Paraconsistent Logic: the View from the Right

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Paraconsistent logic[1] is one of those areas of philosophical logic that has yet to find much in the way of acceptance from a wider audience. By this I mean even a wider audience of philosophical logicians, let alone others. The area remains, in a word, disreputable. For some this is a positive attraction but, for the most part, it has kept away those in need of the services of a paraconsistent account of reasoning, in droves. In this brief essay, I try to reassure potential consumers that it is not in fact necessary to become a radical in order to use paraconsistent logic.

1. Motivation and a Drawing of the Lines of Battle

It is not hard to become disenchanted with the classical account of inconsistency. Anybody who teaches introductory logic knows that, and so do those who have grappled with the possibility of inconsistent sets of obligations or beliefs. But rather than work from these examples, let us reconsider a position in philosophical logic that dates from the beginning of the century and which, for a time, was quite influential (though controversial). Starting in 1918 or so, C.I. Lewis launched an attack on the *Principia* account of 'implication'. In a nutshell, Lewis held that the '⊃' of Russell and Whitehead was too frail a reed to support Russell's reading of it as 'implies'. There were several examples of 'bad' theorems that were intended to bolster this view: the most prominent were the so-called *paradoxes* of material implication.

Everybody has their favorite[2], but the one that Lewis highlighted most often was *ex falso quodlibet*: a false proposition implies any proposition whatever. This was his reading of the classical tautology: $\sim A \supset (A \supset B)$. Lewis thought that any account of 'implies' which had that as a law was wrong. His reasoning was that the intuitive semantics of implication requires it to be the converse of deducibility; i.e., that "A implies B" is true if and only if B is deducible from A. Further, *ex falso quodlibet* is not true of deducibility. In response to these intuitions, Lewis introduced a new connective, strict implication, which does support the deducibility semantics (at least as far as he was concerned). Having done that, he discovered that in his preferred system[3] there remained a modal version of the offending principle according to which a *necessarily* false proposition (strictly) implies any proposition. This discovery did not signal a return to the drawing board[4] however, since Lewis was able to demonstrate

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(to his own satisfaction and that of many others besides) that this modal version is supported by the deducibility semantics.

Lewis' status as a pioneer of modal logic is not in jeopardy, but the connection between his motivation and his logic has been viewed with a jaundiced eye, even by his sympathizers.[5] This is because logicians of his 'school'[6], the 'laws of thought' school, had no very clear grip on the idea of a rule of inference—which notion they mostly took to be unformalizeable[7]. In other words they didn't grasp the significance of "Frege's sign of assertion"[8] viz. ⊢. One way to attack the Lewis program is to observe that implication is a relation, not an operator.[9] Having said that, we can next observe that the relation which holds between A and B exactly when B is deducible from A is just ⊢, so we don't need any funny operators.

At this point the enemies of modal logic are satisfied that we have shown the utter futility and wrong-headedness of the enterprise and want to put this whole sordid chapter in the history of logic behind them. Those of us with less interest in the religious issues however, might want to pause for a moment to see if anything remains of the Lewis motivation once the real nature of the program has been made clear. This becomes the question: Is the classical ⊢, afflicted with *ex falso quodlibet*?

The answer to this question is clearly yes. Consider the classical inference $\{A, \sim A\} \vdash B$. Here we may infer an arbitrary conclusion when all we know is that one of the premises is false. But surely this is wrong, says Lewis. The contrast is with $\{A \wedge \sim A\} \vdash B$. Here we know we have a false premise (in fact, a necessarily false premise), but we also know which one it is, so this inference is correct because it is, in effect, a way of telling us to get rid of the bugger. Put this way, there really does seem to be something about which to complain. For we can all think of contexts in which we don't want to abandon the distinction between what follows and what doesn't, just because we are stuck with bad data from which to reason and we cannot point the finger of blame at one particular datum (or set of data). If we *could* do that, then of course we would get rid of it (or them as the case may be). Alternatively, we see an intuitive difference between these two inferences, a difference which the classical account of deduction fails to respect.

