PART I : TIME

BASIC PROBLEMS IN THE KINEMATICS OF THE ROTATION OF THE EARTH

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1. INTRODUCTION

With the advent of more precise methods for measuring Earth rotation, a number of corrections to the apparent directions in space, to the terrestrial references, and to the rotation axis motion have to be carefully applied. It is the duty of the international Astronomical Union to give recommended or conventional expressions of these corrections in order to avoid inextricable difficulties in discussing the evaluated results. However, this task is not sufficient. The concepts used in the description of the Earth's rotation are somewhat obscured by traditions. They should be purified by removing notions which are not directly relevant.

The following discussion will be restricted to the case where the observations are referred directly to a non-rotating frame given by the directions of stars (or of extragalactic sources; in the following, only the word "star" will be used, designating also these objects). The purpose of the discussion is to look for the minimum requirements for the kinetic study of the rotation of the Earth. It will be considered later how these requirements can be fulfilled without deviating too much from the usual practice in astrometry.

2. FUNDAMENTALS OF EARTH ROTATION MEASUREMENTS

To represent the Earth, a sphere with unit radius is used, where the points Z_i represent the directions of the plumb lines of classical instruments and of the base-lines of interferometers. We first assume that the Z_i have no relative motions, the sphere attached to them is called the <u>terrestrial sphere</u>.

Similarly, a sphere of unit radius can be attached to the star directions, represented by E_j, after correction for the proper motions and for the various effects due to the position of the observer, to his velocity, and to the atmosphere. This sphere is called the <u>non-rotating</u>

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sphere.

Let us assume that that these two spheres are concentric. To describe the rotation of the Earth, it is sufficient to give their relative position as a function of time, t. This can be accomplished by giving the time-series of three parameters $\theta_i(t)$, i = 1, 2, 3, which can be chosen in an infinite number of ways.

This description immediately suggests a method of measurement of the Earth rotation: one has to observe, at successive instants t_1, t_2, ..., the positions of the Z_i among the E_j in order to get $\theta_i(t_1)$, $\theta_i(t_2)$, ... The only condition is that the instants be sufficiently close so that the $\theta_i(t)$ can be interpolated. Such observations are indeed routinely realized with classical instruments, and there is no need to refer to the rotation axis to reduce them, as soon as a definition of the three θ_i is chosen. The results would therefore be completely independent of the errors which may affect the precession and nutation.

However, for dynamical studies, it becomes necessary to refer to the position of the instantaneous axis of rotation, or to the angular momentum axis, or to the figure axis, or to any other axis with a dynamical definition, such as the one proposed by Atkinson (1973). The choice between these axes is only a matter of convenience. The word "pole" will designate the representative point on the spheres of the direction of one of these axes, towards the North.

The principles of the traditional method for measuring the Earth rotation are then as follows:

(a) DATA ASSUMED TO BE KNOWN

The motion of the pole on the non-rotating sphere (i.e. among the E_j) is described by the series A(t), B(t) of the luni-solar precession and nutation.

(b) OBSERVATIONS

The observations locate the terrestrial sphere with respect to the non-rotating sphere (i.e. the Z_i among the E_j) at an instant t_k.

(c) EVALUATION

The position of the pole on the terrestrial sphere (i.e. among the Z_j) is then derived in form of $u(t_k)$, $v(t_k)$, as well as the angular position of the terrestrial sphere, around the polar axis, $\theta(t_k)$.

It is now necessary to give 5 parameters, which is the minimum number of parameters to fix the relative position of the two spheres and one direction relative to them. In current practice, $\theta(t_k)$ is referred to the true equinox of date and the motion of this point appears as a sixth series. But the reference to the moving equinox is quite unnecessary, as we shall see later, and is an example of the complexities due to tradition. A more convenient origin can be chosen on the equator.

In this 5-parameter scheme, the series A(t), B(t) are affected by errors coming from two sources:

- the pole position on the non-rotating sphere at an initial date, t_o, is not perfectly known (for instance, using the present practice, the FK4 pole in 1950.0 does not coincide with the real position of the mean pole at this date):

- the conventional development of the luni-solar precession and nutation from t_o to t contains errors.

Therefore, even when neglecting all other sources of errors, u(t), v(t), θ(t) are also erroneous. However, the orientation of the Earth in space is restored correctly by the use of the values of the five parameters at a date, t, providing that the computations be made rigorously and that these parameters be given with a time resolution such that the shortest terms do not average out (time resolution of a few hours). Consequently, it could serve the interests of the users to tabulate in a single document, not only u, v and θ as functions of time, but also the precession-nutation series A and B employed to derive them.

