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Airborne infection in a fully air-conditioned hospital

III. Transport of gaseous and airborne particulate material along ventilated passageways

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SUMMARY

A mathematical model is described for the transport of gaseous or airborne particulate material between rooms along ventilated passageways.

Experimental observations in three hospitals lead to a value of about 0·06 m.^{2}/sec. for the effective diffusion constant in air without any systematic directional flow. The 'constant' appears to increase if there is any directional flow along the passage, reaching about 0·12 m.^{2}/sec. at a flow velocity of 0·04 m./sec.

Together with previously published methods the present formulae make it possible to calculate the expected average amounts of gaseous or particulate material that will be transported from room to room in ventilated buildings in which the ventilation and exchange airflows can be calculated.

The actual amounts transported in occupied buildings, however, vary greatly from time to time.

INTRODUCTION

A model for assessing the behaviour of an isolation unit comprising a series of rooms opening off a common space has been described previously (Lidwell, 1972). This model was based on the assumption of effectively complete mixing of the air in the several rooms and in the common space with which they communicated. Although this model gave a reasonably good account of the performance of a small burns unit with six patient rooms (Hambraeus & Sanderson, 1972) it was clear that, even in the comparatively short length of the passage way concerned, about 32 m., there were large differences in concentration at different distances from the source of airborne particles entering it. The ward corridor in the part of the hospital studied in the preceding papers was about 100 m. long and any assumption of uniform mixing would clearly have been absurd. A mathematical treatment of diffusion along a ventilated passageway has therefore been developed and applied to this and other situations. Diffusion in this context is transport by any process which follows the diffusion law, that the rate of transport, whether of gas or some property of the medium, across a surface is proportional to the gradient at that point.

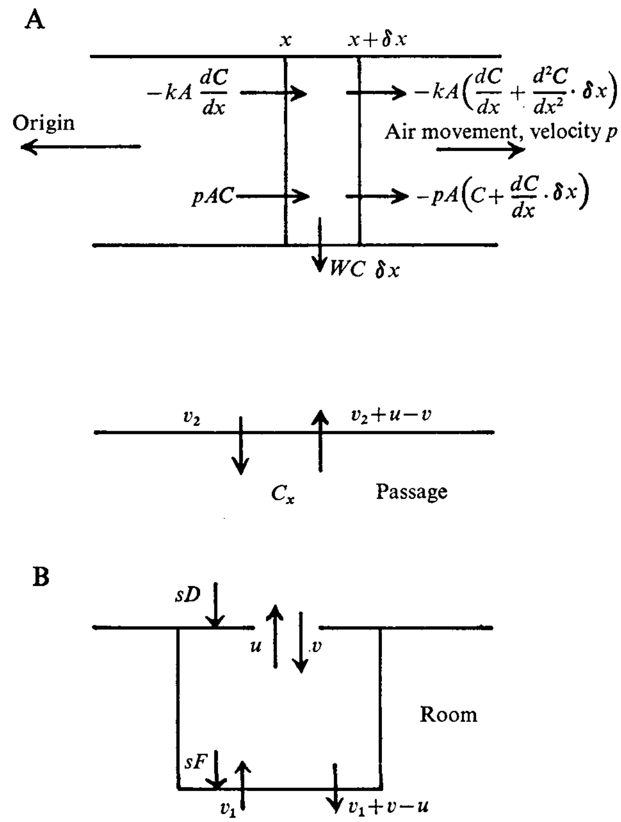

Fig. 1. Calculation of transfer of a tracer along a passage. (A) The quantities of material entering and leaving a thin transverse element situated at distance x from the origin. (B) Exchange with a room which communicates with the passage. The symbols are defined in the text.

AIRBORNE TRANSPORT ALONG A VENTILATED PASSAGEWAY

The model assumed is pictured in Fig. 1. By considering the gain and loss of material for a section of passage of length δx we obtain at equilibrium the following identity:

$$-kA\frac{dC}{dx}+pAC = -kA\left(\frac{dC}{dx}+\frac{d^2C}{dx^2}\cdot\delta x\right)+pA\left(C+\frac{dC}{dx}\cdot\delta x\right)+WC\delta x,$$

whence
$$-kA\frac{d^2C}{dx^2}+pA\frac{dC}{dx}+WC = 0,$$

or
$$\frac{d^2C}{dx^2}-\frac{p}{k}\cdot\frac{dC}{dx} = \frac{W}{kA}\cdot C, \tag{1}$$

where C is the concentration at distance x from the source (considered as a thin plane across the passageway), A is the cross-sectional area of passage, k is the

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diffusion constant and p represents a uniform moveme... the air along the passageway. The lateral loss from the passage, per unit length, W is made up of:

(1) Ventilation into the passage, v_2, where v_2 is the rate of ventilation to the passage per unit length.

