ellipsoid and cylinder. If $x/\lambda = y/\mu = z/\nu$ is a perpendicular, the tangent plane is $\lambda x + \mu y + \nu z = \sqrt{a^2\lambda^2 + b^2\mu^2 + c^2\nu^2}$, and r, the length of the perpendicular, is given by

$$r^2 = \frac{a^2\lambda^2 + b^2\mu^2 + c^2z^2}{\lambda^2 + \mu^2 + \nu^2.}.$$

We have also $$l\lambda + m\mu + n\nu = 0, \qquad (4)$$

since the perpendicular lies in the plane $lx + my + nz = 0$. Consider now the semi-diameter of the ellipsoid when equations are

$$\frac{x}{a\lambda} = \frac{y}{b\mu} = \frac{z}{c\nu} \left(= \frac{r_1}{\sqrt{a^2\lambda^2 + b^2\mu^2 + c^2\nu^2}} \right).$$

Its length is given by $r_1{}^2 = \dfrac{a^2\lambda^2 + b^2\mu^2 + c^2\nu^2}{\lambda^2 + \mu^2 + \nu^2}$, and by (4) it lies in the plane $$\frac{lx}{a} + \frac{my}{b} + \frac{nz}{c} = 0. \qquad (5)$$

Therefore the semi-diameters of the normal section of the cylinder are equal to the semi-diameters of the section of the ellipsoid by the plane (5), and the sections are equal ellipses.

If the tangent at P to the ellipse (Fig. 1) is parallel to $x/l = y/m$, the equations of CN and CR are $lx + my = 0$, $\dfrac{lx}{a} + \dfrac{my}{b} = 0$, and thus the results for the ellipse and ellipsoid are exactly analogous.

R. J. T. Bell.

A Construction by Ruler and Dividers.

1. The following method of cutting off an n^{th} part of a given straight line requires besides a ruler only a pair of dividers or other means of laying off on a given straight line a segment equal to the distance between two given points. In other words, we assume that, besides using a ruler in the usual way, we can mark off from a given straight line AX a segment AB, which shall be equal to the distance between two given points C and D.

2. The construction is as follows:—

Let AB be the given straight line.

Draw any other straight line AX from A, and cut off from AX a succession of equal parts, the $n-1^{\text{th}}$ ending at D, and the $n+1^{\text{th}}$ at C.

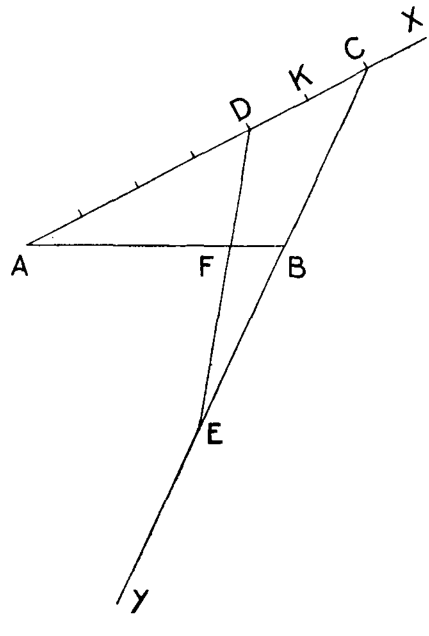
Join CB, and produce it to E, making BE = CB.

https://doi.org/10.1017/S1757748900001274 Published online by Cambridge University Press

Let DE cut AB in F.

Then $FB = \frac{1}{n} AB$.

This may be proved by Menelaus' Theorem, or by joining B to the point K on AX intermediate between D and E, so that BK is $\parallel$ to DF. Therefore $\frac{FB}{AB} = \frac{DK}{AK} = \frac{1}{n}$.



3. To divide AB into n equal parts by a similar method, we may produce CB to γ and cut off from $B\gamma$ $n-1$ parts each equal to CB, and join these points to the corresponding points in KA so that AB becomes divided into n equal parts. In fact, if AD_1, D_1D_2, D_2D_3, etc., are the successive equal segments cut off from AX, and BE_1, E_1E_2, etc., the successive equal segments cut off from $B\gamma$, the straight line D_rE_{n-r} cuts AB in a point F_r such that $AF_r = \frac{r}{n} AB$.

4. Of course the n points of division required in AB could be got by first constructing $FB = \frac{1}{n} AB$ as in the first construction, and then laying off segments equal to it along AB. But, though this construction would be geometrographically simpler than the last, it would have the disadvantage of multiplying the initial error in FB by $n-1$, which might result in the later points of division in the process of dividing AB being affected by two great errors.

Query—What is the relation between the probable errors in the positions of E_r in the two cases?

5. Returning to the construction in 3, we may note that it gives a means of constructing a point F_r in AB such that $AF_r = r \cdot \frac{AB}{n}$.

There is nothing to prevent r from being greater than n in this construction, or less than zero. If r be greater than $n+1$, then D_r will be beyond D_{n+1}, and F_r will be in AB produced.

If r be negative, then D_r will be in DA produced, while E_{n-r} will be in $B\gamma$, so that F_r will lie in BA produced.

Thus we can by ruler and dividers alone divide a segment of a line internally or externally so that the parts are in any commensurable ratio to one another.

6. A modification of the construction of Art. 3 would be got by interchanging the rôles of the lines AD_{n+1} and AB. But this, though requiring fewer points of division, has the disadvantage of failing to give *immediately* the last dividing point required.

R. F. MUIRHEAD.

Elementary proof of the formula for $\alpha^n + \beta^n$ in terms of $\alpha + \beta$ and $\alpha\beta$.*—The proof given by Chrystal uses infinite series, but by a simple modification it may be put in an elementary form.

Write
$$\alpha + \beta = s,$$
$$\alpha\beta = p.$$
$$\alpha^n + \beta^n = s_n.$$
$$sx - px^2 = u.$$

* Chrystal's Algebra, 1st Edition, Chap. 27, Art. 7.