

Appendix A

Physical constants and units

A.1 High-energy physics conversion constants and units

Table A.1. *High-energy physics conversion constants and units*

Quantity	Name	Value
Speed of light	c	$299\,792\,458\text{ m s}^{-1}$
Reduced Planck constant	$\hbar \equiv h/2\pi$	$1.054\,572\,66(63) \times 10^{-34}\text{ J s} =$ $6.582\,122\,0(20) \times 10^{-23}\text{ MeV s}$
Conversion constants	$\hbar c$ $(\hbar c)^2$	$197.327\,053(59)\text{ MeV fm}$ $0.389\,379\,66(23)\text{ GeV}^2\text{ mbarn}$
Units where $\hbar = c = 1$	Mass, energy	$1\text{ eV} = 1.602\,177\,33(49) \times 10^{-19}\text{ J}$ $1\text{ GeV} = 10^3\text{ MeV} = 10^6\text{ keV} = 10^9\text{ eV}$ $1\text{ erg} = 10^{-7}\text{ J}$ $1\text{ eV}/c^2 = 1.782\,662\,70(54) \times 10^{-36}\text{ kg}$
	Length	$1\text{ GeV}^{-1} = 0.197\,327\,053\text{ fm} =$ $0.197\,327\dots \times 10^{-13}\text{ cm}$ $1\text{ in} = 0.0254\text{ m} \quad 1\text{ \AA} = 0.1\text{ nm}$
	Lifetime	$1\text{ GeV}^{-1} = 6\,582\,122\,0 \times 10^{-25}\text{ s}$
	Decay rate	$1\text{ GeV} = (1/6\,582\,122\,0) \times 10^{25}\text{ s}^{-1}$
	Cross-section	$1\text{ GeV}^{-2} = 0.389\,379\,66(23) \times 10^6\text{ barn}$ $1\text{ barn} = 10^{-28}\text{ m}^2 \quad 1\text{ nb} = 10^{-9}\text{ barn}$
	Others	$0\text{ }^\circ\text{C} = 273.15\text{ K} \quad 1\text{ G} = 10^{-4}\text{ T}$ $kT\text{ at }300\text{ K} = [38.681\,49(33)]^{-1}\text{ eV}$ $1\text{ atmosphere} = 760\text{ torr} = 101\,325\text{ Pa}$ $1\text{ dyne} = 10^{-5}\text{ N}$

A.2 High-energy physical constants

A complete list of physical constants is given in PDG [16]. Among them, we have:

Table A.2. Some high-energy physical constants

Observable	Symbol	Value
Electron mass	m_e	0.510 999 06(15) MeV/c ² = 9.109 389 7(54) × 10 ⁻³¹ kg
Muon mass	m_μ	105.658357(5) MeV/c ²
Tau mass	m_τ	1777.03 ⁺³⁰ ₋₂₆ MeV/c ²
Proton mass	m_p	938.272 31(28) MeV/c ² = 1836.152 701(37) m_e
Electron charge	e	1.602 177 33(49) × 10 ⁻¹⁹ C = 4.803 206 8(15) × 10 ⁻¹⁰ esu
Permittivity of free space	ϵ_0	8.854 187 817... × 10 ⁻¹² F m ⁻¹
Fine structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	1/137.035 999 58(52) at $q^2 = m_e^2$ 1/128 at $q^2 = M_Z^2$
Electron anomaly	$a_e \equiv \frac{1}{2}(g_e - 2)$	115 965 218 84(43) × 10 ⁻¹³
Muon anomaly	$a_\mu \equiv \frac{1}{2}(g_\mu - 2)$	116 592 023(151) × 10 ⁻¹¹
Tau anomaly	$a_\tau \equiv \frac{1}{2}(g_\tau - 2)$	0.004 ± 0.027 ± 0.023
Electron radius	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	2.817 940 92(38) × 10 ⁻¹⁵ m
Bohr radius ($m_{nucleus} = \infty$)	$a_\infty = 4\pi\epsilon_0\hbar^2/m_e c^2$ = $r_e\alpha^{-2}$	0.529 177 249(24) × 10 ⁻¹⁰ m
Electron Compton wavelength	$\lambda_e/2\pi = \hbar/m_e c$ = r_e/α	3.861 593 23(35) × 10 ⁻¹³ m
Rydberg energy	$\hbar c R_\infty = m_e c^2 \alpha^2/2$	13.605 698 1(40) eV
Thomson cross-section	$\sigma_T = 8\pi r_e^2/3$	0.665 246 16(18) barn
Bohr magneton	$\mu_B = e\hbar/2m_e$	5.788 382 63(52) × 10 ⁻¹¹ MeV T ⁻¹
Nuclear magneton	$\mu_B = e\hbar/2m_p$	3.152 451 66(28) × 10 ⁻¹⁴ MeV T ⁻¹
Electron cyclotron freq./field	$\omega_{cycl}^e/B = e/m_e$	1.758 819 62(53) × 10 ¹¹ rad s ⁻¹ T ⁻¹
Fermi coupling constant	$G_F/(\hbar c)^2$	1.166 39(2) × 10 ⁻⁵ GeV ⁻²
Weak mixing angle	$\sin^2 \theta_W(M_Z) \overline{MS}$	0.2315(4)
W^\pm boson mass	M_W	80.33(15) GeV/c ²
Z^0 boson mass	M_Z	91.187(7) GeV/c ²
Strong coupling constant	$\alpha_s(M_Z)$	0.118(3)

