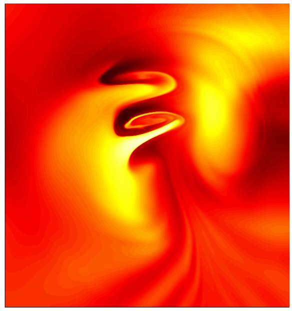
Focus on Fluids journals.cambridge.org/focus

The vortex instability pathway in stratified turbulence



Michael L. Waite†

Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada

The parameter regime of strong stable density stratification and weak rotation is an important one in geophysical fluid dynamics. These conditions exist at intermediate length scales in the atmosphere and ocean (mesoscale and sub-mesoscale, respectively), and turbulence here links large-scale quasi-geostrophic motions with small-scale dissipation. While major advances in the theory of stratified turbulence have been made over the last few decades, many open questions remain, particularly about the nature of the energy cascade. Recent numerical experiments and analysis by Augier, Chomaz & Billant (*J. Fluid Mech.*, vol. 713, 2012, pp. 86–108) present a remarkably vivid illustration of the nonlinear interactions that drive such turbulence. They consider a columnar vortex dipole, which naturally three-dimensionalizes under the influence of strong stratification. Kelvin–Helmholtz instabilities subsequently transfer energy directly to small scales, where the flow transitions into three-dimensional turbulence. This direct link between large and small scales is quite distinct from the usual picture of a turbulent cascade, in which nonlinear interactions are local in scale. But how important is this mechanism in the atmosphere and ocean?

Key words: stratified turbulence, transition to turbulence, vortex breakdown

1. Introduction

Stratified turbulence has a layerwise structure that has inspired a variety of colourful culinary descriptions over the years, such as pancake and blini turbulence (Bretherton 1969). This distinctive layering has long been identified in atmosphere and ocean observations, laboratory experiments and numerical simulations (for a review, see Riley & Lelong 2000), but its origin and dynamics continue to be areas of active research. In a landmark paper, Lilly (1983) proposed a scaling of the Boussinesq equations to investigate the pancake structure of stratified turbulence. In the limit of strong stratification, he showed that the dynamics reduce to decoupled horizontal layers of

† Email address for correspondence: mwaite@uwaterloo.ca

doi:10.1017/jfm.2012.552

two-dimensional turbulence. Decorrelation of the velocity at adjacent levels would lead to independent layers of turbulent motion, and hence pancakes. Lilly (1983) conjectured that such turbulence could account for the atmospheric kinetic energy spectrum via an inverse cascade from (for example) thunderstorm to synoptic scales, a compelling but controversial idea that motivated much of the work on stratified turbulence that followed.

Stratified turbulence theory has advanced significantly over the past 30 years, driven in large part by insights from numerical simulation. For example, the early computations by Herring & Métais (1989) were inconsistent with an inverse cascade, and gradually motivated a major rethink of Lilly's (1983) hypothesis. It is now recognized that energy is transferred from large to small horizontal scales in stratified turbulence, and a cascade phenomenology has been developed to account for the resulting energy spectra (Lindborg 2006). Numerical experiments have shown that the pancake structure breaks down at small scales, since shearing between adjacent layers may lead to Kelvin–Helmholtz (KH) instabilities and a subsequent transition to small-scale turbulence (e.g. Laval, McWilliams & Dubrulle 2003). The layer thickness appears to be set by the buoyancy scale $L_b \equiv U/N$, where U is the characteristic velocity scale and N is the Brunt–Väisälä frequency (Waite & Bartello 2004).

Lilly's (1983) hypothesis for the genesis of layers in stratified turbulence was shaken up by Billant & Chomaz (2000), who showed experimentally and theoretically that layering can emerge through the zigzag instability rather than chaotic decorrelation of layerwise two-dimensional flows. The zigzag instability affects columnar vortices in stratified fluids, and has a dominant vertical wavelength also of L_b. While the laboratory experiments of Billant & Chomaz (2000) remained laminar, numerical simulations have illustrated secondary KH and gravitational instabilities that may transition to turbulence (Deloncle, Billant & Chomaz 2008; Waite & Smolarkiewicz 2008; Augier & Billant 2011). As a result, the evolution of an isolated columnar vortex dipole, which is the subject of the paper by Augier, Chomaz & Billant (2012), exhibits many of the principal characteristics of fully developed stratified turbulence: layering, vertical scales of L_b, and breakdown into small-scale turbulence. Such flows present an idealized framework for studying the nonlinear interactions of stratified turbulence, in which the corresponding physical mechanisms may be readily identified.