This is one way to motivate a paraconsistent account of deduction, and although it has been carefully chosen, let us not forget that it was there to choose. We are about to propose a way of fixing the classical account[10] but we shall carry out this program under the following *conservative principle*:

> Preserve as much as possible of classical logic. In particular, when the tacit assumption of the classical theory of inference holds, viz. that the premise set is consistent, preserve *all* of classical logic.

If we needed something to put on a T-shirt, perhaps "Fix only what's broken" would do. The idea behind this stricture is clear: Many of those who stand in need of our product are practitioners of classical logic. For all (or at least most) of these, the idea of "giving up" classical methods will not be attractive. So what we must do is provide a theory which works like the classical theory , when the classical theory works, and keeps on working when the classical theory gives up. More precisely: our theory should enable a clear distinction to be made between the two cases of inconsistency which we earlier flagged.

The radicals have a different slant on this issue. That classical logic has to go is a given, since it stands exposed as an instrument of exploitation and incorrect thinking. Many radical logicians come to the paraconsistent enterprise with some already prepared non-classical framework within which they prefer to work. Often enough to be mildly ironic, their prior program is an 'updated' version of the original Lewis one, viz. an alternative to the classical account of 'implication'.

To the radicals, our conservative principle is so much classical claptrap and our informing distinction a deluded half-measure. Why settle for making such a feeble distinction and leave the real problem untouched? This problem is the absurd classical account of contradiction. Classically inconsistent sets explode only because bourgeois classical semantics holds, in the face of overwhelming evidence to the contrary, that both A and ~A cannot simultaneously be true! Once we free ourselves from this false classical doctrine, we shall be able to reason from inconsistent sets of premises, in just the same way that we reason from consistent ones. In this brave new world, we will not be able to deduce an arbitrary sentence from {A, ~A} because there will be ways of making both premises true and the conclusion (for at least some conclusions) false. The same, it goes without saying, will hold for the single premise $A \wedge {\sim}A$. So Lewis was just wrong about this, as he was just wrong about the correct theory of implication.

If one's aim is to attack the classical picture of reasoning root and branch, then such a program might well be just the ticket. Lacking that goal however, the ticket in question involves a destination that most would prefer not to visit. Market research clearly shows that except for the radicals, and certain others for whom the science of logic is an occult branch of knowledge, the notion that there are true contradictions is repulsive. The conservative principle is intended to guard us against any similar excess.

2. An Alternative Diagnosis and the Outline of a Theory

We are not as remote from our brethren on the radical left as it might at first appear. Both we and they are concerned with a sort of loophole in the classical conception of inference. According to this conception, an inference holds if and only if it is impossible for the premises to be true and the conclusion false. It seems a cheat then (to introductory students at least) for this relation to hold because (and only because) it is impossible that the premises be true *simplicater*. The radical way to close this loophole, is to fix it so that every premise set, no matter how contradictory, can be made true.[11] Our proposal will be to amend the criterion so that it reads "except for the case in which the premises cannot be simultaneously true in which case ...".

To see how to fill in the ellipsis let us return to the Lewis distinction. What is it about {A, ~A} which makes us think it should be distinct from $\{A \wedge {\sim}A\}$? The answer that occurs naturally, if not most naturally, is that while there is no way of making either set true, there is a way to partition the first so that for each cell of the partition, there is a way of making that cell true. For the second set however, no similar strategy can work. It is not hard to parlay this intuition into a way of measuring the amount or degree of inconsistency which a set of sentences suffers. We can avail ourselves of the fact that classical logic is model complete and replace satisfiability talk with talk of consistency.

Define the predicate CON(Γ, ξ) to hold if and only if there is a family of sets A = $a_1, \ldots, a_{i \in \xi \leq \omega}$ such that each member of the family is (classically) consistent and for each member A of Γ there is some a_k in A such that $a_k \vdash A$.[12] We can now define our measure as the function **i**:

$$\mathbf{i}(\Gamma) = \operatorname{Min}_{\xi}\left[\mathrm{CON}(\Gamma,\xi)\right] \text{ if this limit exists}$$
$$= \infty \text{ otherwise}$$

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The function **i** is named to suggest (degree of) *incoherence*. It is a special case of something called a *level function*, about which we shall have a bit more to say in the sequel. For now we should note that we can distinguish two 'good' degrees, 0 and 1. $\mathbf{i}(\Gamma) = 1$ means that Γ is classically consistent, while $i(\Gamma) = 0$ means that Γ consists entirely of classical tautologies. It seems natural to say that sets of the latter kind are more consistent (less incoherent) than those of the former because the union of a set of tautologies with any consistent set must be consistent. This is not true of an arbitrary consistent set.

