INTERPRETATION OF THE "THIRD HARMONIC" OF THE SOLAR MAGNETIC CYCLE

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ABSTRACT. The peak at $\sim 3/22\ y^{-1}$ seen for all odd-degree axisymmetric modes in the SHF analysis of 'nominal magnetic field' seems to be an artifact of some non-linearity between the field magnitude and the sunspot occurence probability.

1. Introduction

We have an on-going project of analysing in various ways the distribution of the sunspot occurence probability (and a 'nominal' magnetic field defined therefrom), on the sun's surface and in time, using Greenwich data for sunspot groups. We have shown earlier that the spherical harmonic Fourier (SHF) analysis of these quantities yields approximately constant amplitudes and constant phases for their odd-degree axisymmetric SHF components during 1874-1976 (over which the data is available on a magnetic tape provided by H.Balthasar). Detailed results of the SHF analysis are discussed in earlier papers (Gokhale, Javaraiah and Hiremath, 1989, Gokhale and Javaraiah 1990).

In this paper we investigate the behaviour and significance of a third harmonic peak at $\sim 3/22\ y^{-1}$ which is quite significantly present in the spectra of all the odd degree axisymmetric modes with respect to the temporal frequency. These peaks are unlikely to be caused by the asymmetry in the 11 year cycle of the data size, since the magnitude of the nominal field is normalised with respect to the data size in each interval of analysis. Similar periodicity is seen in the SHF analysis of magnetogram data (Stenflo and Vogel 1986) and the hydrogen-alpha spectroheliogram data 1917-1985(Gokhale, et al 1990). It is also seen in the Fourier analysis of solar rotation (Singh and Prabhu, 1985). It is therefore important to see if these 'third harmonic' peaks represent independent periodicity in the nominal magnetic field. For this purpose we have determined the correlation between the amplitudes and phases of the third harmonic components with those of the corresponding first harmonic components. The presence of high corelation between the amplitudes (and a stronger correlation between the phases) of the first and the third harmonics show that the third harmonic peaks do not represent any independent periodicity. It seems they result from some residual nonlinearity between the nominal field and the data size.

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2. Data and Method of Analysis

The data used consists of the heliographic latitudes b_i, longitudes Φ_i and times of observation t_i' of all the sunspot groups observed during 1874-1976 which are included in the ledgers I and II of the Greenwhich Photoheliographic Results. The t_i' are converted to values t_i of the time measured from the zero hour of 1 January 1874. The sunspot occurence probability $p(\Theta, \Phi, t)$ during any sufficiently long time interval (T_1, T_2) is defined as

$$p(\Theta, \Phi, t) = \frac{1}{N(T_1, T_2)} \delta(\mu - \mu_i, \Phi - \Phi_i, \tau - \tau_i)$$

in the neighbourhood of the points (μ_i, Φ_i, τ_i) in the (μ, Φ, t) space, and

$$p(\Theta, \Phi, t) = 0 \qquad \text{elsewhere}$$

where $\mu = \cos\Theta$, $\tau = (t-T_i)/(T_2-T_1)$, δ represents a delta function, $N(T_1,T_2)$ is the number of data points given by sunspot groups during in the time interval (T_1,T_2). It may be noted that this way the data of each sunspot group gets weighted by the number of days it is observed. The 'nominal magnetic field' 'B' (Θ, Φ, t) is defined as

$$'B'(\Theta, \Phi, t) = \pm\, p(\Theta, \Phi, t)$$

where the signs $\pm$ are chosen according to Hale's laws of magnetic polarities, taking care to separate the data from the old and the new cycles during the overlaps of successive cycles.

3. Presence of the "Third Harmonics" with constant amplitudes and phases

In an earlier paper (Gokhale and Javaraiah, 1990) we have plotted the SHF amplitudes of 'B' (Θ, Φ, t) during the interval 1902-1954, as functions of temporal frequency ν, for axisymmetric (m=0) modes of odd degrees of $\ell = 23$. All these plots show a very high 'primary' peak at $\nu = 1/22y^{-1}$ as expected. But all of them also show a smaller "third harmonic" peak at $\sim 3/22\ y^{-1}$, which, in the case of modes $\ell \geq 9$, is comparable to the primary peak. For $\ell = 13$, 15 it is as high as the primary peak. Similar concentration of SHF power at the third harmonic is also seen in the results obtained by Stenflo and Vogel (1986) from the magnetogram data and by Gokhale et al (1990) from the hydrogen-alpha spectroheliogram data 1915-1985. The '7 year' periodicity is present even in the analysis of solar rotation using CaII K spectroheliograms (Singh and Prabhu 1985). We find that for $\ell \geq 13$ the amplitudes and the phases of the third harmonic mode are approximately constant during all the 82 intervals of 22 y length, from 1874-1895 to 1955-1976, each of which is shifted with respective to the previous intervals by one year (Fig.1).

