# FORMATION AND EVOLUTION OF LOW-MASS ALGOLS

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ABSTRACT. We discuss the origin, evolution and fate of low-mass Algols (LMA) that have components with initial masses less than 2.5 $M_{\odot}$. The semi-major axes of orbits of pre-LMA do not exceed 20-25 $R_{\odot}$. The rate of formation of Algol-type stars is ~ 0.01/year. Magnetic stellar winds may be the factor that determines the evolution of LMA. Most LMA end their lives as double helium degenerate dwarfs with $M_1/M_2 \sim 0.88$ (like L870-2). Some of them even merge through angular momentum loss caused by gravitational waves.

## 1. INTRODUCTION

The study of Algols was for a long time, and still remains one of the mainstreams in the investigation of the close binaries (CB). The successful solution of the evolutionary paradox of Algols (Struve, 1948), by Crawford (1955), Morton (1960), Paczynski (1966), Kippenhahn and Weigert (1967), Snezko (1967) and many others gave an inspiration to further studies of CB, which ultimately led to the construction of evolutionary scenarii matching different types of observed CB.

Most evolutionary computations for CB were made under the simple assumption of conservation of total mass and orbital angular momentum of the system. The main results of these computations, which explained the general properties of the Algol family were summarized e.g. by Paczynski (1971), Thomas (1977), Yungelson and Masevich (1983). The most complete list of computations, to our knowledge, was compiled by Kraicheva (1987).

Quite early it became evident that "conservative" assumptions were probably, in most cases, far from reality. Svechnikov (1969) and Popov (1970) had shown that semi-detached CB have systematically lower total mass and momenta than their detached predecessors. Benson (1970) and Yungelson (1973) had found that even accretion of several hundredths of $M_{\odot}$ by the secondary of CB, on a time scale shorter than the thermal one, leads to the overflow of the Roche lobe, formation of a contact system and inevitable mass loss from the system. Detailed study of the evolution of low-mass Algols (LMA) by Iben and Tutukov (1984a) and Kraicheva *et al.* (1986) have shown that a considerable loss of angular momentum can be accomplished by magnetic braking.

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In this paper, within the general scenario approach, we briefly review the origin and evolution of Algols and their place among other types of stars. We pay special attention to the angular-momentum loss and the fate of Algols. We discuss mainly the low-mass Algols, which one may define as ancestors of CB with initial masses of components lower than $\sim 2.5\ M_\odot$. The main property which determines evolution in the Algol stage is the formation of degenerate helium cores after depletion of hydrogen. About 70 per cent of observed Algols belong to this group (estimate based on Svechnikov (1986) catalogue).

## 2. ANGULAR MOMENTUM LOSS BY LOW-MASS ALGOLS

The present-day evolutionary scenarii identify Algols with stars that are in the first Roche-lobe-overflow (RLOF) phase, which occurs either in the main-sequence stage (Case A) or after exhaustion of the hydrogen in the core (Case B). It is usually assumed that the RLOF results from the evolutionary expansion of the star. However, for stars with convective envelopes, another factor may be important, namely, magnetic braking. As first suggested by Huang (1966), the coupling of convection and magnetic field generates a stellar wind which removes the spin-momentum of stars. In binary systems, loss of spin momentum is compensated from the orbital reservoir, thus resulting in decrease of total orbital momentum. The theory of magnetic stellar wind is yet to be developed. However, a semi-empirical approach to the problem is possible: one may apply to components of close binaries the single star rotation-activity relation, transformed into an age-rotation relation.

The usual assumption is that Skumanich's (1972) law, $v \propto t^{-1/2}$, is valid. This relation, which initially was derived for main-sequence F-type stars, has been shown to extend to stars as late as K4 (Rengarajan, 1984). Skumanich's law for synchronously rotating binaries results in the following equation for the rate of angular-momentum loss:

$$\frac{d\,\ell n\,J}{dt} = 10^{-14}\ \frac{R_2^4\ (M_1 + M_2)^2}{\lambda^2\ a^5\ M_1}\quad s^{-1}, \tag{1}$$

where $R_2$ is the radius of the wind-losing star in units of $R_\odot$, $M_1$ and $M_2$ are masses of components in units of $M_\odot$, a is the semi-major axis of the orbit in units of $R_\odot$, $\lambda \sim 1$ is a constant.

The application of Eq. (1) to the description of AML has been suggested for cataclysmic variables (Verbunt and Zwaan, 1981), W UMa stars (Vilhu, 1982) and Algols (Iben and Tutukov, 1984a).