Being able to measure degrees of incoherence is the key to 'fixing' the classical account of inference. We are already used to thinking about inference in *preservationist* terms; in terms, that is, of certain properties of sets which must be preserved or transmitted by the inference relation. In fact, we started down our current path by noticing that the 'truth-preservation' definition of validity had an annoying loophole. In order to achieve a greater precision in this, we shall talk about preservation in terms of the deductive closure operation: $C_{\vdash}(\Gamma) = \{A \mid \Gamma \vdash A\}$. It is clear that $\vdash$ preserves degrees 0 and 1, in the sense that if Γ has either of these two degrees of incoherence, then so does $C_{\vdash}(\Gamma)$. But, we can now see, in order to maintain the Lewis distinction, and jettison *ex falso quodlibet*, we want our inference relation to preserve more than just this.

In fact, another principle occurs to us at this point: *don't make things worse*. With this in mind, let us call an inference relation I, *Hippocratic*, provided: $\mathbf{i}(\Gamma) = \mathbf{i}(C_I(\Gamma))$. Clearly, $\vdash$ fails to be Hippocratic. In fact it converts every set of degree 2 or higher (e.g. $\{A, \sim A\}$ has degree 2), into a set of degree ∞! We can set this to rights, in what is obviously a ' classical' way by means of a construction which is suggested by the definition of **i**.

We shall call the revised notion of (classical) inference *forcing*, and refer to it by the symbol [⊦. The construction goes like this: Suppose $\mathbf{i}(\Gamma) = k$; and let $\mathbf{con}(\Gamma)$ be the class of all families of sets $\mathcal{A}$, of the sort required by the definition of **i**, of 'width' k. We can think of each $\mathcal{A}$ as a sort of decomposition of Γ into subsets each of which is 'appropriate' for using $\vdash$ (although such an oversimplification is not strictly speaking correct).

$$\Gamma \,[\!\vdash A \Leftrightarrow \forall \mathcal{A} \in \mathbf{con}(\Gamma): \exists a_j \in \mathcal{A}: a_j \vdash A$$

Here is an Hippocratic inference relation. One can see, in more general terms, that this construction must preserve degree of incoherence whenever the underlying inference relation preserves level 0 and 1. We also claim that this way of doing things accords with the conservative principle, for consider the following consequences of the definition of [⊦:

$$\varnothing \vdash A \Rightarrow \varnothing \,[\!\vdash A$$
$$\{A\} \vdash B \Rightarrow \{A\} \,[\!\vdash B$$

In the interests of a more general view, we shall call the empty set and all unit sets *singular*. Then we can put this 'bridge' principle: the construction preserves all inferences from singular sets. Notice that this principle requires that we not only agree with $\vdash$ on the laws of thought, but also on all inferences from singletons. One might be inclined to argue here that [⊦ agrees exactly with $\vdash$ on the meaning of the logical words. This would be a pretty non-Quinean thing to say however, so perhaps we'd be better off to steer clear of controversy.

The move to a non-classical logic often involves giving things up, which is why non-classical logicians have such a selling job to do: the naive and untutored general-

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ly resist giving up the principles in the learning of which they have invested time and effort. Let us come clean at the outset then—it would do our cause no good to be found out later: The adoption of $[\vdash$ will require us to give up certain classically valid inferences and it will throw certain others into a new light.

Those whose specialty is not formal logic will be most concerned with the operator rules. Since certain of the classical rules are central to its non-Hippocratic nature these must undergo a sea change. Perhaps most prominent is the rule of *conjunction introduction, aka adjunction*. In its classical guise this rule has the form: $\Gamma \vdash A$ and $\Gamma \vdash B \Rightarrow \Gamma \vdash A \wedge B$. Clearly it won't do, since by its means we can move from a set of degree less than ∞ to a set of degree ∞. Nor are we the first by any means to suggest that allowing the deduction of arbitrary conjunctions of conclusions correctly arrived at might well give us something untoward.[13] Our contribution in this area is *not* the suggestion that we give up adjunction.[14] What we suggest instead is that the classical adjunction rule is not stated with proper generality.