(2) Sedimentation (of particles) onto the floor of the passage, sD, where s is the velocity of sedimentation of the particles and D is the width of the passage.

(3) Loss due to exchange of air with rooms along the corridor, u', where

$$u' = u - \frac{uv}{v_1 + sF + v},$$

summed over all the rooms opening onto the passageway and expressed per unit length of passage, where u = the rate of airflow out from a room into the passage, v the rate of airflow into the room from the passage, v_1, is the rate of air supply to the room and F its floor area.

Hence

$$W = v_2 + sD + u - \frac{uv}{v_1 + sF + v} \quad (2)^*$$

(expressed per unit length of passage).

The solution of equation (1) is of the form

$$C/C_0 = K_1 e^{m_1 x} + K_2 e^{m_2 x}, \quad (3)$$

where C_0 is the value of C at the origin, K_1 and K_2 are constants and

$$m_1 = \sqrt{\left(\frac{W}{kA} + \frac{p^2}{4k^2}\right)} + \frac{p}{2k} \quad (3\text{A})$$

and

$$m_2 = -\sqrt{\left(\frac{W}{kA} + \frac{p^2}{4k^2}\right)} + \frac{p}{2k}. \quad (3\text{B})$$

Since $C/C_0 = 1$ when $x = 0$, $K_1 + K_2 = 1$.

* By considering the model as depicted in Fig. 1B then, for the passage,

$$\begin{aligned} WC_x &= C_x(v_2 + u - v) - C_R u + C_x sD + C_x v \\ &= C_x(v_2 + u + sD) - C_R u \end{aligned}$$

and for the room

$$\begin{aligned} vC_x &= C_R(v_1 + v - u) + C_R u + C_R sF \\ &= C_R(v_1 + v + sF), \end{aligned}$$

where C_R is the concentration in the room.

whence

$$WC_x = C_x(v_2 + u + sD) - \frac{uv}{v_1 + v + sF} . C_x$$

or

$$W = v_2 + sD + u - \frac{uv}{v_1 + v + sF}.$$

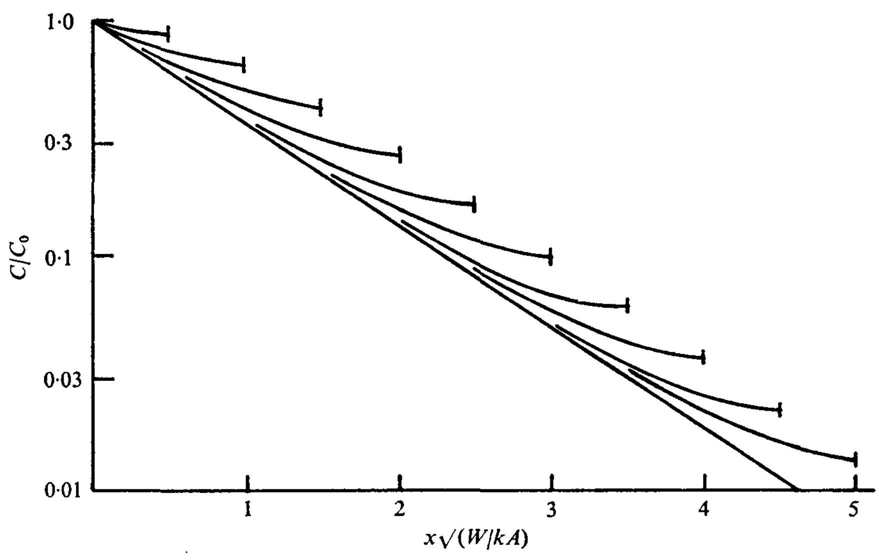

Fig. 2. The concentration gradient produced along passages of finite length closed at the end. The curves are derived from equations (3) and (5) in the text. The underlying straight line from which the individual curves deviate as the ends of the respective passages are approached corresponds to $C/C_0 = e^{-x\sqrt{(W/kA)}}$ (see text).