Skumanich's law was derived for single stars. However, Basri _et al._ (1985) found that synchronized dwarf components of CB follow the same rotation-activity relationship as single stars, but in subgiants the activity of chromospheres is even an order of magnitude higher. Iben and Tutukov (1984a) suggested applying Eq. (1) to secondaries of LMA with convective envelopes. At present, such an application is no more than a useful conjecture that still awaits proof. To our knowledge, the only Algol-type star with a measured magnetic field is Algol itself (Lestrade et al., 1986). The usual indicators of photospheric magnetic activity for LMA are practically unobservable in Algols, because of the low brightness of the secondaries compared to the primaries. However, the presence of deep convective envelopes in secondaries allows us to conjecture that coupling of convection with rotation generates a magnetic field, or amplifies the relic field.

The physical properties of LMA secondaries closely resemble those of secondaries in RS CVn type systems which are known for their magnetic activity. Some RS CVn stars are indeed semi-detached and, but for their small orbital inclinations, would appear as Algols. One certain example of such a system is BH CVn (Hall, 1981; Little-Marenin, 1986) in which the discovery of the active chromosphere of the KO IV secondary was made possible by the relative coolness of the F2 IV primary and the absence of visible emission from the accretion disk. According to White and Marshall (1983), the X-ray luminosity of Algols corresponds to that of single stars and components of wide binaries, extrapolated to the rotation velocities of Algols. Olson (1985) found variations of the brightness of the cool component of U Cep, suggestive of a magnetic-activity cycle. Therefore, in what follows we take Eq.(1) as a crude approximation describing angular momentum loss unaccompanied by mass loss, and we use the results of Iben and Tutukov (1984a) and Kraicheva et al., who applied this Equation to LMA.

## 3. THE ORIGIN OF ALGOLS

Let us now consider the range of initial masses and separations of components in systems that produce the observed Algols. By "observed Algols", we mean here the semi-detached systems with hot primaries and cool secondaries whose features allow us to estimate their photometric and spectroscopic orbits, and which are listed in the catalogues of absolute dimensions of close binaries like those of Giuricin et al. (1983) or Svechnikov (1969, 1986). To estimate the birthrates of Algols produced in different ways, we shall employ the formula based on results of the study of statistical properties of binary stars (Popova et al. 1982).

$$\partial^3 \nu \approx 0.1\, \partial \log a \; M_1^{-2.5}\, \partial M\, \partial q \;\; yr^{-1}, \qquad (2)$$

where a is the separation of components in units of $R_\odot$, $M_1$ is the mass of primary in units of $M_\odot$ and $q = M_2/M_1$.

Presumably, due to star-formation conditions, binaries with orbital semi-major axes $a/R_\odot \lesssim 6\,(M_1/M_\odot)^{1/3}$, do not form. Because of this, most stars with $1.5 \lesssim M_1/M_\odot \lesssim 4$ fill their Roche lobes only after the hydrogen exhaustion in their cores (see Fig. 1 where ZAMS and TAMS lines are drawn). However, components of CB with $M_1 \lesssim 1.5\ M_\odot$ may become closer than the above-mentioned limit as a result of magnetic braking, caused by the stellar wind from convective envelopes. This permits the

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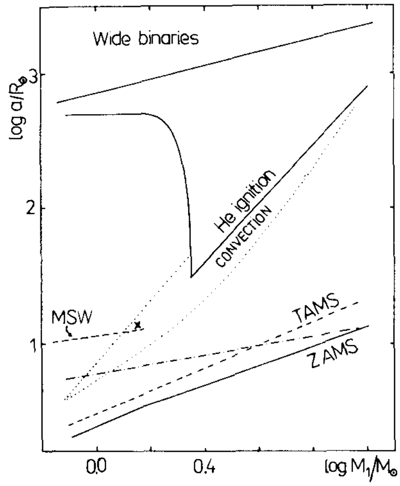


Fig. 1. Diagram of initial primary mass - initial semi-major axis of orbit for CB. Dash-dotted line is the lower limit of a new-born CB. MSW-line is the upper border of the region where MSW brings components into contact in their main-sequence life time. Dotted lines are the upper border of region populated by observed LMA and the border of deep convective envelopes of primaries. Possible position of the predecessor of L870-2 is marked by the cross.