This property is important in the cases where only a precise orientation of the Earth is needed. The definition of the pole and its location on the non-rotating sphere is immaterial, at least in theory. It is nevertheless useful, for practical reasons and for dynamical studies, that all the motions of the pole which can be modeled be taken into account (See Appendix I).

As already stated, all these considerations are only valid when the Z_i have no relative motions. If they have non-negligible relative motions they must be corrected for. The terrestrial sphere must then be attached to the Z_i in "a prescribed way" which has some arbitrary character. Similarly the relative motions of the apparent places of the E_j must be corrected. These corrections are not trivial when an accuracy of $0\overset{''}{.}001$ is to be ensured. Appendix I lists some of them and points out some difficulties encountered in expressing their precise values.

3. THE NON-ROTATING REFERENCE SYSTEM

A non-rotating reference system is realized by a great circle and a reference point σ_o on it (Figure 1). This system is fixed among the directions of the stars and corrected for proper motions. The pole, S_o, of the circle defines the direction OZ_o; OX_o is along $O\sigma_o$; and OY_o completes the direction triad. The motion of the pole P will be described by the time series, d = S_oOP, and E = σ_oS_oP.

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The instantaneous system OXYZ will be chosen so that OZ is along OP. The condition is imposed, that, when P moves on the non-rotating sphere,

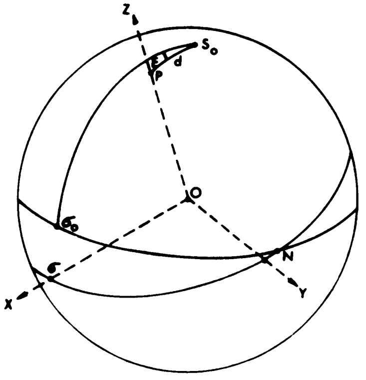

Figure 1. Non-rotating reference system.

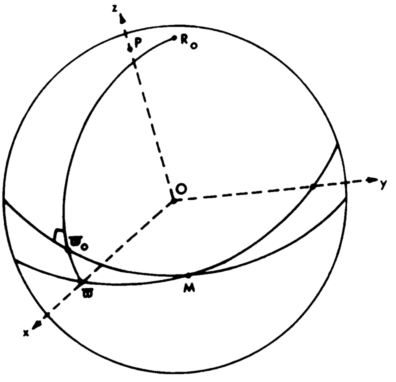

Figure 2. Terrestrial reference system.

the instantaneous system has no component of rotation around OZ. Let σ be the point where OX intersects the sphere. To realize the above condition it would be necessary to fix an origin on the equator at a date, t_o, then to derive σ at a date t, taking into account the whole history of the motion of P between t_o and t. Practically σ would be obtained by giving the quantities $\sigma_o N$ and $\sigma N - \sigma_o N$, N being the ascending node of the equator in the great circle of pole S_o. These quantities depend on E and d only. The nutation terms being small, the computations can be done for the precession only, and σN can be used on the true equator instead of the mean equator. σ will be called the non-rotating origin. One of the coordinates of a star is the usual declination, but the other differs from the conventional right ascension; it will be called the instantaneous ascension.

Atkinson has already proposed to use on the true equator a reference point freed from the nutation. In the above system, it is proposed to remove also the effects of luni-solar and planetary precessions.

4. THE TERRESTRIAL SYSTEM OF REFERENCE

The terrestrial system of reference is similarly defined by a great circle and a reference point ϖ_o on it (Figure 2). The pole, R_o, of the circle gives the direction Oz_o; Ox_o is along $O\varpi_o$; and Oy_o completes the direction triad. This system is realized by the coordinates attributed to the Z_i and their variations. As the secular motion of P is slow, and the periodic components are small, it is possible to locate R_o so that P keeps close to it.

The instantaneous system has its Oz axis along the direction of the moving pole P. In order to cancel any component of rotation around Oz, the reference point $\overline{\omega}$ in the equator is obtained by $\overline{\omega}M = \overline{\omega}_o M$, where M is the ascending node of the equator in the circle of the reference plane $x_o O y_o$; Ox is along $O\overline{\omega}$. $\overline{\omega}$ is equivalently defined by the intersection of $R_o\ \overline{\omega}_o$ with the equator.

The present practice follows the above principle. For instance the initial latitudes and longitudes of the BIH, and their variable systematic corrections materialize a system of reference attached in a prescribed way to the Z_i. However, the relative positions of the Z_i are poorly known and the position of $\overline{\omega}_o$ is very uncertain. The consequence is that precise geodetic networks are obtained with undetermined rotations, especially around Oz_o.