2. Overview

Augier *et al.* (2012) employ high-resolution numerical simulations to investigate the evolution of a columnar vortex dipole with strong stratification and weak viscosity. Reynolds numbers are larger than those attainable in the laboratory, but are naturally orders of magnitude smaller than those in the atmosphere and ocean. The dipole is perturbed to excite the zigzag instability, which eventually undergoes secondary instabilities and, ultimately, a breakdown into turbulence. The analysis of this transition is mainly spectral, and a particular emphasis is placed on the poloidal–toroidal decomposition of the velocity. This decomposition separates the velocity into horizontally rotational (toroidal) and divergent (poloidal) components, which, in the linear regime, correspond to vortical modes and gravity waves, respectively. Since the dipole is entirely toroidal, the poloidal kinetic energy is a useful quantity that readily identifies perturbations and secondary instabilities. It is this careful and detailed spectral analysis that separates the work of Augier *et al.* (2012) from previous studies on this problem, and allows them to untangle the steps of the transition to turbulence.

https://doi.org/10.1017/jfm.2012.552 Published online by Cambridge University Press

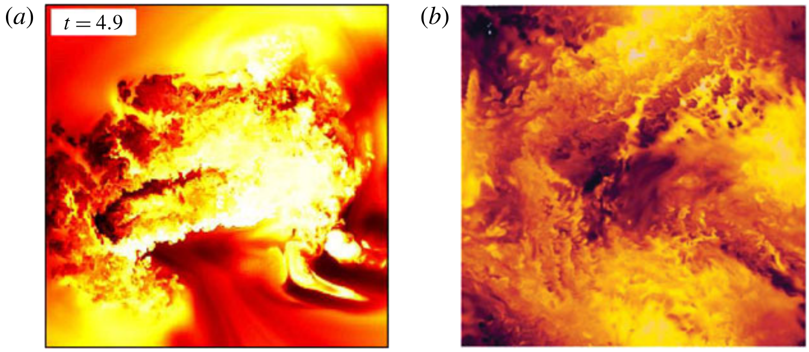


FIGURE 1. (*a*) Horizontal slice of density after transition of the dipole to small-scale turbulence. (*b*) Horizontal slice of density in an analogous simulation of fully developed stratified turbulence. (Panel (*a*) is adapted from Augier *et al.* (2012); panel (*b*) is adapted from Brethouwer *et al.* (2007).)

The breakdown into turbulence is shown to occur in three stages. First, there is exponential growth of perturbations associated with the zigzag instability, which acts to shear the dipole into layers. This growth occurs at large horizontal scales (i.e. the vortex scale) and vertical scales around L_b, as expected. As the dipole begins to shear, regions of reduced Richardson number develop, which are susceptible to KH instabilities (Deloncle *et al.* 2008). These secondary instabilities appear after a few turnover times, driving a sudden transfer of kinetic energy from the vortex scale down to the buoyancy scale. The physical space fields at this time exhibit the distinctive structure of KH billows (see the figure alongside this paper's title, which shows a horizontal slice of density).

The final step in this transition is the destabilization of the KH billows into small-scale turbulence. This breakdown is identified by a broadening of the kinetic energy spectrum at sub-buoyancy scales. The density field at this time shows a large patch of turbulence embedded inside the dipole (figure 1*a*), which is reminiscent of simulations of randomly forced stratified turbulence (e.g. figure 1*b*; from Brethouwer *et al.* (2007)). This similarity is remarkable, considering the very different large-scale flows in these two studies: an isolated vortex dipole and forced strongly stratified turbulence. In fact, the one-dimensional energy spectra at this stage are in surprisingly good agreement with theoretical predictions for stratified turbulence (Lindborg 2006). In the horizontal there is a $-5/3$ power law, while in the vertical the spectrum shallows from -3 to $-5/3$ below the Ozmidov scale (figure 2 in Augier *et al.* 2012). There is a significant deficit of kinetic energy at horizontal scales between the vortex and buoyancy scales, which is probably due to the idealized nature of the large-scale flow; with fully developed stratified turbulence at large scales, the Lindborg (2006) cascade would presumably fill this gap.