LONG PASSAGEWAYS

For a passage of infinite length (and uniform conditions)

$$C \to 0 \quad \text{as} \quad x \to \infty$$

and we have
$$K_1 = 0, \quad K_2 = 1.$$

Hence
$$C/C_0 = \exp\left(-\sqrt{\left(\frac{W}{kA}+\frac{p^2}{4k^2}\right)}+\frac{p}{2k}\right). \tag{4}$$

SHORT PASSAGEWAYS

If the passage in one direction from the origin is of finite length, a, then equating the quantity entering this to that lost gives the identity

$$-kA\left(\frac{dC}{dx}\right)_{x=0} = \int_0^a WC\,.\,dx.$$

Equation (3) then leads to the relation

$$-kA(m_1K_1+m_2K_2) = W[K_1e^{m_1x}/m_1+K_2e^{m_2x}/m_2]_0^a.$$

Since there can be no uniform movement of air along a passage with a closed end $p = 0$ and

$$m_1 = \sqrt{\left(\frac{W}{kA}\right)}, \quad m_2 = -\sqrt{\left(\frac{W}{kA}\right)}.$$

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Hence

$$-\sqrt{(WkA)}(K_1-K_2) = \sqrt{(WkA)}(K_1 e^{a\sqrt{(W/kA)}} - K_2 e^{-a\sqrt{(W/kA)}}) - \sqrt{(WkA)}(K_1-K_2)$$

and

$$K_1 e^{a\sqrt{(W/kA)}} = K_2 e^{-a\sqrt{(Wk/A)}}, \tag{5}$$

while, as before,

$$K_1+K_2 = 1.$$

In Fig. 2, C/C_0 is shown as a function of $x\sqrt{(W/kA)}$ for various values of $a\sqrt{(W/kA)}$. It can be seen from this figure that C/C_0 departs substantially from $e^{-x\sqrt{(W/kA)}}$ only at distances less than $\sqrt{(W/kA)}$ from the end of the passage and that for values of $a > 1{\cdot}5\ \sqrt{(kA/W)}$ the terminal value of

$$C \simeq 2C_0 e^{-a\sqrt{(W/kA)}}.$$

CONCENTRATION AT THE ORIGIN

For purposes of calculation the material entering the passage from the source room is assumed to be evenly distributed over a plane section of the passage at the origin, $x = 0$. The amount moving in the downwind direction is then

$$-kA\left(\frac{dC_a}{dx}\right)_{x=0} + pAC_0$$

and the amount moving in the upwind direction is

$$-kA\left(\frac{dC_b}{dx}\right)_{x=0} - pAC_0,$$

where C_a is the concentration in the downwind direction and C_b that in the upwind direction.

Equating these to the amount of material entering the passage

$$u_S C_S - v_S C_0 = -C_0 kA\{(m_1 K_1 + m_2' K_2)_a + (m_1 K_1 + m_2 K_2)_b\},$$

where u_S is the rate of airflow out from the source room into the passage, v_S the rate of airflow into that room from the passage and C_S in the concentration in the source room. Hence

$$\frac{C_S}{C_0} = -\frac{kA}{u_S}\left\{(m_1 K_1 + m_2 K_2)_a + (m_1 K_1 + m_2 K_2)_b + \frac{v_S}{kA}\right\}$$

If the passage is long in both directions $K_2 = 1$, $K_1 = 0$ and from equations 3A and 3B

$$(m_2)_a = -\sqrt{\left(\frac{W}{kA} + \frac{p^2}{4k^2}\right)} + \frac{p}{2k}, \quad (m_2)_b = -\sqrt{\left(\frac{W}{kA} + \frac{p^2}{4k^2}\right)} - \frac{p}{2k}$$

and

$$\frac{C_S}{C_0} = \frac{2kA}{u_S}\left\{\sqrt{\left(\frac{W}{kA} + \frac{p^2}{4k^2}\right)} + \frac{v_S}{2kA}\right\}. \tag{6}$$

Table 1. *Transport of airborne particles along the passage in a burns unit*

Distance from source room, x (in m.)	C/C_0	$a\sqrt{(W/kA)}$	$\sqrt{(W/kA)}$
−10	Entry end	—	—
−7·5	0·221	2·4	0·24
0	(1·00)	—	—
+7·5	0·089	7·0	0·32
+15·0	0·055	4·25	0·19
+22	Inner end		

The values of $a\sqrt{(W/kA)}$ have been estimated from the data given in Fig. 2 by choosing that curve which intersects the given value of C/C_0 at the appropriate value of x/a, where a is the distance from the source room to the end of the passage in the direction concerned.