What we require is a formulation of the rule which takes proper account of degrees of incoherence. Here is the one which does that:

$$\Gamma [\vdash A_1 \ \& \ldots \ \& \ \Gamma [\vdash A_{i(\Gamma)+1} \Rightarrow \Gamma [\vdash \bigvee (A_i \wedge A_j) \ (1 \leq i \neq j \leq i(\Gamma)+1)$$

This rule must preserve level, and when the premise set has degree 1 (i.e. the degree of 'ordinary' consistent sets), it collapses into the classical adjunction rule. Similarly, the ordinary rule of *conditional elimination* (or, as it is improperly called), *modus ponens* is no respecter of degrees. Consider, for example, the premise set $\{A, A \rightarrow (B \wedge \sim B\}$, which has degree 2. Good old MP used here will generate a closure which has degree ∞. But, again, when the premise set has degree no higher than 1, we can cross as many of these conditional bridges as we come to.

The case of the negation rules—classically both are forms of *reductio ad absurdum*—is a bit stranger. Here we have the rules as is, without that is, any restriction on degrees, but they are trivial. It certainly is true that: $\Gamma \cup \{A\} [\vdash \bot \Rightarrow \Gamma [\vdash \sim A$, but in the absence of adjunction in its simple minded classical form, we shall not be able to discharge the hypothesis unless Γ contains $\bot$, or A is $\bot$.[15] This could lead officials of the logical consumers protection bureau to complain that our logic does not meet advertised specs, since *reductio* is very closely aligned with the classical approach. In answer we say that our approach is consistent with the true meaning of *reductio* which is not faithfully represented in the above rule but rather in: $i(\Gamma \cup \{A\}) > i(\Gamma) \Rightarrow \Gamma [\vdash \sim A$. And, of course, the latter is a correct rule for $[\vdash$.

In summary, what we must give up on this conservative approach are only certain kinds of inferences from classically inconsistent sets. When inconsistency has not reared its ugly head we give up nothing! This is a crucially important point. Since classical logic simply collapses in the face of inconsistent premise sets, it would be better to say that there are actual gains rather than that we have no losses.

Those whose study is the theory of deductive inference are liable to be far more concerned over the fate of the structural rules under this new scheme, than of the operator rules. Here we have some reassurance to offer, and some explaining to do. First the reassurance: we do indeed have [cut], which in the current pseudo-natural deduction context looks like:

[cut] $\Gamma, A [\vdash B$ and $\Gamma [\vdash A \Rightarrow \Gamma [\vdash B$

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and we also have the (true[16]) rule of reiteration:

[R] $A \in \Gamma$ implies Γ [⊢A

What we don't have, is the rule of monotonicity or dilution; at least we don't have it in its classical form: $\Gamma \vdash A \Rightarrow \Gamma \cup \Delta \vdash A$. The reason this fails is that the result of dilution might be a set of higher degree which would require more 'splitting up' of the premise set, and so an inference might be lost. Of the versions that we *do* have (the correct versions, we are tempted to say) one mentions degrees:

[mon] Γ [⊢A and $i(\Gamma \cup \Delta) = i(\Gamma) \Rightarrow \Gamma \cup \Delta$ [⊢A

and the other mentions what we earlier called singular sets (in the current setting, unit sets and the empty set):

[mon*] Γ [⊢A and SING(Γ) $\Rightarrow \Gamma \cup \Delta$ [⊢A

So this logic of ours might be labeled non-monotonic.[17] This makes for an interesting comparison with what is usually called non-monotonic logic in the AI literature. In that setting, one often specifies bits of 'default reasoning' which takes the form of allowing certain conclusions to be drawn so long as the default assumptions hold. If new premises arrive (somehow) which contradict (or ' override' as some say) the assumptions, then the conclusions drawn 'by default' must be withdrawn. So in the AI approach we are told which inferences are at risk, but in our limited form of the principle of monotonicity, we are told rather which inferences are *not* at risk.

