

# Measuring and Decoding Gravitational-Inertial Modes in Intermediate- and High-Mass Stars

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**Abstract.** This talk discussed the basics of gravito-inertial asteroseismology as recently developed for stars born with a convective core. Photometric space missions originally built for exoplanet hunting, notably *Kepler*, have opened up the low-frequency regime of stellar oscillations and revealed a larger diversity in variability than anticipated prior to the era of high-precision space photometry. The talk explained the basics of forward seismic modelling based on gravito-inertial modes, which probe the deep stellar interior. It described how a hierarchical fitting approach allows us to derive the near-core rotation period, the amount and shape of convective core overshooting, and the level of chemical mixing in the radiative envelope for stars born with a convective core and burning hydrogen in their core. A summary of the current status, covering the mass range  $1.4 \lesssim M \lesssim 5 M_{\odot}$ , is provided here through references to numerous recent papers.

**Keywords.** Stars: interiors, stars: evolution, stars: oscillations (including pulsations), stars: rotation, methods: statistical, methods: numerical

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## 1. Introduction

Stars are the building blocks of galaxies, clusters, associations, and exoplanetary systems. Models of their evolution are therefore a basic ingredient of many studies in contemporary astrophysics. Now that we can obtain high-precision time-series photometric data from space, we find that stellar models show major shortcomings in terms of mixing and angular momentum, even during the ‘simple’ core-hydrogen burning phase of their evolution. That is particularly the case for stars born with a convective core. Asteroseismology (the study and interpretation of non-radial stellar oscillations) offers a new method to evaluate and calibrate stellar models, because it uses *observational evidence coming from the stellar interior rather than relying on surface properties alone*.

Asteroseismology of core-hydrogen burning low-mass stars ( $M \lesssim 1.4 M_{\odot}$ ) relies on stochastically-excited solar-like pressure-mode oscillations with periods of a few minutes. For such modes, the pressure force is the dominant restoring force. These modes are connected with sound waves in the stellar interior. They reveal a characteristic *frequency spacing*, whose value is determined by the mean density of the star. This so-called large frequency spacing allows us to obtain the mass and radius of distant stars, by scaling the frequencies of the solar oscillations and assuming that the input physics of the solar model offers a good approximation for other low-mass pulsators.

The evolved stages of low- to intermediate-mass stars ( $M \lesssim 3 M_{\odot}$ ), covering shell-hydrogen burning and core-helium burning, offer us the opportunity to exploit dipole mixed modes – a major asset revealed by the *Kepler* data of red giants (see [Chaplin & Miglio \(2013\)](#), [Hekker & Christensen-Dalsgaard \(2017\)](#) for recent reviews). Mixed modes have periods of a few hours and reveal a pressure-mode character in the stellar

envelope but a gravity-mode character in the deep interior of evolved stars. That enables us to deduce not only the burning stage – see [Bedding \*et al.\* \(2011\)](#) and [Mosser \*et al.\* \(2014\)](#) – but also the interior rotation properties as revealed from dipole mixed modes, as discovered by [Beck \*et al.\* \(2012\)](#) and [Mosser \*et al.\* \(2012\)](#). Because of the scaling relations for pressure modes, stars subject to them can be studied as asteroseismic ensembles, as is the case for thousands of low-mass dwarfs and red giants in the *Kepler* database.

Gravity-mode oscillations with periods of half to a few days occur in main-sequence stars born with a well-developed convective core and a radiative envelope ( $M \gtrsim 1.4 M_{\odot}$ , and spectral types from early F all the way to O). For such modes, buoyancy is the dominant restoring force. They correspond to gravity waves revealing characteristic *period spacings*. While such spacings were known from ground-based photometry for white dwarfs (see, e.g., [Kawaler \*et al.\* \(1999\)](#) for their probing power in such stellar remnants), they were first discovered in main-sequence stars by [Degroote \*et al.\* \(2010\)](#) and [Pápics \*et al.\* \(2012\)](#) from 150-day CoRoT light curves. Although the CoRoT data were suitable for discovering the period spacings for a few stars, the precision of the mode periods was insufficient for deducing unambiguous mode identification, which subsequently prevented the details of the interior physics of models representing such stars to be evaluated. That opportunity had to await the 10 times longer light-curves observed by the *Kepler* mission.

