The case of linear disturbances is readily treated by the thin tube equations, though the algebra is tedious. Linear motions satisfy an equation of the Klein-Gordon type for amplitude $Q(z,t)$, related to velocity $v(z,t)$ (see Rae and Roberts 1982):

$$\frac{\partial^2 Q}{\partial t^2} - c_T^2(z) \frac{\partial^2 Q}{\partial z^2} + \Omega_{\text{saus}}^2(z) Q = 0. \quad (24)$$

Here c_T is the tube speed (defined in eqn (1)) and $\Omega_{\text{saus}}^2(z)$ is a complicated expression involving the equilibrium profile of the tube and its atmosphere. In deriving equation (24) it was assumed that $p_e(z,t)$ is in fact the undisturbed external gas pressure field.

If the atmosphere is *isothermal* ($\Lambda_0 = \text{constant}$) the coefficients in the Klein-Gordon wave equation (24) become constants, with Ω_{saus} given by (Defouw 1976; Roberts and Webb 1978)

$$\Omega_{\text{saus}}^2 = \left(\frac{\rho}{4} - \frac{2}{\gamma}\right) \omega_a^2 - \left(\frac{3}{2} - \frac{2}{\gamma}\right) \left(\frac{\beta}{\beta + \frac{2}{\gamma}}\right) \omega_a^2, \quad (25)$$

where $\omega_a \equiv c_0/2\Lambda_0$ is the acoustic cutoff frequency of the atmosphere and $\beta \equiv 2\mu_0 p_0(z)/B_0^2(z)$ is the usual plasma beta (a constant). It is possible to identify the first term on the right of equation (25) as arising from the *geometry* (shape) of the expanding tube and the second term as arising from the *elasticity* of the tube. Accordingly, the frequency Ω_{saus} is reduced by the elasticity of the tube. A *rigid* tube corresponds to $\beta = 0$.

Observe that in the absence of stratification ($g=0$) $\Omega_{\text{saus}} = 0$, and we obtain the simple wave equation with dispersion relation $\omega^2 = k^2 c_T^2$. The thin tube equations, then, relate to the slow sausage mode; in fact, they relate to the slow surface wave.

Kink modes may also be described by thin tube equations, though of a somewhat different form from equations (14)-(18); see Spruit (1981). Indeed, Spruit's thin tube equations for the kink mode also lead to a Klein-Gordon equation (see also Roberts 1986a):

$$\frac{\partial^2 Q}{\partial t^2} - c_k^2 \frac{\partial^2 Q}{\partial z^2} + \Omega_{\text{kink}}^2 Q = 0, \quad (26)$$

with

$$\Omega_{\text{kink}}^2 = \frac{c_k^2}{4\Lambda_0^2} \left(\frac{1}{4} + \Lambda_0' \right). \quad (27)$$

As with equation (24), the external pressure field is assumed to be its unperturbed value. In the absence of stratification ($g=0$), equation (26) reduces to the simple wave equation, with dispersion relation $\omega^2 = k^2 c_T^2$.

What is the significance of the occurrence of the Klein-Gordon equation? In an isothermal atmosphere both Klein-Gordon equations have constant coefficients, and so both have dispersion relations of the form

$$\omega^2 = k^2 c^2 + \Omega^2 \quad (28)$$

for propagation speed c ($= c_T$ or c_k) and frequency Ω ($= \Omega_{\text{saus}}$ or Ω_{kink}). A dispersion relation of the form (28) possesses *propagation cutoff*: high frequencies ($\omega > \Omega$) propagate vertically ($k^2 > 0$) whereas low frequencies ($\omega < \Omega$) are evanescent ($k^2 < 0$). (This is much the same as for the vertical propagation of sound waves in an isothermal atmosphere, with c there becoming the sound speed c_s and Ω the acoustic cutoff ω_a .) Impulsively generated disturbances lead to a wave front propagating with the speed c , behind which an oscillating *wake* arises (Rae and Roberts 1982).

It is of interest to compare the sausage and kink modes for an isothermal atmosphere (Spruit and Roberts 1983; Roberts 1986a). Taking photospheric values of $\Lambda_0 = 125$ km and $c_0 = v_A = 7.5$ km s⁻¹, we find that both modes e-fold in a distance of 500 km (four scale heights), to be compared with a vertically propagating sound wave which e-folds in two scale heights (250 km). The propagation speeds of the two modes are similar, about 5.3 km s⁻¹ for the sausage mode and 4.5 km s⁻¹ for the kink mode. However, the frequencies $\Omega_{\text{saus}}/2\pi$ and $\Omega_{\text{kink}}/2\pi$ are quite different, being 4.8 mHz (period 208s) for the sausage mode but only 1.4 mHz (period 700 s) for the kink mode. (The sausage mode's frequency is close to the acoustic cutoff frequency.) This marked difference in the two frequencies has led Spruit (1981) to suggest that kink modes are likely to be more readily generated and propagated in the solar atmosphere than sausage modes.