4. Interpretation

It may be thought that the 'third harmonic' peaks result from the asymmetry of the 11 year half cycles of the data size. However this is not

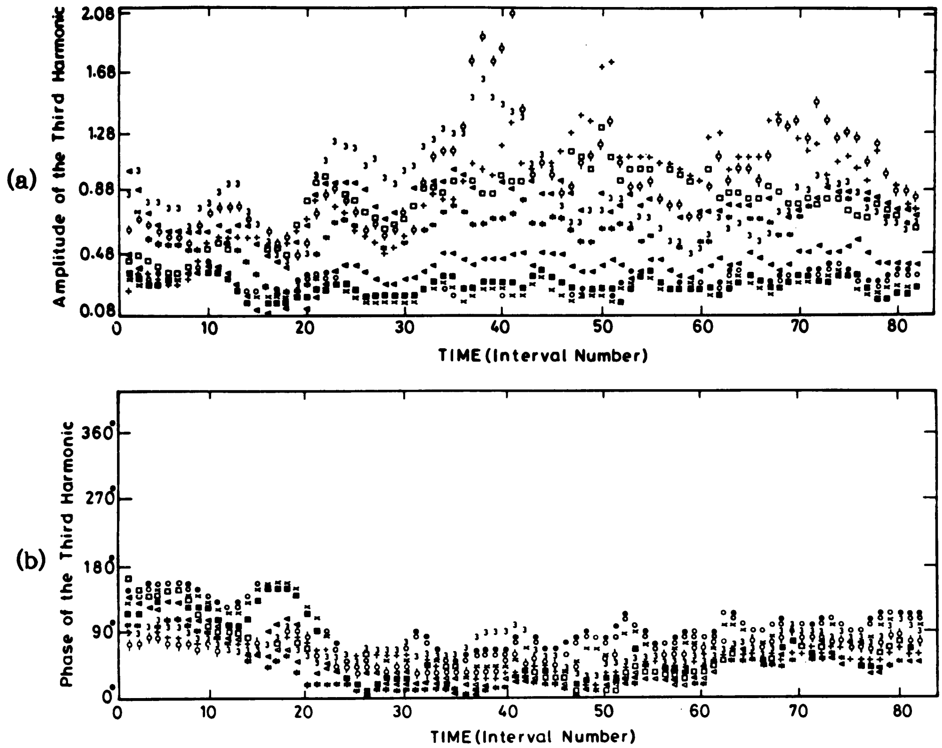

Figure 1 (a) Amplitude ratio A3/A1 and (b) the phase difference δ 3- δ 1 as functions of time represented by the interval number. Symbols: ℓ =1: filled circle, ℓ=3: open circle,.......... ℓ =21: letter 'phi'.

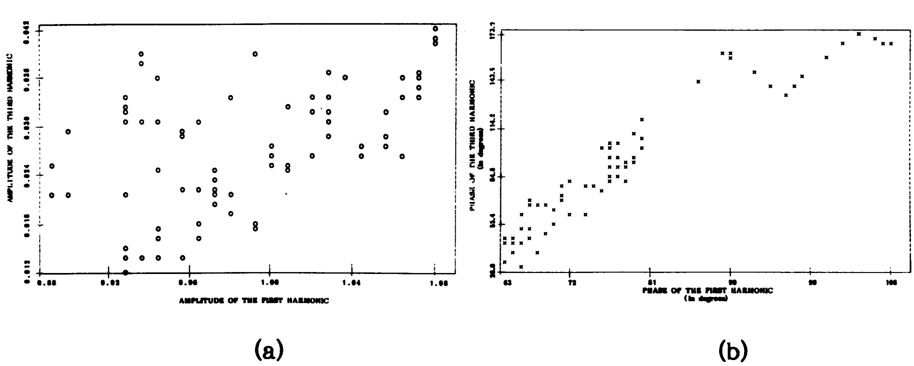

Figure. 2 (a) Amplitude A3 of the third harmonic versus amplitude A1 of the first harmonic, and (b) phase δ3 of the third harmonic versus phase δ1 of the first harmonic, for the mode ℓ =1, m=0.

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true, since the nominal field 'B' is normalised to the size of the data as shown by the factor $1/N\ (T_1, T_2)$ in equation (1). In order to see if the third harmonic is an independent SHF mode in the time dependence of 'B' we plotted for each odd degree ℓ, the amplitudes A3 (and the 'initial' phases $\delta 3$) of the third harmonic mode against the amplitude A1 (initial phase $\delta 1$) of the first harmonic mode. We find, that for each odd degree upto $\ell = 13$ (upto which we have done our computations), there is a strong correlation between the amplitudes (and the initial phases) of the third and the first harmonics. In figures 2(a) and 2(b) we illustrate the correlations A3 versus A1 and $\delta 3$ versus $\delta 1$ for the mode (ℓ=1, m=0).

For all the odd modes upto ℓ= 13, the correlation coefficients are ~ 0.49 between A1 and A3 and ~0.85 to 0.95 between $\delta 1$ and $\delta 3$.

This shows that the third harmonic peaks in the SHF power are unlikely to represent an independent set of modes. They result most probably from some non-linearity between the magnitude of the analyzed function 'B' (Θ, Φ, t) and the true sunspot occurence probability.

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DISCUSSION

DALSGAARD: The presence of the third harmonic may simply reflect the non-sinusoidal shape of the sunspot index as function of time. This is supported by the constancy of the amplitude ratios and phase difference. The situation is equivalent to Fourier analysis of Cepheid light curves.

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GOKHALE: Certainly one does expect a third harmonic due to the non-sinusoidal nature of the sunspot cycle, or due to a non-linear dependence of the 'data' variable (P) on a basic variable such as the magnetic field (B). That is why we have tested the correlation of the amplitudes and phases of the third harmonic with respect to those of the first harmonic. However, we have *normalized* the sunspot probability (p) to the *total* number of data points in each 22-year sample, thereby filtering out the time-dependence of the data size (if p depends linearly on B). Since we still see the third harmonic, with different amplitudes for different odd ℓ (≤ 9), but the amplitude and phase for each ℓ correlates with those of the first harmonic for the same ℓ, we conclude that there must be some non-linearity between p and B. As for the independence of the third and first harmonics for $\ell > 9$, further analysis will be needed.