5. THE ROTATION OF THE EARTH IN THE NON-ROTATING REFERENCE SYSTEM

We already defined the parameters

- E and d, which describe the motion of P on the non-rotating sphere,
- u and v, which describe the motion of P on the terrestrial sphere (the usual coordinates of the pole).

It remains to define the angular position around OP. This is obtained by the angle $\overline{\omega} O \sigma$, reckoned positive westward, which will be called the <u>stellar angle</u> and denoted by θ. Following its definition, θ expresses the sidereal rotation of the Earth, and its time derivative gives directly the angular velocity of the Earth in space.

As θ increases linearly if the rotation of the Earth is uniform, UT1 has the form

$$\mathrm{UT1} = k(\theta - \theta_o),$$

the coefficient k being a constant chosen so that a day of UT1 is close to the duration of the mean solar day. Appendix II gives the expression of UT1 according to the above principles, with the present system of constants.

6. DISCUSSION

Before reaching the conclusions, additional comments will be made on the non-rotating reference system and the definition of UT1.

6.1. On the non-rotating reference system

A catalogue of stars defines its own pole and equinox, which do not coincide, in general, with the real pole and equinox. Although small, the departure of the catalog pole from the real pole is significant

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(it gives rise, in particular, to systematic errors, $\Delta\delta_\alpha$). The separation of the equinoxes may be large. In other terms, the accuracy defect of the practical definition of the equatorial system is larger than the internal defects of the catalog.

This unhappy situation cannot be expected to improve, particularly for the equinox. Already existing methods of classical astrometry, such as studies of the data of time and latitude services, locate the relative positions of the stars and the pole position fairly well, but they do not locate the equinox. The project of space astrometry (European Space Agency, 1976) does not locate the pole nor the equinox. Interferometry does not locate the equinox. It is therefore illogical to attach the whole coherent system to poorly measured directions and to rotate this system from time to time, to adjust it to the last evaluations of these directions.

The proposed system, definitively linked to the directions of stars, would be especially useful in the future, when the internal coherence of the catalogs will be improved. It also has the advantage of emphasizing the experimental character of the figures which express the position of the celestial pole and also of the pole of the ecliptic.

It can be considered that the pole and equinox of the future FK5 will be for some time the best estimates of the positions of these points among the fundamental stars. But it will not remain so, and it is suggested that corrections for these points be issued instead of rotating the whole system.

6.2. On the definition of UT1

UT1 has presently no clear definition. It is often presented as a form of the mean solar time, which has no theoretical nor operational advantages, but shows a discouraging complexity. In fact, only the sidereal rotation of the Earth has to be known, and UT1 could be merely cancelled, retaining only the stellar angle defined above.

As it would be unrealistic (on account of its broad use and relation to the UTC system) and somewhat unpractical (on account of the rapid variation of the stellar angle with respect to TAI) to abandon UT1, its definition as a function of the stellar angle is proposed in the conclusions.

7. CONCLUSIONS, RECOMMENDATIONS

From the considerations listed in this paper result the following proposals.

7.1. Symposium No. 82 should indicate what is the <u>definition of the pole</u> which seems suitable (pole of rotation, pole of angular momentum, pole proposed by Atkinson, ...).

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7.2. The Symposium should adopt the expressions of a few corrections for effects entering in the study of the rotation of the Earth, which are fairly well known and recommend their general use (Earth tides, diurnal nutation, relativistic deflection of light, variation of UT1 due to zonal tides). Furthermore, accurate methods of reduction should be recommended, the aim being uncertainties smaller than $0\rlap{.}''001$.

7.3. Concerning the definition of the space reference system, it is suggested

(a) that this system be attached to the directions of stars or extragalactic sources without reference to the position of the pole and of the equinox of some date,

(b) that the coordinates of the pole of rotation be given in this system by two time series,

(c) that, similarly, the coordinates of the pole of ecliptic be given in this system.

The four parameters stated in (b) and (c) contain all the basic information for fundamental astrometry.