What observational evidence is there for the existence of tube waves in the photosphere? Solanki and Roberts (1990) have argued that the asymmetry observed in the Stokes profile formed in photospheric lines may be explained as a consequence of tube waves. The waves are of relatively strong amplitude, about 1 km s⁻¹, and contribute significantly to the heating of the chromosphere. Evidence for the existence of waves in flux tubes also comes from observations by Deubner and Fleck (1990), who report the existence of waves on the chromospheric magnetic canopy propagating along the canopy and ultimately *down* the converging funnels provided by the tubes! It would seem that photospheric flux tubes are both sources for tube modes and receptors for canopy modes.

4.2 Nonlinear Waves

The stratification in the upper photosphere and above makes it likely that tube waves will grow in amplitude and possibly form shocks. There are, however, a number of effects working against the tendency for wave amplitudes to grow. The photosphere-chromosphere is a region of the solar atmosphere where nonadiabatic effects (radiative losses) are likely to be significant, and such effects work to limit the amplitude of waves as their energy is irreversibly dissipated in the form of heat. Also, it is known that in the absence of gravity flux tubes can support solitons (Roberts and Mangeney 1982; Roberts 1985b; Weisshaar 1989), nonlinear waves in which the amplitude is in balance with dispersive effects. These dispersive effects arise from the inertia of the tube's surroundings, an effect evident in the linear theory (Section 2.2) and included in thin flux tube theory by retaining the effect of $p_e(z,t)$ in equation (17). (The derivations of the Klein-Gordon equations (24) and (26) for sausage and kink modes neglects this effect.) How important all these effects are likely to be in a fully stratified, radiative flux tube is presently unknown and will no doubt require detailed numerical simulations (e.g. Herbold *et al.* 1985; Musielak *et al.* 1989) to fully assess.

Nonlinear effects are reviewed in detail by Ryutova (1990a,b).

5. CONCLUSIONS

The magnetic flux tubes we have described here are basically simple structures, involving sharp changes in density and/or pressure. The role of the magnetic field is to provide a channel connecting one region of the solar atmosphere with another; such channels are elastic. Despite this simplicity the waves that such tubes support are not especially simple to describe. This complexity springs from the fact that in magnetohydrodynamics there are three basic waves in a magnetic medium and additionally that a tube can duct these modes in essentially two ways (as a surface wave or a body wave). In reality, of course, photospheric and coronal magnetic flux tubes will be more intricate objects than the simple models explored here. Nonetheless, we trust that some of our descriptions remain and that the simple concept of a tube, grounded in the work of Faraday, is also a guide to the reality of magnetism in the solar atmosphere.

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DISCUSSION

UBEROI: In your last part of the talk you pointed out an analogy with Pekeris's work on surface waves in the ocean. The source in the Pekeris work is located inside the ocean bed (I think). In your case is the source inside or outside? If it is outside does this analogy hold good?

Comment: Regarding your existence of surface waves for $v_A > v_{Ae}$ I like to point out that I have published a paper in 1982 in Solar Physics studying the parametric analysis of the dispersion equation and giving the parameters (ρ, B) for which the surface waves will exist and if these parameters are not suitable surface waves will not exist.

ROBERTS: Concerning your question about Pekeris waves, Roberts *et al.* (1984) assumed that the source was located in the coronal loop wave-guide, say at the bottom of the loop. Locating the source off centre or outside the loop would of course give slightly different results. But one would expect a flare, say, to generate waves across a number of loops, and each loop would exhibit a dispersive wave-packet behaviour that is of the *general* form as given for a symmetric source. It would be interesting to see what differences do in fact arise but the aim in Roberts *et al.* was more to exhibit the impulsive signature of a rapid pulsation, and that is most simply done in the case of a symmetric disturbance.

Regarding your comment, thank you for the reference [Uberoi (1982), *Sol. Phys.* 78, 351]. In my paper in 1981 [Roberts (1981a)] I pointed out that *two* magnetoacoustic waves may arise at a single interface, one fast and one slow surface mode. The slow surface wave is generally present but the fast surface wave exists only under certain conditions, as you have also noted. This topic is being investigated further (see Miles and Roberts 1989; Jain and Roberts, in preparation).

RUDERMAN: What do you say about the thin tube approximation conveniency in the solar atmosphere?

ROBERTS: I think that the thin tube approach is very valuable for photospheric tubes and gives a good guide to the behaviour of the slow surface (sausage) mode and the slow kink mode. Body modes, however, have a different lateral structure and so need to be treated differently; curiously, though, slow body waves have phase-speeds that

are very close to the thin tube speed, c_T . It should also be kept in mind that the external pressure field on the boundary of the tube, sometimes neglected in analytical and numerical studies of stratified tubes, is important in some circumstances and so should be retained in the thin tube equations. Indeed, its retention gives dispersive corrections to wave speeds and these are important for nonlinear studies. In fact the thin tube equations, with the external pressure term included, lead to flux tube solitons, as shown, for example, in Roberts and Mangeney (1982) and Roberts (1985).

AL-KHASHLAN: What other things can magnetic flux tubes do besides supporting a variety of magnetohydrodynamic waves?

ROBERTS: I have of course concentrated on the wave aspects of a flux tube but a tube, in general terms, transmits stresses and strains and also may provide a duct for flows and for spicules. Coronal flux tubes are also liable to ideal mhd instabilities, ballooning modes, etc. - see, for example, work by A W Hood. Such instabilities of coronal tubes and coronal arcades may be important for prominence eruption.