When $a=1$, the equation becomes

$$y-x-y^3+x^3y^2+x^2y^3=0,$$

$$\text{i.e.}\quad (y^2+xy-1)(x^2y-y+x)=0.$$

In this case therefore the curve consists of the hyperbola $y^2+xy=1$ and the cubic $x^2y-y+x=0$, and is shown in Fig. 2.

R. J. T. Bell

Graphical Trisection of Circular Arc.—Let O be the centre of a given circle of radius a; and OA, OB the containing radii of the arc which is to be trisected.

Along OA set off $OE=\frac{1}{4}OA$, and draw BF parallel to OA and equal to $\frac{5}{4}a$.

Then a line EP parallel to OF will pass through a point P on the circumference such that $\text{arc}\,AP=\frac{1}{3}\text{arc}\,AB$ approximately.

For, supposing $AOB=3\phi$, the equation to EP is

$$y(5+4\cos 3\phi)=4\sin 3\phi(x-\tfrac{1}{4}a).$$

This will be exactly satisfied by $x=a\cos\phi$, $y=a\sin\phi$

if $\qquad 5\sin\phi+4\sin\phi\cos 3\phi=4\sin 3\phi\cos\phi-\sin 3\phi,$

or if $\qquad 5\sin\phi-4\sin 2\phi+\sin 3\phi=0.$

Now

$$5(\phi-\tfrac{1}{6}\phi^3+\tfrac{1}{120}\phi^5)-4(2\phi-\tfrac{8}{6}\phi^3+\tfrac{32}{120}\phi^5)$$
$$+(3\phi-\tfrac{27}{6}\phi^3+\tfrac{243}{120}\phi^5)=0$$

if $\qquad \phi^5=0.$

Therefore the error is of the fifth order of small quantities if ϕ be small.

For an arc of 60° the error amounts to $2'$; and for an arc of 45° or less the error is less than $1'$.

[The above was suggested by a construction given by C. S. Bingley, Esq., F.C.I.S., in "Knowledge," November 1911.]

R. F. Davis

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