3. Future

The buoyancy scale is a relatively tiny scale in geophysical fluid dynamics ($O(1)$ km in the atmosphere, smaller in the ocean). As a result, the instabilities and transitions described here would often go unresolved in numerical simulations. Transfers of energy into and below the buoyancy scale may need to be parametrized in practice, and the work of Augier *et al.* (2012) represents a major contribution to the physical understanding that is necessary for designing a parametrization. Of course, these simulations are quite idealized, so more work remains to be done. An important next step would be the consideration of vortices with finite Rossby number that are more representative of the atmospheric mesoscale and oceanic sub-mesoscale. Do similar transitions to turbulence occur for such vortices? Given the apparently wide generality of the zigzag instability, it seems reasonable to guess that the results of this paper will be quite generic.

This work also raises fundamental questions about the nature of the energy cascade in stratified turbulence. The primary driver of the transition to turbulence is KH instability of the large-scale vortex, which transfers energy directly from the vortex scale into the buoyancy scale. This mechanism is inherently non-local in scale, since the separation between vortex and buoyancy scales is large for strongly stratified flows. Similar non-local interactions and KH instabilities have also been observed in simulations of fully developed stratified turbulence (e.g. Waite 2011). But the stratified turbulence cascade phenomenology of Lindborg (2006) is based on the hypothesis of local interactions between vortices larger than L_b. It is likely that both processes are active in stratified turbulence: a local cascade as well as a non-local KH link to the buoyancy scale. Augier *et al.* (2012) make a compelling case for the importance of the KH mechanism in vortex dipoles – and, by extension, stratified turbulence – but its significance compared to the Lindborg (2006) cascade is a key question that remains to be answered.

References

AUGIER, P. & BILLANT, P. 2011 Onset of secondary instabilities on the zigzag instability in stratified fluids. *J. Fluid Mech.* **682**, 120–131.

AUGIER, P., CHOMAZ, J.-M. & BILLANT, P. 2012 Spectral analysis of the transition to turbulence from a dipole in stratified fluids. *J. Fluid Mech.* **713**, 86–108.

BILLANT, P. & CHOMAZ, J.-M. 2000 Experimental evidence for a new instability of a vertical columnar vortex pair in a strongly stratified fluid. *J. Fluid Mech.* **418**, 167–188.

BRETHERTON, F. P. 1969 Waves and turbulence in stably stratified fluids. *Radio Sci.* **4**, 1279–1287.

BRETHOUWER, G., BILLANT, P., LINDBORG, E. & CHOMAZ, J.-M. 2007 Scaling analysis and simulation of strongly stratified turbulent flows. *J. Fluid Mech.* **585**, 343–368.

DELONCLE, A., BILLANT, P. & CHOMAZ, J.-M. 2008 Nonlinear evolution of the zigzag instability in stratified fluids: a shortcut on the route to dissipation. *J. Fluid Mech.* **599**, 229–239.

HERRING, J. R. & MÉTAIS, O. 1989 Numerical experiments in forced stably stratified turbulence. *J. Fluid Mech.* **202**, 97–115.

LAVAL, J.-P., MCWILLIAMS, J. C. & DUBRULLE, B. 2003 Forced stratified turbulence: successive transitions with Reynolds number. *Phys. Rev.* E **68**, 036308.

LILLY, D. K. 1983 Stratified turbulence and the mesoscale variability of the atmosphere. *J. Atmos. Sci.* **40**, 749–761.

LINDBORG, E. 2006 The energy cascade in a strongly stratified fluid. *J. Fluid Mech.* **550**, 207–242.

RILEY, J. J. & LELONG, M.-P. 2000 Fluid motions in the presence of strong stable stratification. *Annu. Rev. Fluid Mech.* **32**, 613–657.

WAITE, M. L. 2011 Stratified turbulence at the buoyancy scale. *Phys. Fluids* **23**, 066602.

WAITE, M. L. & BARTELLO, P. 2004 Stratified turbulence dominated by vortical motion. *J. Fluid Mech.* **517**, 281–308.

WAITE, M. L. & SMOLARKIEWICZ, P. K. 2008 Instability and breakdown of a vertical vortex pair in a strongly stratified fluid. *J. Fluid Mech.* **606**, 239–273.