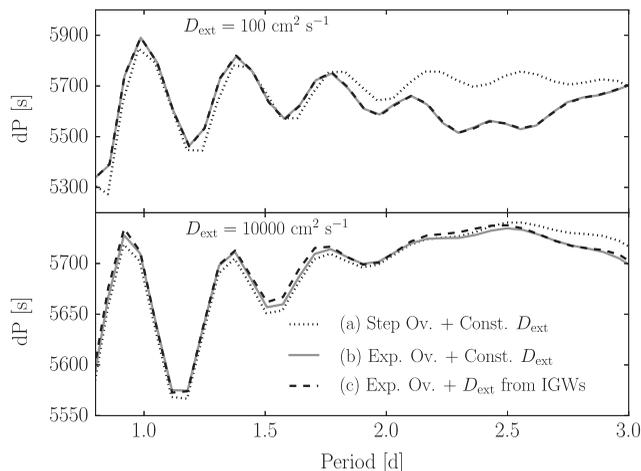
Two extensive review papers on low-mass star asteroseismology have been published by [Chaplin & Miglio \(2013\)](#) and [Hekker & Christensen-Dalsgaard \(2017\)](#), so here we focus on gravity modes and their probing power of stellar interiors as derived from *Kepler* data.

## 2. Gravito-Inertial Asteroseismology

Stars born with a well-developed convective core and a radiative envelope live much shorter lives and have different interior structure compared to low-mass stars born with a radiative core and a convective envelope. In the absence of magnetic activity caused by an envelope dynamo, O, B, A and F-type stars tend to be rapid rotators. Their evolution is considerably affected by their interior rotation and the angular momentum transport processes it induces, as well as by the extent and shape of convective core overshooting. Extensive theory has been developed for almost half a century to describe rotationally-induced instabilities and mixing processes (see [Maeder \(2009\)](#), for a review), but none of those phenomena is well calibrated by observations of a star's surface properties. As a consequence, models of the evolution of O–F stars employ numerous free parameters to describe these uncalibrated processes (see [Heger \*et al.\* \(2000\)](#) and references therein).

Gravity-inertial asteroseismology offers a new way of evaluating the theory of rotating stars with convective cores, and can lead to the level of precision required to calibrate the physical processes in stellar interiors. At present this is only feasible for stars in the core-hydrogen burning phase without mass loss, owing to a lack of mode identification for stars with  $M \gtrsim 5 M_{\odot}$ . The *Kepler* data of B–F-type stars made it obvious that ground-based data are not suitable for detecting the required low-frequency gravito-inertial modes, as they have amplitudes in the range 10–10 000 ppm; see, e.g., [Van Reeth \*et al.\* \(2015\)](#) and [Pápics \*et al.\* \(2017\)](#). The excitation of low-frequency gravito-inertial modes in intermediate- to high-mass rotating stars of  $M \gtrsim 1.4 M_{\odot}$  is well understood in terms of the opacity mechanism acting on the partial ionisation zone of Fe-like elements and/or the flux blocking mechanism – see [Szewczuk & Daszyńska-Daszkiewicz \(2017\)](#) and [Bouabid \*et al.\* \(2013\)](#) for excitation computations in these two cases, respectively.

A major conclusion which was derived from the *Kepler* data of B–F-type pulsators is that the density of excited low-frequency gravito-inertial modes is far larger than anticipated from ground-based data. Moreover, the occurrence of series of Rossby or Yanai modes of consecutive radial order is the norm rather than the exception in fast



**Figure 1.** Period differences ( $dP$ ) for dipole modes of consecutive radial order, versus their period  $P$ . The figure is based on three stellar Models, each with a different treatment of core overshooting and envelope mixing, as indicated by the different line styles. See text for details.

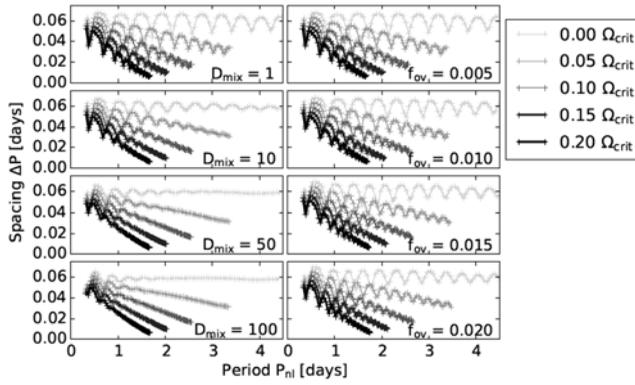
rotators; see Saio *et al.* (2018) and Van Reeth *et al.* (2018). The majority of these low-frequency modes have too low an amplitude to be detectable in ground-based photometry.