3. The Virtues of Conservatism

The sketch just given is enough to give the flavor of the theory being proposed, though it does not touch on many matters of importance to logicians.[18] Rather than repair these deficiencies, we shall consider the adjudication of the conservative approach on a somewhat larger scale. For a theory to succeed, it must have more than niceness (which we have in spades); it must offer generality and fruitfulness as well. Whatever the benefits, so far uncanvassed, under the latter heading, the prospects under the former might be thought a bit thin. After all, the whole thrust of our approach has been to effect a patch on the classical account of deduction. This is rather misleading however.

While it is true that our conservatism restricted what changes we could make in ⊢, the technique by which such changes are effected turns out to be quite general and flexible.[19] The function **i**, which assigns degrees of incoherence, is got by a method which generalizes quite easily to produce what we have elsewhere called *level* functions. What we require in the general case is two predicates of sets Φ and Ψ (which we assume to be non-empty) such that Φ (the general counterpart of consistency) and Ψ—the 'singularity' predicate are assumed to be downward monotonic (preserved by subsets), while neither is assumed to be preserved by supersets. Once we go through this generalization, we can show that the corresponding generalization of the construction which fixes a given inference relation will be Hippocratic provided that the starting relation preserves level 0 and 1.[20] We can also see that every fixed relation will have the structural rule [R] provided that all unit sets are singular, and each will be compatible with [cut] in the sense that the result of adding [cut] to any Hippocratic relation will also be Hippocratic. Finally it can be shown that the result of adding the classical version of dilution will, under quite general conditions, produce a relation which is not Hippocratic.[21]

This might seem like a catalog of arcana rather than evidence of generality, but it is through playing with the predicates Φ and Ψ that one can obtain those variations on the forcing relation which have had the most appeal to those interested in applications. One gets a more pointed form of the consistency predicate e.g., by letting it be defined by: $\Phi(\Gamma) \Leftrightarrow \Gamma \nvdash B$ for every $B \in \Sigma$, rather than $\Gamma \nvdash \bot$. Here Σ is some non-empty set (the set of appalling and repulsive sentences which one must bar from conclusionhood no matter what). Once adjusted by the corresponding construction, we get from this a species of inference appropriate to the case when we like some but not all of the consequences of a theory, but cannot find (at least right away) some 'nice' modification of our theory of inference which will block the nasties.

Similarly we might have an application in which the construction using **i** looses too many 'good' inferences because premise sets get 'broken up' too finely. In much of our reasoning about physical theory for example, we shall expect certain natural 'clumps' of principles always to be available together no matter how high the degree of our premise set turns out to be. All we require in order to guarantee this, is to require the clumps in question to be singular.[22]

Finally we consider the question of fruitfulness. It is too soon to get the judgment of history on this score, but the cupboard is far from bare. One of the first places we should expect a theory of inference to show some application muscle is in the field of logic. This has certainly been the case with our account, principally in the area of modal logic. Standard, or as some call it, *normal* modal logic is axiomatized by the addition of the following to the classical rules:

$[R\Box]\ \Gamma \vdash A \Rightarrow \Box[\Gamma] \vdash \Box A$

where $\Box[\Gamma]$ refers to the set $\{\Box B | B \in \Gamma\}$.

If we replace the $\vdash$ on the left side of this rule by $[\vdash$ we get a modal logic which resembles the normal variety except for the principle of complete aggregation $[K]\ \Box A \wedge \Box B \rightarrow \Box(A \wedge B)$. These modal logics make an interesting and worthy study in their own right, but have also turned up in the study of the existing systems **S1** and the system **E** of H.B. Smith.[23]

Notes

[1]I think the term is da Costa's (one of the pioneers of the subject). I first heard it in the presentation of a a paper by Wolf and da Costa at the Pittsburgh meeting of the Society for Exact Philosophy in 1978.

[2]My personal favorite is: $(A \supset B) \vee (B \supset C)$. It is mildly ironic that this is required to be a tautology in order for the Lewis modal logic S1 to have a semantics (or at least the semantics that it in fact has).

[3]The logic **S2**. The logic **S1** has only that version of this 'paradox' in which the main connective is material rather than strict implication.