In a previous paper (Foord & Lidwell, 1975*a*) estimates were made of a notional ventilation rate to the passage, treated as if there were complete mixing within it. On this basis

$$\frac{C_S}{C_0} = \frac{V_{ES}+u_S}{u_S},$$

where V_{ES} is the notional ventilation rate in relation to the part of the passage immediately outside the source room door. Then from equation (6)

$$V_{ES} = 2kA\left\{\sqrt{\left(\frac{W}{kA}+\frac{p^2}{4k^2}\right)}+\frac{v_S}{2kA}\right\}-u_S. \tag{7}$$

COMPARISON WITH OBSERVED DATA

For these calculations the suffixes S and R are used to denote quantities relating to the source and receiving rooms respectively. C_0 is the concentration in the passageway immediately outside the source room and C_x that outside a receiving room at distance x along the passage.

1. *Uppsala burns unit*

The results obtained for transport of airborne particles between the room and along the passage of this unit have been described (Hambraeus & Sanderson, 1972).

From the data given, W can be estimated as the sum of the corridor ventilation and sedimentation (exchange with the rooms was negligible). Hence

$$W = \frac{1100}{2600\times 32}+0{\cdot}005\times 3 = 0{\cdot}025 \text{ m.}^3\text{/sec./m.}$$

The data for the passage concentrations are given in Table 1 from which an average value for $\sqrt{(W/kA)}$ can be estimated as 0·25/m. The cross-sectional area of the passage was approximately 8 m.^{2} and hence

$$k = 0{\cdot}025/(8\times 0{\cdot}25^2) = 0{\cdot}050 \text{ m.}^2\text{/sec.}$$

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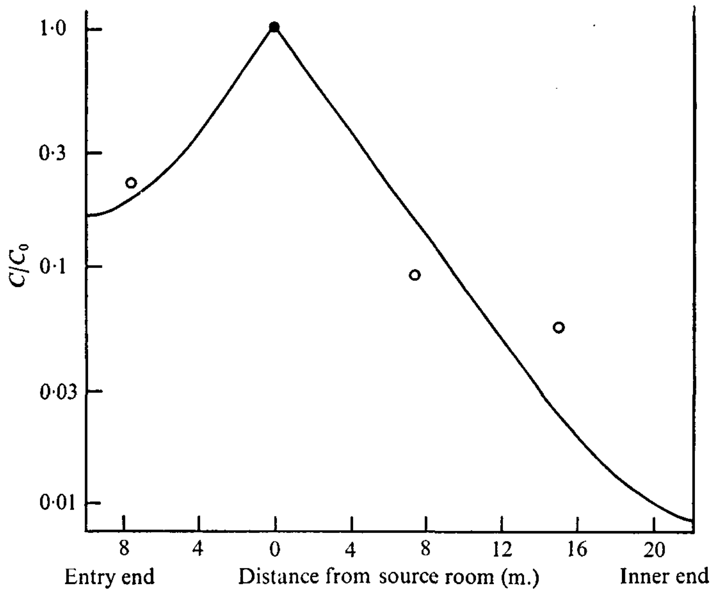

Fig. 3. The variation in concentration of particle tracer along the passage of the Burns Unit at the Uppsala University Hospital (Hambraeus & Sanderson, 1972). The full lines show the calculated curves. The experimentally observed values are indicated by open circles.

From Table 2 in the paper cited

$$u_S = 17/3600 = 0{\cdot}0047 \text{ m.}^3/\text{sec.}$$

As there was no uniform airflow along the passage, which was closed at both ends, $p = 0$ and, since v_s was small, equation (6) leads to

$$\frac{C_S}{C_0} = 2\sqrt{(WkA)}/u_S = 42{\cdot}5.$$

This compares well with the value of 45 given in Table 1 (line 2) of the same paper.

By using equation (5) values for K_1 and K_2 along the passage in both directions can be calculated and the concentration profile along the passage deduced. This is shown in Fig. 3.

