CORRESPONDENCE

The Joint Editors — 3 *October* 1951

The Journal of the Institute of Actuaries Students' Society

Building Society mortgages

Sirs,

G. J. Knapman developed in an interesting letter (*J.S.S.* **10**, 159–60) a formula for calculating the net single premium at rate of interest i for an assurance to cover the amount outstanding under a Building Society mortgage subject to a rate of interest j, referring to a valuable paper of H. A. Gosden (*J.S.S.* **7**, 174–6) and proposing to construct three columns C'_x, M'_x, D'_x for the range of ages over which these policies extend. In a previous paper 'Amortización y Seguro de Vida' published in the *Revista del Colegio de Ingenieros de Venezuela*, no. 151–2, Caracas, 1944, assuming $x+n \leqslant 75$, I developed a formula which needs only the construction of two columns

$$G_x = C^i_x a^j_{\overline{75-x}|}; \quad H_x = \sum_{t=0}^{75-x} G_{x+t}.$$

The net single premium then becomes

$$H_{x:\overline{n}|} = \frac{H_x - H_{x+n} - (M_x - M_{x+n})\, a^j_{\overline{75-x-u}|}}{D^i_x(a^j_{\overline{75-x+1}|} - a^j_{\overline{75-x-n+1}|})}$$

Yours faithfully,

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Annuities with a guaranteed term

Sirs,

The value of the guarantee part of the Guaranteed Term Life Annuity is discussed by Hymans (*J.S.S.* **10**, 231) and he derives

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a neat method of approximating to the exact values where calculations are required for various terms and rates of interest. In similar calculations I have used a simple rule, which around the usual retirement ages is usually accurate enough for practical purposes. The rule is: *The value of the guarantee varies as the square of the term.*

Thus, for example, the value of a 3-year guarantee is approximately 36% of the value of a 5-year guarantee.

Using this rule, if accurate values of the guarantee are calculated for a 10-year term at each rate of interest and age the values for shorter terms may be taken as n^2% of the 10-year values. The method gives very reasonable approximations to the required values and certainly the labour of calculation is light.

The rule can be demonstrated using Hymans's formula. He shows that the value of the extra payments to make a life annuity payable for n years certain can be expressed as UV, where U is independent of i and n, and V is independent of x; and also that $V=\bar{b}_{\overline{n}|}-\bar{a}_{\overline{n}|}$, where $\bar{b}_{\overline{n}|}$ is a continuous annuity at an adjusted rate of interest, positive or negative. If we expand V in powers of n we have

$$V=\tfrac{1}{2}n^2(\delta-\delta')-\tfrac{1}{6}n^3(\delta^2-\delta'^2)+\tfrac{1}{24}n^4(\delta^3-\delta'^3)-\ldots.$$

This shows that to a first approximation the value of the guarantee varies as the square of the term.

In the example given, we have $i=\cdot035$, $j=\cdot0268$, $\delta=\cdot034401$, $\delta'=-\cdot026447$, $n=5$.

Hence

$$V=12\cdot5\times\cdot060848-20\cdot83\times\cdot000484+26\cdot04\times\cdot000059\ldots$$

$$=\cdot7520.$$

This agrees closely with ·7523 the value obtained by Hymans. It will be seen that the first term, depending on n^2, is very much greater than subsequent terms of the expansion.

Incidentally, another excellent approximation to the benefit we are considering is $\tfrac{1}{2}n^2v^{3n/4}(\mu l)_{x+3n/8}/l_x$.

However, I make no claims that this formula has advantages for practical work, as the interpolation for fractional values of v and

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(μl) may be rather tiresome. The snag with some otherwise excellent approximations, including this one, is that it may be nearly as much work to determine the approximate as the exact value.

Yours faithfully,

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Annuities with a guaranteed term

Sirs,

The article by Mr Hymans under the above title (*J.S.S.* **10**, 230) prompted me to calculate specimen values of the additions to be made to the value of a continuous annuity in order to arrive at the value of a continuous annuity payable for various minimum guaranteed periods. Examples of the values of $\bar{a}_{\overline{n}|}-\bar{a}_{x:\overline{n}|}$ based on the $a(m)$ ultimate table are given below:

Term	Age	Rate of Interest per annum					
		$2\frac{1}{2}\%$	3 %	$3\frac{1}{2}\%$	4 %	$4\frac{1}{2}\%$	5 %
5	60	·255	·250	·247	·243	·238	·235
	65	·370	·363	·357	·351	·345	·340
	70	·533	·523	·514	·506	·498	·490
	75	·811	·797	·784	·772	·759	·747
10	60	1·026	·992	·961	·931	·901	·873
	65	1·453	1·407	1·360	1·318	1·277	1·238
	70	2·066	2·000	1·936	1·877	1·818	1·762

It will be seen that the ratio of the cost of the guarantee at one rate of interest to the corresponding cost at another rate of interest is practically independent of age; e.g. the cost on a 5 per cent basis