Bull. Aust. Math. Soc. **106** (2022), 167–169
doi:10.1017/S0004972722000533

TRANSVERSALITY, REGULARITY AND ERROR BOUNDS IN VARIATIONAL ANALYSIS AND OPTIMISATION

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(Received 26 April 2022; first published online 9 June 2022)

2020 *Mathematics subject classification*: primary 49J53; secondary 65K10, 90C31.

Keywords and phrases: variational analysis, transversality, metric regularity, error bounds.

Transversality properties of collections of sets, regularity properties of set-valued mappings and error bounds of extended-real-valued functions lie at the core of variational analysis and optimisation because of their importance for stability analysis, constraint qualifications, qualification conditions in coderivative and subdifferential calculus, and convergence analysis of computational algorithms. The thesis is devoted to the investigation of several research questions related to the aforementioned properties and their applications in several optimisation problems. The main tools of the analysis are standard techniques of modern variational analysis (see [2, 10, 11, 13]).

Quantitative analysis of transversality properties of collections of sets is investigated in the convex and nonconvex, linear and nonlinear settings by employing conventional tools of generalisation differentiation. The quantitative relations between transversality and regularity properties of set-valued mappings as well as nonlinear extensions of the new transversality properties of a set-valued mapping to a set in the range space are discussed. These results are presented in recent publications [1, 3–6, 14].

We study theoretical and applied aspects of a new property called 'linear semi-transversality of collections of set-valued mappings' in metric spaces [9]. The property can be seen as a generalisation of the conventional semitransversality of collections of sets and the negation of the corresponding stationarity which is a weaker property than the extremality of collections of set-valued mappings [12]. Necessary and sufficient conditions in terms of primal and dual objects and quantitative connections with the semiregularity of set-valued mappings are formulated. The results are applied to optimality conditions for a multiobjective optimisation problem with geometric constraints.

Thesis submitted to Federation University in December 2020; degree approved on 24 May 2021; principal supervisor Alexander Kruger, associate supervisor David Yost.

We examine a comprehensive (that is, not assuming the mapping to have any particular structure) view on the regularity theory of set-valued mappings and clarify the relationships between the existing primal and dual quantitative necessary and sufficient conditions including their hierarchy [7]. The typical sequence of regularity assertions, often hidden in the proofs, and the roles of the assumptions involved in the assertions, in particular, on the underlying space (general metric, normed, Banach or Asplund) are exposed. As a consequence, we formulate primal and dual conditions for the conventional metric regularity and subregularity properties as well as stability properties of solution mappings to inclusions.

We propose a unifying general framework of quantitative primal and dual sufficient and necessary error bound conditions covering linear and nonlinear, local and global settings [8]. The function is not assumed to possess any particular structure apart from the standard assumptions of lower semicontinuity in the case of sufficient conditions and (in some cases) convexity in the case of necessary conditions. Employing special collections of slope operators, we introduce a succinct form of sufficient error bound conditions, which allows one to combine in a single statement several different assertions: nonlocal and local primal space conditions in complete metric spaces and subdifferential conditions in Banach and Asplund spaces. As a consequence, the error bound theory is applied to characterise subregularity of set-valued mappings, and calmness of the solution mapping in convex semi-infinite optimisation problems for which linear perturbations of the objective function and continuous perturbations of the right-hand side of the constraint system are allowed.

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https://doi.org/10.1017/S0004972722000533 Published online by Cambridge University Press