2. *Greenwich District Hospital*

Transport of tracer gas

The variations of C/C_0 along the passage in both directions from source rooms in different positions in the ward units studied are shown in fig. 7 of a preceding paper (Foord & Lidwell, 1975*a*). As the passages were relatively long the formula of equation (4) has been used for purposes of computation and the values of (d ln C/dx) are given in Table 2. From equation (4) it can be immediately seen that the product of the values (d ln C/dx) in the two directions, i.e. upwind and downwind (p negative and positive respectively), is equal to W/kA. Similarly the difference of these two values is equal to p/k. Values for V_{ES} have then been

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Table 2. *Transport of tracer-gas along ward corridor*

	W (m.^{3}/sec./m.)	u (m.^{3}/sec.)	v (m.^{3}/sec.)	$d \ln C/dx$		W/kA (m^{-2})	k (m.^{2}/sec)	p/k (m^{-1})	p (m./sec.)	V_{ES} (m.^{3}/sec.)	
Series 1				A → C	C → A					calc.	obs.
A	0·0446	0·140	< 0·005	0·52	0·12	0·062	0·090	0·40	0·036	0·33	0·30
B	0·0359	0·100	0·046	0·38	0·16	0·061	0·074	0·22	0·016	0·26	0·30
C	0·0315	0·080	0·070	0·24	0·24	0·058	0·068	0·00	< 0·001	0·25	0·25
Series 2											
A	0·0356	0·092	0·040	0·28	0·25	0·070	0·064	0·03	0·002	0·22	0·36
B	0·0322	0·091	0·057	0·32	0·13	0·042	0·099	0·19	0·019	0·28	0·35
C	0·0316	0·081	0·062	0·092	0·33	0·030	0·131	0·24	0·031	0·34	0·60

The symbols have the meaning given in the text. Values of W were calculated according to equation (2). All values of W, u, v and ($d \ln C/dx$) are averaged for each ward unit. Two values of ($d \ln C/dx$) are given, for transport from end A of the ward area towards end C and vice versa. A = 8 m.^{2}. Values of k, p and V_{ES} (calc.) have been obtained as described in the text. The values of V_{ES} (obs.) are derived from the data of Fig. 7 (Foord & Lidwell, 1975*a*).

Table 3. *Calculation of the particle loss factor for transport between rooms in Greenwich District General Hospital*

Quantity	Series 1		Series 2	
u (m.³/sec.)	0·107		0·088	
v (m.³/sec.)	0·039		0·053	
v_1 (m.³/sec.)	0·205		0·195	
p (m./sec.)	0·017		0·017	
k (m.²/sec.)	0·077		0·098	
	Gas	Particle	Gas	Particle
W (m.³/sec./m.)	0·036	0·054	0·033	0·051
$-d \ln C/dx$ (average for two directions)	0·266	0·316	0·223	0·269
$d/dx\ (\log_{10}\alpha''')$	0·116	0·137	0·097	0·117
α'	3·5	4·1	4·8	5·6
α''	6·2	13·9	4·7	10·3
$d/dx\ (\log_{10}(\alpha_p'''/\alpha_g'''))$	0·021		0·020	
$\log_{10}(\alpha_p'/\alpha_g')$	0·063		0·067	
$\log_{10}(\alpha_p''/\alpha_g'')$	0·350		0·340	
$\log_{10}$(particle loss factor)/m.	$0·413+0·021x$		$0·407+0·020x$	
$\log_{10}$(particle loss factor)/room interval, $(X) = 6·5$ m.	$0·413+0·130X$		$0·407+0·136X$	

The symbols are defined in the text. Values of u, v, p and k are taken from Table 2 averaged over all the ward units. Values of v_1 are derived similarly from those given in Table 1 of a previous paper (Foord & Lidwell, 1975*a*), by equating $v_1+v(1-1/\alpha')$ to the product of the ventilation rate and the room volume. v_2, the ventilation to the passage, was at the rate of about 0·014 m.³/sec./m. In addition to the exchange of air between patient rooms and the passage, which can be calculated from the values of u and v given above there was also exchange between the passage and the service rooms which opened off it. No measurement of this was obtained but from the number of these rooms and their ventilation arrangements an estimate of 0·008 m.³/sec. per metre of passage length was made and this has been included in u' when calculating the values of W by means of equation (2). $d \ln C/dx$ can then be calculated from equation (4).