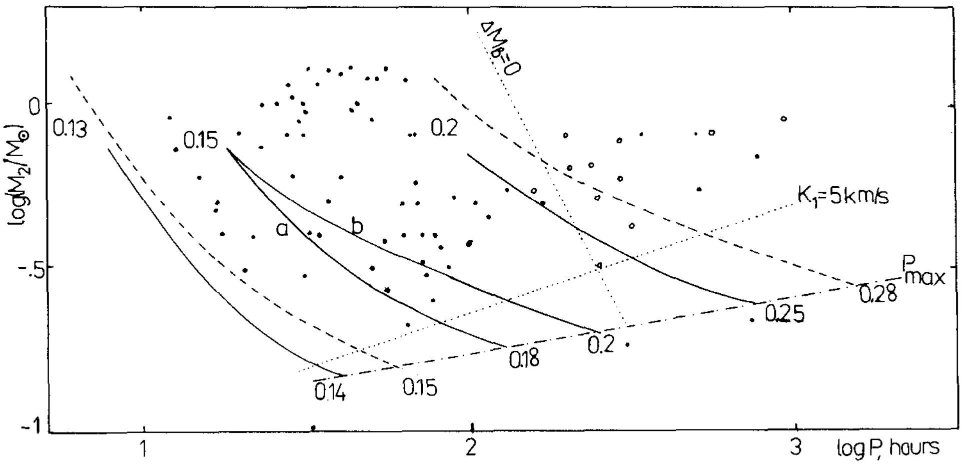


Fig. 2. The period - mass diagram for secondaries of systems with total mass 1.8 $M_⊙$ (continuous line) and 3 $M_⊙$ (dashed line) evolving with AML as given by Eq.(1). Numbers at the beginning and the end of tracks are initial and final values of $M_{He}$ (in $M_⊙$). Track b for initial $M_{He}$ = 0.15 $M_⊙$ is computed under conservative assumptions. Dotted lines are limits set by selection effects as discussed in the text. Points and circles are, respectively, positions of observed LMA and systems with undersize subgiants.

region $a \lesssim 6\ R_\odot$, $M_1 \lesssim 1.5\ M_\odot$ in the initial $M_1$ - a space to be populated (Fig. 1). Popova et al. (1982) estimated that this region is populated predominantly by W UMa stars and that their space density is about 60 times lower than for wider systems. Also Bopp and Fekel (1984) and Young et al. (1987) have found that binaries with M-dwarf components would have periods less than about 5 days (i.e. 7 - 10 $R_\odot$) only if dwarfs had active chromospheres and hence, possibly, magnetic stellar winds.

If $M_1 \approx M_2 = M \lesssim 1.5\ M_\odot$, one gets from Eq.(1) for the time scale of AML: $\tau_{SW} \approx 1.65 \times 10^5\ M_1\ a^5\ R_2^{-4}\ (M + M_2)^{-2}\ \lambda^2$ yrs (a,R,M - in solar units). Assuming that the lifetime of low-mass stars is $10^{10}\ (M/M_\odot)^{-3}$ yrs and the mass-radius relation for them is $R/R_\odot \approx 0.92\ (M/M_\odot)^{0.9}$ one finds that magnetic braking brings binaries with $a/R_\odot \lesssim 11.2\ (M_1/M_\odot)^{0.32}$ for $M \approx M_\odot$ (MSW-line in Fig. 1) into contact. Even wider systems may come into contact through the evolutionary expansion of their envelopes. Eq. (2) combined with the limits given by the ZAMS- and MSW-lines in Fig. 1 gives the birthrate of systems brought into contact by AML during the life-time of their primaries as $\sim 7.10^{-3}$ per year. These systems become either W UMa stars or Algols with small (0.13 - 0.15 $M_\odot$) cores. Svechnikov and Taidakova (1983) estimate, after taking into account the geometrical and photometrical probabilities of discovery, that the space densities of W UMa stars and sd-systems in the vicinity of the Sun are $1.6\text{x}10^{-4}$ and $1.2\text{x}10^{-4}$ pc$^{-3}$, respectively. (We sum up sd-stars and so-called systems with "undersize" subgiants because the latter are most probably also demi-detached (Hall and Neff, 1979; Budding, 1985)). The average total mass of stars in the sd-group is $\sim 2.5\ M_\odot$; thus most sd-systems belong to the low-mass group of Algols. Taking the radius of the Galaxy $\sim$ 10 kpc and its scale-height $\sim$ 200 pc one obtains $1.10^7$ and $0.75\text{x}10^7$ W UMa stars and Algols, respectively, in the Galaxy. As we cannot distinguish which initial separations of stars produce one or another type of systems, we estimate that they all live about $2.5\ 10^9$ yrs. Thus W UMa stars evolve on the nuclear time scale. For LMA, this estimate is several times higher than that from the evolutionary calculations which assume AML by magnetic braking. Although all estimates are so crude that the agreement may be considered satisfactory, extrapolation of Skumanich's law to Algols may overestimate the efficiency of AML.