7.4. Concerning the realization of the space reference system, it is suggested

(a) that, for the near future, the FK5 be considered as the official realization of the system mentioned in 7.3(a), and that, at its standard date, its pole and its equinox (which, together with the conventional value of the inclination of the ecliptic, give the pole of ecliptic) be the basis of the coordinates stated in 7.3(b) and 7.3(c);

(b) that, later, no attempt be made to adjust the pole and equinox of the FK5 to the observed positions of these directions; the improvement of the FK5 should consist of

- the reduction of internal inconsistencies in positions and proper motions at the standard epoch,
- a rotation rate affecting the proper motions, if required, in order to link it to a better observed non-rotating reference system (for instance linked to extragalactic sources);

(c) that, if a significant departure of the observed pole and equinox from the pole and equinox of the FK5 is found, the position and motion of these directions in the FK5 system should be given, without rotating the FK5 system.

7.5. Concerning the instantaneous equatorial reference system, it is recommended that a non-rotating origin be adopted on the instantaneous equator, instead of the conventional equinox. This point should coincide with the mean equinox of the FK5 at its standard epoch.

7.6. Concerning the definitions of Universal Time:

(a) The hour angle of the non-rotating origin from the prime meridian, here denoted by stellar angle, should be the basis of the definition of UT1. A self contained definition of UT1 could be

> "UT1 is an angle which is proportional to the sidereal rotation of the Earth, the coefficient of proportionality being chosen so that UT1, in the long term, remains in phase with the alternation of day and night. In some applications, UT1 can be considered as a non-uniform time scale."

(b) The definition of UT2 should be abolished.

(c) It could be useful to define a new form of UT, in which the variations due to zonal tides are removed.

7.7. Concerning the orientation of the Earth in space, in order to provide users with all the data they need in a single document, it is recommended that the services dealing with the rotation of the Earth tabulate not only UT1-TAI and the coordinates of the pole in the Earth linked system, but also the coordinates of the pole in the space reference system, which are used in the reduction of the observations.

APPENDIX I

CORRECTIONS TO THE TERRESTRIAL AND APPARENT SPACE REFERENCES AND TO THE ROTATION OF THE EARTH

We will deal in this Appendix with the various corrections which are needed to remove the relative motions of the terrestrial references and of the space references as seen from the Earth. We will also consider the terms of the rotation of the Earth which can be modeled with an accuracy matching the precision of the measurements.

In classical astrometry the final precision of the measurements of Earth rotation is of the order of 0".01. It is thus satisfactory that the corrections be known with uncertainties of a few 0".001. The recent decisions taken by the International Astronomical Union meet this requirement (but the relative positions of the stars and of the zeniths are not known to this level of precision except for Earth tides and for some relativistic effects).

With the advent of methods which could lead to an observational precision of 0".001, or better, what appeared previously as known corrections becomes a subject of study, and there is little that can be done presently. There are, however, a few terms for which the current practice can be improved, and only these terms will be considered here.

1. Corrections for relative motions of the terrestrial references

In order to derive the corrections due to Earth tides, it was recommended that clinometers, gravimeters, strain-meters..., be placed in the vicinity of astronomical instruments. But this recommendation had little effect. This failure can probably be explained by the difficulty of interpretation of the data. On the other hand, it is not always felt necessary to have local measurements. A possible issue could be to recommend that, in the absence of reliable measured effects of the Earth tides, conventional corrections be applied with adopted values of h, k, ℓ, without phase lag, or local corrections. It is too early to adopt a model of plate motions.

2. Corrections for the apparent relative motions of celestial sources

The complete correction for proper motions of stars would require the knowledge of the ratio of radial velocity/distance, in order to take care of the foreshortening effect. In some cases, this effect is not negligible.

The aberration should be computed so that the geometric direction is obtained by adding to a unit-vector along the apparent direction, the vector, $-\vec{V}/c$ ($\vec{V}$ is the velocity of the observer, c the velocity of light). The reverse method (unit-vector along the geometric direction) is often used. The difference between the two methods amounts to 0"001 for the annual aberration. It should also be remembered that the component of the velocity of the Earth perpendicular to the ecliptic and the perturbations in longitude give rise to aberrations of the order of 0"001.

In classical astrometry, the relativistic deflection of light in the gravitational field of the Sun is usually omitted. However, even for night observations, this deflection is of the order of a few 0"001. This effect has been studied by Brandt (1974). Kimura (1935) already attempted to apply this relativistic correction to the data of the International Latitude Service. Interferometric measurements should be also corrected for the effects of the gravitational field of the Sun, in addition to the relativistic effects which appear in the computation of the time delay and fringe rate (Thomas, 1974).