### 3. Seismic Modelling Approach

Forward seismic modelling considers the observed oscillation frequencies of identified modes and tries to fit them with those predicted from stellar models, for various assumptions of the input physics. The minimal free parameters to compute the models with fixed choice of the input physics are the stellar mass  $M$ , the initial hydrogen and metal mass fractions ( $X, Z$ ), and the age represented by the central hydrogen mass fraction  $X_c$  in the case of core-hydrogen burning. For intermediate- and high-mass stars *it is essential to take into account the Coriolis force in the computations of gravito-inertial mode properties when performing forward seismic modelling*, even for moderate rotators. That complicates matters with respect to the case where the force can be omitted, as for pressure modes in low-mass sun-like stars or red giants. Gravito-inertial asteroseismology must hence be done in at least a 5-D modelling scheme, where the 5<sup>th</sup> parameter is the rotation period  $P_{\text{rot}}$  of the star. Moreover, unlike low-mass stars born with a radiative core, stars with a convective core are subject to the phenomenon of core overshooting, adding a necessary 6<sup>th</sup> parameter to be estimated.

The complication in forward seismic modelling of stars with a convective core is largely compensated by the major asset it brings: a direct measurement of the near-core rotation period and chemical mixing from the fitting of mode trapping and rotational mode frequency shifts. The former phenomenon is illustrated in Fig. 1, where we show the period-spacing patterns as a function of mode period for gravity modes of the same degree  $l$  and consecutive radial order  $n$ , for three different non-rotating stellar Models with the same mass of  $3.25 M_{\odot}$ ,  $X = 0.71$ ,  $Z = 0.014$ , and evolutionary stage (central hydrogen fraction  $X_c = 0.50$ ). What differentiates the Models, and hence their gravity modes, is their mixing profile  $D_{\text{mix}}(r)$ :

(a) Model (a) has a fully mixed zone adjacent to the convective core caused by core overshooting in the shape of a step function having  $D_{\text{mix}}(r) = D_{\text{mix}}(r_c)$  over an overshoot distance given by  $\alpha_{\text{ov}} = 0.15 H_p$ , where  $r_c$  is the position of the core boundary



**Figure 2.** Period-spacing patterns for a rigidly rotating stellar model of  $3 M_{\odot}$  with central hydrogen fraction  $X_c = 0.5$ , for various rotation rates, exponential overshooting values  $f_{ov}$ , and mixing properties  $D_{mix}$  (in  $\text{cm}^2 \text{s}^{-1}$ ) as indicated in the legends.

adopting the Ledoux criterion. There is also additional constant diffusive chemical mixing throughout the entire radiative envelope equal to  $D_{ext} = 100 \text{ cm}^2 \text{ s}^{-1}$  (upper panel) or  $10\,000 \text{ cm}^2 \text{ s}^{-1}$  (lower panel);

(b) Model (b) has an exponentially decaying diffusive core overshooting as of  $r_c$  outward, described by the parameter  $f_{ov} = 0.015$  at the core boundary (see Paxton *et al.* (2011) for a definition of this overshoot prescription); here as well there is additional constant diffusive mixing in the radiative envelope equal to  $D_{ext} = 100 \text{ cm}^2 \text{ s}^{-1}$  (upper panel) or  $10\,000 \text{ cm}^2 \text{ s}^{-1}$  (lower panel);

(c) Model (c) is the same as Model (b) regarding the core overshooting, but here there is diffusive chemical mixing in the radiative envelope due to internal gravity waves (IGW) according to the mixing profile computed by Rogers & McElwaine (2017), with value equal to  $D_{mix} = 100 \text{ cm}^2 \text{ s}^{-1}$  (upper panel) and  $10\,000 \text{ cm}^2 \text{ s}^{-1}$  (lower panel) at the overshoot zone boundary where the radiative envelope starts.

It can be seen in Fig. 1 that the gravity modes of Models (a) and (b) have different period spacings owing to the difference in overshooting description. A typical uncertainty of the measured period spacings for a nominal *Kepler* light-curve and gravity modes of period around 2.5 d amounts to some 50 s. Hence, the modes shown in the upper panel of Fig. 1 can reveal which shape of core overshooting, a step function or an exponentially decaying function, is most appropriate. This discrimination becomes harder as more envelope mixing  $D_{ext}$  comes into play. The gravity modes of Models (b) and (c) have almost equal periods (full versus dashed lines in Fig. 1). That is not surprising given that they probe the near-core region where  $D_{mix}$  is the same for both Models. However, owing to the different shape of  $D_{mix}(r)$  in the envelope – constant  $D_{mix}$  versus rising  $D_{mix}(r)$  from IGWs – the chemical mixing has a different efficiency in changing the surface abundances. In particular, the difference in surface nitrogen abundance between Models (b) and (c), along with their period-spacing properties, enables us to distinguish in principle between those two Models for the envelope mixing (Pedersen *et al.* 2018).