$\alpha' = C_S/C_0$ is obtained from equation (6) and $\alpha'' = C_x/C_R$ as given by Fig. 1, i.e. $(v_1+v+sF)/v$. The sedimentation rate for the particles, s, has been taken as 0·006 m./sec., see Foord & Lidwell (1975*b*). D, the width of the passage was 2·8 m., A, its cross-sectional area, 8 m.² and F, the floor area of a patient room, 47 m.², see Foord & Lidwell (1975*b*).

calculated according to equation (7). The results of these calculations together with the appropriate values of W, u and v are given in Table 2.

Comparison of particle and gas transport

In a previous paper (Foord & Lidwell, 1975*b*) the effects of particle loss during transport between rooms in the ward units has been discussed. Good correlation was obtained with the transit time. In addition the particle loss factor was also correlated with the distance apart of the two rooms (fig. 5 in that paper).

The value of W, equation (2), includes a term for loss by sedimentation. It is therefore possible to deduce the difference in transport between gas tracer and particles which would be expected on the basis of the model proposed in this paper. Using the known values of air supply to patient rooms and passage and the previously estimated values for air exchange between these s and the passage the concentration gradient along the passage can be calculated from equation (4). The 'particle loss factor' has been defined earlier (Foord & Lidwell, 1975*b*) as

$$(C_S/C_R)_{\text{particle}}/(C_S/C_R)_{\text{gas}} = \alpha_{\text{particle}}/\alpha_{\text{gas}}.$$

This can be broken down into three components

$$\frac{(C_S/C_0)_p}{(C_S/C_0)_g} \cdot \frac{(C_x/C_R)_p}{(C_x/C_R)_g} \cdot \frac{(C_0/C_x)_p}{(C_0/C_x)_g} = \frac{\alpha_p'}{\alpha_g'} \cdot \frac{\alpha_p''}{\alpha_g''} \cdot \frac{\alpha_p'''}{\alpha_g'''}.$$

Equation (4) gives estimates of $d \ln C/dx$ for both gas and particle tracers. The difference between the two values, with change of base, then gives $\log_{10} (\alpha_p'''/\alpha_g''')$ per unit length of passage. Values of α_p' and α_g' can be obtained from equation (6). The values of α_p'' and α_g'' follow directly from the relationship shown in Fig. 1 since

$$\alpha'' = C_x/C_R = (v_1 + sF + v_R)/v_R.$$

The results of all these computations are given in Table 3 together with the values of the particle loss factor derived from them. The logarithms of these values are shown as a function of the distance along the passage between the rooms or as a function of the number of rooms apart for the Greenwich inter-room spacing of 6·5 m. This is compared with experimental data in fig. 5 of the previous paper (Foord & Lidwell, 1975*b*). It will be seen that the calculated values indicate a somewhat higher particle loss factor than that deduced from the experimental data but the variability in the data is much greater than the discrepancy. On the basis of both the experimental and the calculated results it would seem that a good approximation to the particle loss factor is given by the relation:

$$\text{particle loss factor} = 0{\cdot}30 + 0{\cdot}11X,$$

where the source and recipient rooms are X rooms apart (1 room interval $\simeq$ 6·5 m.).

3. *Isolation unit St Mary's Hospital*

Some measurements with the particle tracer, not reported elsewhere, were made in the isolation unit described by Williams & Harding (1969). This consisted of two groups of patient rooms, each group opening off a ventilated lobby area, linked by a passage about 45 m. long. There was a perceptible movement of air along the passage from west to east. The width of the passage was 1·8 m. and the ventilation supply small. Hence, $W \simeq 0{\cdot}005 \times 1{\cdot}8 = 0{\cdot}009$ m.^{3}/sec./m., being the loss due to sedimentation of the particles.

Measurements of the variation in particle concentration along the passage were made in both directions and led to the following values:

$$\left(\frac{d \ln C}{dx}\right)_{\text{E}\to\text{W}} = 0{\cdot}092, \text{ in the east to west direction,}$$

$$\left(\frac{d \ln C}{dx}\right)_{\text{W}\to\text{E}} = 0{\cdot}253, \text{ in the west to east direction,}$$

hence $W/kA = 0{\cdot}0233$ and $p/k = 0{\cdot}161$.