The upper limit of the region producing LMA for $M_1 > 1.5\ M_\odot$ may be formally determined by the condition of stable mass-exchange

$$\frac{d\ \ln R}{d\ \ln M} > \frac{d\ \ln R_L}{d\ \ln M},$$

where $R_L$ is the radius of equivalent Roche lobe. If this condition is violated, mass loss by the donor proceeds on the dynamical time scale, and the removal of mass causes more mass to overflow the Roche lobe. Mass-exchange also occurs on the dynamical time scale, for stars with deep convective envelopes if $q > 0.6$ or, for stars with radiative envelopes, if $q > 2.1$ (Paczynski and Sienkiewicz, 1972; Tutukov et al., 1982; Hjellming and Webbink, 1987). In agreement with expectations, all observed LMA have $q \lesssim 0.6$. Such rapid mass exchange results in direct transformation of a star into a helium white-dwarf without an intervening Algol stage. Taking for $R_L$ the expression suggested by Iben and Tutukov (1984b): $R_L \approx 0.52\, a\, (M_1/(M_1 + M_2))^{0.44}$ one gets the condition of stable mass exchange

$$q < \frac{1}{2}\frac{d \ln R}{d \ln M} + 0.78$$

Employing the values of

$$\frac{d \ln R}{d \ln M}$$

for adiabatic mass-loss from stars with $M_1 \lesssim 3\, M_\odot$ given by Hjellming (1988), one gets for $q \approx 1$ the upper border of the pre-Algol region which is drawn in Fig. 1. It practically coincides with the line at which the depths of convective envelopes reach $\sim 50$ per cent of the stellar radii. This allows us to extend the borderline to masses higher than $\sim 3\, M_\odot$. For stars with $M_1 \lesssim 1.5\, M_\odot$ this limit is lower than the limit of stellar-wind influence. However, there is one more way to estimate the borders of the region producing LMA. The radii and luminosities of stars with degenerate helium cores are uniquely determined by the mass of the core $M_{He}$. Iben and Tutukov (1984b) give the following relation-ships (in solar units):

$$R = 10^{3.5}\, (M_{He}/M_\odot)^4, \quad L = 10^{5.6}\, (M_{He}/M_\odot)^{6.5} \qquad (3)$$

As the estimates of initial and present $M_{He}$ of observed secondaries of LMA by Iben and Tutukov (1984a) and Kraicheva et al. (1986) show, they are confined to 0.14 - 0.25 $M_\odot$ for stars with initial masses 1 - 2.5 $M_\odot$. This results in an upper limit for the pre-LMA region which is somewhat higher than that resulting from estimates based on radii derivatives (Fig. 1). There are at least two possible explanations of the existence of this limit. It is usually assumed that mass exchange on the dynamical time scale inevitably leads to the formation of a common envelope resulting in the loss of the whole donor envelope, or even in the merger of components. However, one may conjecture that for mass-exchange rates corresponding to still relatively shallow convective envelopes (about half of the stellar radius) evolution inside common envelope does not lead to the loss of whole envelope, and the donor emerges from it as a normal semi-detached subgiant. This gives some credence to results of the usual conservative computations.

Another explanation of the absence of LMA with massive helium cores is merely that it is a selection effect. By definition, the primary is the more luminous component, although some exceptions are known. Assuming that, after the common-envelope stage, the mass of primary increases to $\alpha$ M, where $1 \lesssim \alpha \lesssim 2$, one finds that a new-born sd-system is not recognizable as an Algol if

$$P > 10^{2.14}\, (\alpha M_1/M_\odot)^{3.7} / (M_2/M_\odot)^{0.5} \text{ hours.} \qquad (4)$$

This limit corresponds to $M_{He} \approx 0.14\, (M/M_\odot)^{0.62}\, \alpha^{0.62}$. As the luminosity of the primary does not change significantly in the mass-exchange phase,

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the same estimate of limiting $M_{He}$ is valid when $M_{He}$ is initially lower but increases in the course of evolution. Thus, the system still exists as a semi-detached one, but for us it disappears as an Algol. It becomes a Kantian "Ding an sich". The single giant or subgiant observed instead of Algol would have peculiarly high rotation velocity. In Fig. 2 we show the corresponding limit for $\alpha$ M = 2 $M_{\odot}$.