As was shown by Moravveji *et al.* (2016) and Van Reeth *et al.* (2016), any forward seismic modelling of gravity modes in the sub-inertial regime of a rotating star, with the aim of deriving the level and shape of both the core overshooting and the envelope mixing, must rely on the value of the near-core rotation period  $P_{rot}$ . Indeed, the rotation of the star sets the ‘tilt’ that occurs in the period-spacing pattern. That is illustrated in Fig. 2, where we show the dipole gravity mode patterns predicted for models of  $3 M_{\odot}$  with various rotation rates (expressed as a percentage of the critical rotation frequency  $\Omega_{crit}$ ), various levels of exponential diffusive core overshooting, and different levels of constant

diffusive envelope mixing. Basing their work on patterns similar to those in Fig. 2, Van Reeth *et al.* (2016) and Ouazzani *et al.* (2017) developed a method of estimating  $P_{\text{rot}}$  from the slope of the spacings for F stars. The method by Van Reeth *et al.* (2016) has meanwhile been applied to pulsating B stars by Pápics *et al.* (2017) and Zwintz *et al.* (2017). It was also generalised recently to treat the case of non-rigid rotation (Van Reeth *et al.* 2018). A summary of the core-to-envelope rotation rates for 67 intermediate-mass stars with  $1.4 \lesssim M \lesssim 5 M_{\odot}$ , rotating from a few percent up to 50% critical, is available in Fig. 1 of Aerts *et al.* (2017).

With  $P_{\text{rot}}$  determined for tens of stars, the next step is to exploit the morphology of the observed period-spacing patterns, including the structure of the dips. That requires performing a minimisation process between observed and predicted gravito-inertial mode periods in a high-dimensional free parameter space (at least 5-D). A good forward modelling scheme for rotating gravito-inertial mode pulsators can be summarised as follows:

(a) deduce the periods  $P_i$  of gravity modes, in both the super-inertial and sub-inertial regime, and select those  $P_i$  of  $i = 1, \dots, N$  that constitute a period-spacing pattern;

(b) use the slope of the observed period-spacing pattern ( $P_i, \Delta P_i$ ) to identify the degree  $l$  and azimuthal order  $m$  of the modes  $P_i$  and to estimate  $P_{\text{rot}}$  with the method of Van Reeth *et al.* (2016); it requires a sparse grid of stellar models for an appropriate mass range, and the computation of their gravity-mode periods;

(c) compute a dedicated multi-D fine grid of stellar models that include not only the four basic free parameters mass, age and initial chemical composition (X,Y) but also various levels and shapes of core overshooting and envelope mixing. For the derived  $P_{\text{rot}}$ ,  $l$ ,  $m$ , compute their gravity-mode periods in an inertial frame of reference to identify the radial orders  $n_i$  of the detected modes. Subsequently, select the most likely stellar models using maximum likelihood estimation, along with the most likely values of  $M$ ,  $X_c$ ,  $X$ ,  $Z$ ,  $f_{\text{ov}}$ , and  $D_{\text{mix}}$  in the radiative envelope, together with their uncertainties.

Given the computational challenges of the above procedure, a simpler version has so far been applied to a few B and F stars; see Kurtz *et al.* (2014), Saio *et al.* (2015), Moravveji, *et al.* (2015, 2015), Kallinger *et al.* (2017). Full applications that will lead to high-precision estimates of mass, age, core overshooting and envelope mixing are currently under way using *Kepler* and BRITE data for O, B, A and F stars.

#### 4. Ongoing and Future Research

Since step (c) in the modelling scheme outlined in Sect. 3 has not yet been done for the majority of intermediate-mass and high-mass stars in the *Kepler* database for which period-spacing patterns have been detected, our current focus lies on that step in the forward modelling. Aside from the near-core rotation period already provided in Aerts *et al.* (2017), it will deliver estimates of the value and shape of the core overshooting and also of the envelope mixing across the mass range  $1.4 \lesssim M \lesssim 5 M_{\odot}$ . It will reveal the relationships between mass, rotation, age, core overshooting and envelope mixing.

Major future progress in gravito-inertial asteroseismology, for both single and binary stars, is expected from large ensembles of O–F stars to be monitored during a year or longer by NASA’s TESS mission (its Continuous Viewing Zones, Ricker *et al.* (2016), to be launched in 2018) and by the PLATO mission (Rauer *et al.* (2014), to be launched in 2026). Their long-duration space photometry will be complemented by ground-based spectroscopic surveys (e.g., Kollmeier *et al.* (2017) for the SDSS V programme) that will achieve detailed asteroseismic modelling as described in Sect. 3 for thousands of single and binary intermediate- and high-mass stars of various metallicities.

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