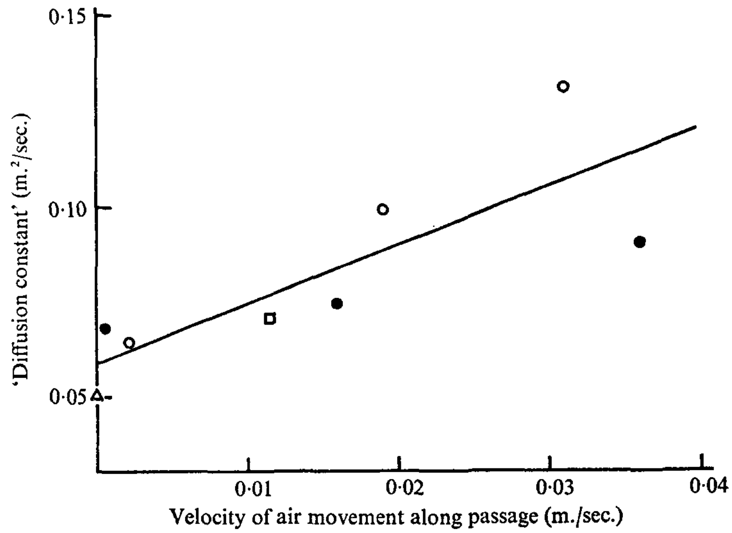

Fig. 4. Values of the diffusion constant in passages in relation to the overall velocity of air movement along the passage. The circles give the values obtained from the obervations made at the Greenwich District General Hospital. Open circles, series 1; filled circles, series 2. The triangle is derived from observations made in the Uppsala Hospital and the square from those made at St Mary's Hospital, London.

Since the cross-sectional area of the passage, A, was 5·5 m.^{2} this gives k = 0·070 m.^{2}/sec. and p = 0·011 m./sec.

DISCUSSION

The model proposed is able to give a coherent picture of the transport of both gas and particle tracers along the ward corridor in the Greenwich Hospital. The data from the other two situations are too limited to give any confirmation of the value of the method taken individually. It is, however, worth noting that the air drift velocities deduced are reasonable in magnitude. The highest value is 0·036 m./sec. (approximately 7 ft./min.) which is consistent with the fact that all the air movements were below the velocities measurable by normal anemometry.

The values for the 'diffusion constant' from the several determinations have been plotted together in Fig. 4 against the value of the air-drift velocity. There appears to be a consistent relation between the two quantities, the 'diffusion constant' rising from about 0·06 m.^{2}/sec. in still air to double that value at a drift velocity of about 0·04 m./sec.

Although the consistency of these results is notable it must be emphasized that the variability of the experimental observations from which they have been deduced was very high so that the possible errors of interpretation are considerable. However, taken with the methods developed earlier (Lidwell, 1972) the present work does give a basis for estimating the extent of transport of gases or particles between rooms in ventilated buildings along passage ways of substantial length, e.g. up to 100 m. or more. When or if such calculations are made it must be borne in mind that the actual transfers which take place will almost certainly vary by large factors from time to time. The variations will probably lead to an approximately log-normal distribution of the amount of transferred material with standard deviations up to 2·0 or more (logarithms to base 10). The calculations made here relate only to the median values of such transfers.

REFERENCES

FOORD, N. & LIDWELL, O. M. (1975*a*). Airborne infection in a fully air-conditioned hospital. I. Air transfer between rooms. *Journal of Hygiene* **75**, 15.

FOORD, N. & LIDWELL, O. M. (1975*b*). Airborne infection in a fully air-conditioned hospital. II. Transfer of airborne particles between rooms resulting from the movement of air from one room to another. *Journal of Hygiene* **75**, 31.

HAMBRAEUS, A. & SANDERSON, H. F. (1972). The control by ventilation of airborne bacterial transfer between hospital patients, and its assessment by means of a particle tracer. III. Studies with an airborne particle tracer in a ward for burned patients. *Journal of Hygiene* **70**, 299.

LIDWELL, O. M. (1972). The control by ventilation of airborne bacterial transfer between hospital patients and its assessment by means of a particle tracer. II. Ventilation in subdivided isolation units. *Journal of Hygiene* **70**, 287.

WILLIAMS, R. E. O. & HARDING, L. (1969). Studies of the effectiveness of an isolation ward. *Journal of Hygiene* **67**, 649.