Thus, it remains unclear whether LMA with massive cores are simply unobserved due to selection effects or whether donors with large $M_{He}$ merge with accretors inside common envelopes (producing FK Com-type stars? Tutukov and Yungelson, 1987), or donors become white dwarfs directly.

The distribution of LMA over the difference of $M_{bol}$ shows that most of them have $M_{bol_1} - M_{bol_2} \approx -1^m$ to $-3^m$. For stars just leaving the main sequence, $M_{He} \approx 0.13\ M^{0.22}$ (Iben and Tutukov 1984a). Thus, the magnitude difference is given by $\Delta M_{bol} > -10 \log a - 6.41 \log M - 0.4$. For $\alpha \approx 1.5$, M = 1.5 $M_{\odot}$ this just gives $\Delta M_{bol} \approx 3^m\!.3$. Thus, crudely, LMA are confined to $-3^m \lesssim \Delta M_{bol} \lesssim 0$.

Above, we estimated the possible number of Algols produced by stars with $M_1 < 1.5\ M_{\odot}$. Let us now consider the contribution of more massive systems. The ancestors of systems with $1.5 \lesssim M_1 \lesssim 2.5\ M_{\odot}$ appear as LMA if the initial semi-major axes of their orbits are confined to $6\ (M/M_{\odot}) \lesssim a/R_{\odot} \lesssim 5\ (M/M_{\odot})^{2.2}$ (see Fig. 1). Calculations by Iben and Tutukov (1984a) and Kraicheva et al. (1986) indicate that after AML causes the RLOF, and the system passes through a short common envelope stage, evolution proceeds on the time-scale which is much longer than the time scale of AML, because mass exchange, which moves components apart, counterbalances the effect of AML. The life-time of the system as a semi-detached one is determined by the remaining main-sequence life time of the accretor. Within the uncertainty caused by the possible increase of the mass of accretor we may set $t_{exc} = 10^{10} / (M_1/M_{\odot})^{-3}$ yrs. With this $t_{exc}$ we get $\sim 4 \times 10^6$ Algols in the Galaxy with initial $1.5 \lesssim M_1 \lesssim 2.5\ M_{\odot}$. This number is quite comparable to the total number of Algols ($\sim 7.5 \times 10^6$). It is, therefore, possible that this mass interval produces most of LMA. The remainder are ancestors of systems with initial $M_1 \lesssim 1.5\ M_{\odot}$.

Systems with initial mass $2.5 \lesssim M_1 \lesssim 4\ M_{\odot}$ are observed as Algols if RLOF occurs prior to He ignition. The life-time of observed systems in this case very strongly depends on the time RLOF begins and on the mass-ratio of the components. It varies from $\sim 10\ t_{KH}$ for early Case B and $q \approx 1$ to $\sim 0.1 t_{KH}$ for very late Case B. Taking for this mass interval $R_{TAMS} \approx 10^{0.2}\ (M/M_{\odot})^{0.7}\ R_{\odot}$ and $L_{TAMS} \approx 10^{0.18}\ (M/M_{\odot})^4\ L_{\odot}$ we get $t_{KH} \approx 10^{7.1}\ (M/M_{\odot})^{-2.7}$ yrs. With $t_{exc} \approx 5\ t_{KH}$ this results in $1.2 \times 10^4$ Algols with initial mass $2.5 \lesssim M_1 \lesssim 4\ M_{\odot}$ in the Galaxy, or a space density of $\sim 2 \times 10^{-7}\ pc^{-3}$.

When the initial mass $M_1 > 4\ M_{\odot}$, systems evolving in the Case A of mass exchange begin to appear. Integrating the Eq. (2) with limits on a given by the line $6\ (M/M_{\odot})^{1/3}$ and TAMS-line (Fig. 1) and assuming the main-sequence life-time $10^{9.4}\ (M/M_{\odot})^{-2.2}$ (Iben and Tutukov, 1986a) one obtains $5 \times 10^4$ Algols evolving in Case A. As we had already mentioned, the mass-exchange time varies from $\sim 10\ t_{KH}$ in early Case B to $\sim 0.1\ t_{KH}$

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in very late Case B. Setting $t_{exc} = \alpha$ t one obtains the number of Case B Algols equal to $5 \times 10^2$ $\alpha$. This agrees with the well-known predominance of Case A Algols among observed massive systems $(M_1 + M_2) \gtrsim 7\ M_\odot$, (Giuricin and Mardirossian, 1981).