3. Corrections for the rotation of the Earth

3.1. Definition of the pole and motion of the pole

Following the proposal by Atkinson (1973) to replace the instantaneous pole of rotation, which is the present axis of reference, by a new axis freed from the forced diurnal nutation with respect to the Earth, many discussions within the IAU led to contradictory recommendations. We will not comment on the possible choices in this paper, but only recall that, according to the definition of the pole, some terms appear either in the

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motion of the pole on the terrestrial sphere or on the non-rotating sphere. Anyway, the forced diurnal nutation has to be taken into account. The elasticity of the Earth changes only slightly its amplitude, as shown theoretically by McClure (1973). This nutation has been found in the observations (McCarthy, 1976).

Besides the precession-nutation, for which the IAU has prepared improved numerical coefficients, and the "sway" which can reach $0\overset{''}{.}001$, no other motions of the pole can be modeled sufficiently well for the computations.

3.2. Variations of UT1 due to zonal tides

These variations (Woolard, 1959; Pil'nik, 1970) are real irregularities of the rotation of the Earth; in principle, they should require no correction. However, the terms with the shortest periods may cause some difficulties in the interpretation of measured values of UT1, if the time resolution is not sufficient. For instance, the terms with periods 13.7 d and 27 d have amplitudes of the order of 0.8 ms; they produce problems in handling the 5-day averages of UT1 determined by the BIH with random uncertainties (1σ) of 1 ms. In addition, terms with long periods have large amplitudes (0.15 s for the term of 18.6 years) and they must be removed before investigating the unmodeled variations of UT1.

Several of the short period terms have been experimentally found by many authors, in particular by Pil'nik (1970). It was observed that the 13.7 d term had an amplitude which did not correspond to the theory (Guinot, 1974); but subsequent studies (Guinot, not published) indicate that the discrepancy might be due to an additional term with a period of 13.70 d of unknown origin.

It might be advisable to define a form of UT (UT3 ?) which would be UT1 corrected for the zonal tide effects. If this suggestion is accepted, the conventional expression of the correction should be given.

3.3. Suppression of UT2

The seasonal variation of UT1 is variable from year to year (see, for instance: Okazaki, 1975, 1977; Lambeck and Cazenave, 1973). The conventional value of UT2, obtained from UT1 by addition of periodic annual and semi-annual components is therefore of little use. It is proposed to cancel the definition of UT2.

APPENDIX II

DEFINITION OF UT1

The present definition of UT1 is contained in the well known expression giving the mean sidereal time at 0h UT:

$$T = 6^h\ 38^m\ 45\overset{s}{.}836 + 8640\ 184\overset{s}{.}542\ t + 0\overset{s}{.}0929\ t^2,$$

t being measured in Julian centuries of 36525 days of UT1 from 1900 January 0, 12h UT1.

We assume that the non-rotating reference system is the mean equatorial system on 1900 January 0, 12h UT1. On the mean equator of date, the non-rotating origin is then derived from the mean equinox of date by a rotation $\zeta_A + z_A - s$ eastward, where s is a small quantity integrated along the path of the mean pole. Therefore the stellar angle is

$$\theta = T - \zeta_A - z_A + s.$$

With the present system of constants

$$\zeta_A + z_A = 46\ 085\overset{''}{.}06\ \tau + 139\overset{''}{.}73\ \tau^2 + 36\overset{''}{.}32\ \tau^3$$

$$s = 36\overset{''}{.}28\ \tau^3 - 0\overset{''}{.}04\ \tau^4$$

τ being reckoned from 1900.0, in units of 1000 tropical years. Expressing $\zeta_A + z_A$, with t as unit, disregarding the negligible effects of the non identical origins for t and τ and for the irregularity of the Earth rotation,

$$\zeta_A + z_A - s = 307\overset{s}{.}2403\ t + 0\overset{s}{.}0932\ t^2.$$

Thus, the expression of θ at 0h UT is

$$\theta = 6^h\ 38^m\ 45\overset{s}{.}836 + 8639\ 877\overset{s}{.}302\ t - 0\overset{s}{.}0003\ t^2.$$

The small term in t^2 results from small inconsistencies in the system of constants. Omitting this term, the general expression of UT1 has the form

$$UT1 = k(\theta - \theta_o)$$

given section 5.

This definition of UT1 leads to the same value of UT1 as the usual expression, if the instantaneous ascension is employed instead of the usual right ascension.

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DISCUSSION

F.P. Fedorov; Do you propose that the definition of the pole that was adopted in Kiev a year ago, specified by the instantaneous axis, be replaced by Atkinson's definition?

B. Guinot: The instantaneous rotation pole is not especially convenient for the reduction of astronomical observations, or for theoretical work. Although I did not specify in my paper which point should be adopted for the terrestrial and celestial pole, I believe now that Atkinson's pole is the best.