Proceedings of the Edinburgh Mathematical Society (1995) **38**, 357–360 ©

BOOK REVIEWS

STEINMETZ, N. *Rational iteration: complex analytic dynamical systems* (de Gruyter Studies in Mathematics Vol. 16, de Gruyter, Berlin, New York 1993) x+190 pp., 3 11 013765 8, about £60.

Although work on the iteration of polynomial and rational functions on the complex plane dates back to 1918–19 with the pioneering papers of Fatou and Julia, there has been a tremendous resurgence of interest in this area in the last few years. Partly, at least, this is due to the development of the computer, which has enabled practical examination of iterates and the associated Julia and Mandelbrot sets, with the observation of many beautiful and intricate features inviting theoretical explanation.

This book provides a self-contained account of the theory of rational iteration, including the Fatou–Julia theory, dynamics on the Fatou set (including the existence of rotation domains and Sullivan's Theorem on the non-existence of wandering domains), the geometry and dynamics of the Julia set, and miscellaneous topics such as the connectivity of the Mandelbrot set and Lyubich's invariant measure. The book assumes a reasonable knowledge of complex function theory, including Montel's Theorem on normal families and Carathéodory's Theorem on the boundary correspondence of conformal mappings. The properties of quasi-conformal mappings required for Sullivan's theorem are also taken on trust.

The book is readable and mathematically precise, though occasionally the treatment seems a little terse. The text is backed up by well chosen exercises and it is illustrated by many grey-tone illustrations depicting Julia sets and invariance domains of various functions.

The book could be the basis for a postgraduate course on rational iteration and is also suitable for researchers seeking an acquaintance with the area. Inevitably it stops short of the sophisticated methods developed in very recent years such as the Yoccoz puzzle pieces and invariant line families. Nevertheless, it raises the reader to a level of knowledge at which such topics of contemporary research become accessible.

The book should be considered alongside Alan Beardon's *Iteration of rational functions* (Springer-Verlag, 1991), which covers a remarkably similar range of topics at the same level and with a similar audience in mind. The books differ in style: Beardon's book is more discursive and there is more motivation for the development by way of specific examples; Steinmetz's book is mathematically more concentrated, though not at the expense of readability. In their individual ways both books make an excellent job of achieving their aims of providing attractive treatments of rational iteration to bridge the gap between the basic theory of complex functions and the research frontiers of rational iteration theory.

K. J. FALCONER

SCHMIDT, R. *Subgroup lattices of groups* (de Gruyter Expositions of Mathematics Vol. 14, de Gruyter, Berlin, New York 1994) 576 pp., 3 11 011213 2, £139.38.

Early in one's study of groups one tries to pay less attention to arguments involving elements and concentrate more on the subgroup structure of groups. The study of the lattice $L(G)$ of subgroups of a group G may be thought of as the logical consequence of this approach, investigating a group by looking only at the structure of the lattice of subgroups.

The first paper in the subject was by Röttlander in 1928 and was motivated by the lattice isomorphism described in the Galois correspondence between the subgroups of the Galois group and the intermediate field extensions of a Galois extension. This leads immediately to the question of the extent to which $L(G)$ determines the group G. Röttlander showed that there are nonisomorphic groups G_1, G_2 and a lattice homomorphism $\phi: L(G_1)\rightarrow L(G_2)$ such that $|U:V|=|U^{\phi}:V^{\phi}|$ whenever $V\leqq U\leqq G$.

The two basic questions in the theory are:

(I) Given a lattice L satisfying property $\mathscr{X}$ what can one say about groups G with $L(G)\cong L$?

(II) Given a class of groups $\mathscr{Y}$ what can one say about the lattice $L(G)$ for $G\in\mathscr{Y}$?

An early example of a result of type (I) is Ore's theorem that if $L(G)$ is distributive then G is locally cyclic. Further results concern M-groups or groups with modular subgroup lattice. Iwasawa characterized both locally finite and nonperiodic M-groups. For periodic groups the author has given a characterization in terms of extended Tarski groups.

In M-groups every subgroup is modular but modular subgroups arise throughout in more general situations. One usually wishes to get information about the normal structure of a group but if N is a normal subgroup of G and ϕ is a projectivity from G to $\bar{G}$ (i.e., a lattice isomorphism from $L(G)$ to $L(\bar{G})$) then the image V^{ϕ} need not be a normal subgroup of $\bar{G}$. However the image is modular and this leads to consideration of the structure and embedding of M/M_G and M^G/M_G for a modular subgroup M of a group G.

There are many examples given of the second type of result in which well-known classes of groups are characterized by giving conditions on the lattice of subgroups.

For finite groups one can characterize simple, perfect, soluble and supersoluble groups. The author remarks that Ore's theorem and this characterization of finite supersoluble groups are the only results known in which the lattice-theoretic and group-theoretic conditions both seem to be natural conditions. Most of these characterizations have been extended to infinite groups in recent years, results being obtained for simple, perfect, polycyclic, finitely generated soluble groups, etc. However there is still no lattice-theoretic characterization for the class of soluble groups.

The above can only give a taste of the results contained in this book and can give little impression of the range of techniques which have been developed.

The only previous volume devoted to this topic is that by Suzuki in the *Ergebnisse* series from 1956. That ran to 92 pages with 3 pages of references. This book is six times the length with 18 pages of references, reflecting the development of the subject over the last forty years.

Most group theorists will find results and techniques of interest here but the aim of the book is as a reference for specialists in the area. It will be particularly important for graduate students working in the area and suggests a number of possible directions for further investigation.

M. J. TOMKINSON

ROSENBLUM, M. and ROVNYAK, J. *Topics in Hardy classes and univalent functions* (Birkhäuser Advanced Texts, Birkhäuser, Basel, Berlin, Boston 1994) 264 pp, 3 7643 5111 X, £36.

It was generally believed that with the proof of the Bieberbach conjecture by Louis de Branges the principal motivation for the study of extremal problems for univalent functions had gone. Instead, the subject has been stimulated by the revelation of the close connection between such problems and the much studied problems of operator theory. The appearance of the generalized Jacobi polynomials like some *deus ex machina* in de Brange's original proof is now seen as perfectly natural. Indeed the polynomials never appear explicitly in the authors' presentation, the crucial step being a positivity result for the generalized hypergeometric function $F(a,b,c,d,e\,|\,x)$. This result is due to Askey and Gasper, but the history of such positivity results is long, going back beyond Fejér to the masters of the 19th century.

The book is essentially in two parts and all of the above is clearly set forth in the last three chapters. The crucial early idea was due to Löwner, who realised that much important