Let us now discuss the position of LMA in the period-secondary-mass diagram (Fig. 2). It is clearly limited by several lines. From the low-period side, the line of minimal helium cores $\sim 0.13\ M_\odot$ of stars with $M \approx 0.8\ M_\odot$ which exhaust hydrogen in the cores in the Hubble time, sets the limit. At the top, the area is limited by the values of $M_2$ corresponding to the end of rapid mass-exchange or to the common envelope stage, namely $\sim 0.6$ of the initial mass of the present secondary. From the side of large periods the border is set, as discussed above (Eq. 4), by the common-envelope formation line or by the line on which the luminosity of the donor begins to exceed that of the accretor. It is drawn in Fig. 2 for $\alpha M = 2\ M_\odot$. The lowest values of $M_{He}$ (or maximum attainable values of P) correspond to $M_{He} = M_2$. From Eq. (3) and Kepler's law it follows that $P_{MAX} \approx 10^{6.2} (M_2/M_\odot)^{5.3}$ hours. However, it is easily seen that we just do not observe LMA with that low $M_{He}$. The reason possibly is that, with the increase of P, the semi-amplitude of radial velocity of the primary $K_1 \approx 604\ M_2\ P^{-1/3}\ M_t^{-2/3}$ km $s^{-1}$ (masses are in $M_\odot$, P - in hours) declines below the detection limit, which is several km $s^{-1}$ for typical A-class primaries. Thus, although we still observe eclipses, we are not able to determine the spectroscopic orbit. In Fig. 2 this detection limit is drawn for K = 5 km $s^{-1}$, $M_1 = 2\ M_\odot$. In Fig. 2 we show also some of the parametrized evolutionary tracks for LMA computed by Kraicheva et al. (1986), assuming AML by magnetic braking.

## 4. EVOLUTION OF LOW-MASS ALGOLS

The incorporation of MSW into the computations of evolution of close binaries leads to some changes in the presently familiar picture. The time scale of the growth of He-core of the secondary is $t \approx 10^{5.36} (M_{He}/M_\odot)^{-5.6}$ yrs (Iben and Tutukov, 1984b). Comparing this with Eq. (1) one finds that AML influences the evolution of donors with $M_{He} \lesssim 0.25\ M^{0.1}\ \alpha^{0.2}\ M_\odot$ or periods $P \lesssim 360\ (M/M_\odot)^{0.1}\ \alpha^{-1.2}$ hours. The change of trend of evolutionary tracks due to MSW is illustrated in Figs. 2 and 3. The MSW shortens the evolutionary life-time by several times (however, the life-time still exceeds that of the accretors or is at least comparable to it). The MSW severely limits the growth of helium cores, thus decreasing the mass of final products. In the HR-diagram the familiar tracks rising to high luminosities are absent, the change of radii is comparatively small.

The tracks cover most parts of the region of the HR-diagram populated by observed (Svechnikov, 1986) secondaries of LMA (Fig. 3). However, there are some secondaries with lower effective temperatures than expected from the evolutionary tracks. It is possible that their $T_e$ are underestimated. E.g. the coolest secondaries from the compilation of Giuricin et al. (1983) are omitted as uncertain from the more critical Svechnikov (1986) catalogue. Besides, the estimates of $T_e$ by Svechnikov