[4]The drawing board in question was getting rather worn out by this time anyway. Lewis' first attempts to formalize strict implication had collapsed into classical logic

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and once that problem had been solved he discovered that his logic had the 'wrong' form of transitivity principle.

[5]Both W.T. Parry and R. Barcan-Marcus

[6]In this group belong not only Russell but also Quine. Strange bedfellows indeed.

[7]In this connection, Judith Pelham's recent paper (Pelham 1993), is most illuminating.

[8]The phrase is Wittgenstein's. He gives an account of going to visit Russell over a period of several days before the first war, with the express purpose of puzzling out what this "sign" meant. The two of them finally decided that it was meaningless. This may be shocking but it isn't surprising given what Russell has to say (Russell 1903) concerning assertion.

[9]Quine uses this mode of attack (among many others of course) but he does not follow through as we do in the next sentence.

[10]In fact, we do more. The method we propose can 'fix' any account of inference that meets certain minimal requirements.

[11]In the usual sense that set of sentences is true provided all of its member sentences are true. One might take the position here (starting a competing radical school) that this betrays deeply embedded classical thinking. We really ought to construct an alternative to the classical theory of true sets, such that individual sentences inherit their truth from the fact that they are members of true sets rather than the contrary way of doing things. It might even be the case, on this alternative radical proposal, that one need not embrace true contradictions. To the best of my knowledge, this new school is without members.

[12]In other places this sort of predicate is defined in terms of partitions or covering families. This works, and is to some extent more intuitive, but it fails to generalize to the entire class of level functions. In addition, it fails to account for those Γ such that CON(Γ, 0) which must be dealt with by a 'convention'.

[13]Such, for example, is Henry Kyburg's diagnosis of what goes wrong in the lottery paradox.

[14]It is, to my mind, rather unfortunate that paraconsistent logic in our style has come to be called "the non-adjunctive approach".

[15]In fact, another way of saying that an inference relation respects the Lewis distinction is to say that if Γ has degree ∞, then it must contain a singular subset with that degree (where only sets of degree ∞ explode).

[16]In most introductory logic books what is *called* the rule of reiteration, is an amalgam of this rule and the rule of monotonicity or dilution.

[17]R. Sylvan has remarked in a survey of non-monotonic logics that we can comfortably take "non-monotonic" as a synonym for "non-deductive". If forcing really is to be regarded as non-monotonic then, I trust I have made this view rather more uncomfortable.

18 Perhaps most prominent is the issue of characterizing [$\vdash$ by means of a set of rules. In some recent work, Bryson Brown makes an important contribution to this

subject by providing a much simpler proof that the rules published (Schotch and Jennings 1989), are correct. The original (unpublished) proof by Schotch and Jennings is a real brute.

[19]And, we should not fail to point out, the method generalizes to virtually any inference relation. We are not restricted to the classical realm by anything but market surveys.

[20]There is a bit more at stake here than just Hippocraticality, we might wish to concentrate on properties Φ which satisfy certain 'nice' conditions e.g., compactness. It can also be shown that this is 'transmitted' by the construction in the sense that if Φ is compact, then its corresponding level function is level-compact. By this is meant that if $\ell(\Gamma) = n$ then $\ell(\Delta) = n$ for Δ some finite subset of Γ (where ℓ is a level function). This result is due to Blaine d'Entremont.

[21]This is quite important for the adjudication of a suggestion of D. Lewis in (Lewis 1983) He suggests in that essay that his approach is akin to ours, but it is easy to see that the logic he proposes has the classical dilution rule, meets the conditions referred to, and hence fails to be Hippocratic.

[22]In "On detonating" this species of forcing is called A-forcing, and it's presentation there is rather different. This is largely because the reformulation of level functions in terms of the predicate Ψ had not then appeared.

[23]Both results are thus far unpublished but details are to be found in Doyle, R., Mares, E., Schotch, P.K. et al. "S1: the Final Chapter". This **S1** result is not without its embarrassing aspects since it was Max Cresswell who discovered that Schotch, Sylvan, and sundry others were in error in asserting that complete aggregation (in its strict implication form) holds in this mysterious and least understood of the Lewis logics.

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