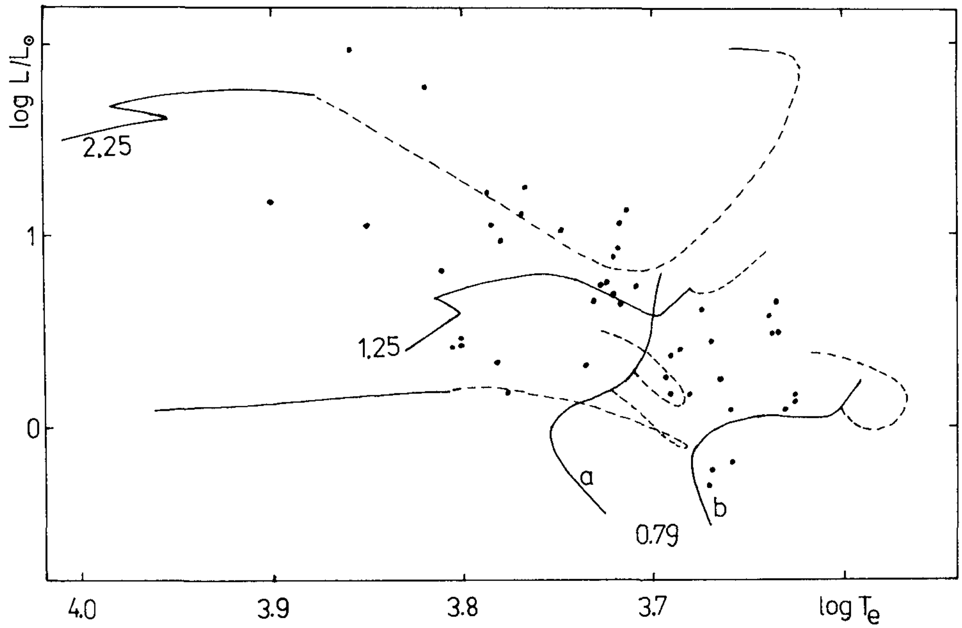


Fig. 3. Secondaries of LMA in the L - $T_e$ diagram. Evolutionary tracks for 1.25 $M_\odot$ (Iben and Tutukov, 1984) and 0.79 $M_\odot$ are computed assuming momentum loss by MSW. The evolutionary track for 2.25 $M_\odot$ (Giannone and Giannuzzi, 1972) is a "conservative" one. Continuous lines are portions of tracks with constant mass, dotted ones correspond to semi-detached stage. Track b for 0.79 $M_\odot$ is computed with reduced mixing length.

are up to 0.1 in log Te higher. On the other hand, the cold region of the HR-diagram can possibly be reached by decreasing the value of mixing-length in the convective envelopes of secondaries. This is indicated by evolutionary track computed by us for a star of 0.79 $M_\odot$ with $\alpha = \ell/Hp = 0.5$ (Fig. 3), as compared to tracks with "standard" value $\alpha = 1.5$.

## 5. THE FATE OF LOW-MASS ALGOLS

The semi-detached phase of evolution ends up with the exhaustion of the hydrogen envelope of the donor, which is turned into a white dwarf, or by evolutionary expansion of the accretor, leading probably to a new contact phase (this important branch of the evolutionary scenario is still uninvestigated). The RLOF by the former accretor inevitably leads to the formation of a common envelope from which the system emerges as a double degenerate He-dwarf. The transformation of the semi-major axis of the orbit in the common envelope stage may be described by a law suggested by Tutukov and Yungelson (1981):

$$\beta \frac{M_{He_1} M_{He_2}}{2 a_f} \approx \frac{M^2}{a} \quad (5)$$

where $M_{He_1}$, $M_{He_2}$ are the masses of older and younger dwarfs, a and $a_f$ are the initial and final values of semi-major axis, M is the mass of components (we assume $M_1 \approx M_2$), $\beta$ is the parameter of the order of unity describing the efficiency of energy expenditures. Combining (5) and (3) one gets for the mass ratio of dwarfs a remarkably rigorous expression (Tutukov and Yungelson, 1988a):

$$q \approx 0.88\, a^{1/67}\, M^{-4/67}\, \beta^{-1/67} \quad (6)$$

This value of q is very close to estimate of $q \approx 0.89$ in the close-binary white-dwarf system L870-2 (Saffer et al., 1988) which is most probably a post-LMA system. The uniqueness of the final value of q may help to discriminate post-LMA among other close-binary dwarfs, when more are discovered. For L870-2, it is possible to derive the present masses of its components and their separation, if the older white dwarf has mass 0.26 - 0.31 $M_\odot$ and the initial masses of components of the system were 1.4 - 1.7 $M_\odot$. The present mass of the older dwarf corresponds to $M_{He} \approx 0.2\ M_\odot$ at the moment of RLOF. The possible position of the system at the beginning of the Algol stage is shown in Fig. 1 by a cross. Thus, L870-2 is an ancestor of the widest of observed LMA.

The evolution of post-LMA is determined by the AML via gravitational-wave radiation. The merger time of a system of two dwarfs is

$$T_M \approx 1.5 \cdot 10^8\, a^4\, M_1^{-1}\, M_2^{-1}\, (M_1 + M_2)^{-1} \text{ yrs}$$

(a and M - in solar units). Following the evolutionary changes of masses and separations of components one may estimate that post-LMA close binary dwarfs merge if the mass of older white dwarf is less than $\sim 0.2\ M_\odot$ (Tutukov and Yungelson, 1988a). Thus, although L870-2 will not merge in the Hubble time, ancestors of most presently observed LMA will merge, producing, probably, single helium stars with $M \approx 0.3 - 0.6\ M_\odot$. This is one of the ways to form helium OB-subdwarfs (Tutukov and Yungelson, 1988b).

In connection with the final stages of evolution of Algols let us mention that remnants of LMA with masses 0.13 - 0.3 $M_\odot$ have luminosities from $\sim L_\odot$ to $\sim 100\ L_\odot$. In the horizontal parts of the tracks, after detachment from the Roche lobe, these remnants cross the region of the HR-diagram occupied by hot subdwarfs and may account for at least part of those objects. In the course of contraction there may occur several bursts of nuclear burning in the envelope of a helium dwarf (Iben and Tutukov, 1986b). During the outbursts, dwarfs with luminosities 100 $L_\odot$ - 2000 $L_\odot$ cross the Hertzsprung gap several times.

## 6. CONCLUSION

Let us now list some problems arising in Algol studies.

1. The phase of stable mass exchange corresponds to $q \lesssim 0.6$, while unevolved systems have $q \approx 1.2 - 1.5$. Observed LMA have $q \lesssim 0.6$. What are the observed counterparts of systems with $0.6 \lesssim q \lesssim 1.5$?

2. The calculations of evolution of LMA secondaries predict considerable change of their surface chemical composition: more than two orders of magnitude increase of $He^3$ and about the same decrease of $C^{12}$ content. There are some indications of decreased N/C ratio in several Algols (Parthasarathy et al., 1979). Further studies of the chemical composition of transferred matter may give an additional check on the evolutionary picture.

3. Last but not least, although application of Skumanich's law improves our ability to explain the properties of LMA, we must recall that it is still a formal extrapolation of the braking law for single main-sequence stars. A theory of stellar wind and empirical data on rotation of Algols are still in needed.

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## DISCUSSION

Plavec, congratulating Yungelson on his paper, asked for clarification of the term "undersize subgiant". Yungelson replied that he had in mind a number of systems, with periods greater than about ten days, in which the secondaries apparently do not quite fill their Roche lobes. He recognized, however, that many would regard these as regular Algols with badly determined parameters. Plavec also commented on two stars outside the "permitted" area in Yungelson's Figure 2, pointing out that the elements of these two systems (AL Gem and DN Ori) are poorly determined. Yungelson replied that the $P_{max}$ line in the Figure is not rigorously determined, being based on somewhat simplified calculations. He agreed that the positions of the two stars in the diagram are not well-known, but if the positions are roughly correct, the systems are rare examples of ones observed just before the secondary is detached from the Roche lobe.

Rucinski pointed out that Skumanich's law may depend differently (i.e. less steeply) on the period for rapidly rotating stars, such as are found in Algols, and asked how this might affect the calculations. Yungelson accepted that the exponent in the law is uncertain but pointed out that the application of the law, in the form he had adopted, to cataclysmic variables, had proved successful. A less steep dependence on the period would lead to larger minimum masses of the white dwarfs and a longer time-scale of evolution. Possibly the end of evolution might be exhaustion of the donor's envelope rather than overfilling of the primary's Roche lobe.

Eggleton asked if it is possible to measure spectroscopically the mass-ratios of binaries such as FF Aqr and AY Cet, that contain a red giant and a white dwarf, so that the masses could be checked against Yungelson's computations. Guinan replied that this had been done for FF Aqr with IUE, by Etzel and collaborators, and a mass of about 0.6 $m_{\odot}$ was found for the OB subdwarf (the result is not yet published). Hall pointed out that 29 Dra and HD 185510 were also suitable systems for this purpose.

In response to a query from Walker about astrometric observations of L870-2, Yungelson intimated that he had based his arguments on results in a preprint of a paper by R.A. Saffer *et al.* (now published, *Astrophys. J.*, **334**, 947, 1988). Livio commented that, although this pair is extremely interesting, the masses might be higher than given in the preprint. Yungelson replied that even if the masses of L870-2 were too large for helium dwarfs, the discovery of the system was exciting, since the theory of stellar evolution had predicted the existence of such close binary stars.

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