A.3 CKM weak mixing matrix

In the electroweak standard model $SU(2)_L \times U(1)$, where both quarks and leptons left-handed doublets and right-handed singlets, the quark mixing matrix can be represented as:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad (\text{A.1})$$

where from weak decays, the mixing matrix has the value:

$$\begin{pmatrix} 0.9745 - 0.9757 & 0.219 - 0.224 & 0.002 - 0.005 \\ 0.218 - 0.224 & 0.9736 - 0.9750 & 0.036 - 0.046 \\ 0.004 - 0.014 & 0.034 - 0.046 & 0.9989 - 0.9993 \end{pmatrix} \quad (\text{A.2})$$

In the Wolfenstein parametrization:

$$\begin{aligned} V_{us} &\simeq \lambda, & V_{ub} &\simeq \lambda^3 A(\rho - i\eta) \\ V_{cb} &\simeq \lambda^2 A, & V_{td} &\simeq \lambda^3 A(1 - \rho - i\eta) \end{aligned} \quad (\text{A.3})$$

A.4 Some astrophysical constants

Table A.3. Some astrophysical constants

Observable	Symbol	Value
Newton gravitation constant	G_N	$6\,672\,59(85) \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}$
Astronomical unit	AU	$1.495\,978\,706\,6(2) \times 10^{11} \text{ m}$
Tropical year(equinox to equinox)	yr	$31\,556\,925.2 \text{ s}$
Age of the universe	t_0	$15(5) \text{ Gyr}$
Planck mass	$\sqrt{\hbar c/G_N}$	$1.221\,047(79) \times 10^{19} \text{ GeV}/c^2$
parsec(1AU/1 arc sec)	pc	$3.085\,677\,580\,7(4) \times 10^{16} \text{ m} = 3.262 \dots \text{ ly}$
light year	ly	$0.306\,6 \dots \text{ pc} = 0.9461 \dots \times 10^{16} \text{ m}$
Solar mass	M_\odot	$1.968\,92(25) \times 10^{30} \text{ kg}$
Solar luminosity	L_\odot	$3.846 \times 10^{26} \text{ W}$
Solar equatorial radius	R_\odot	$76.96 \times 10^8 \text{ m}$
Earth mass	M_\oplus	$5.973\,70(76) \times 10^{24} \text{ kg}$
Earth equatorial radius	R_\oplus	$6.378\,140 \times 10^6 \text{ m}$
Hubble constant	H_0	$100 h_0 \text{ km s}^{-1}\text{Mpc}^{-1} = h_0 \times (9.778\,13 \text{ Gyr})^{-1}$
Normalized Hubble constant	h_0	$0.5 \leq h_0 \leq 0.85$
Critical density of the universe	$\rho_c = 3H_0^2/8\pi G_N$	$2.775\,366\,27 \times 10^{11} h_0^2 M_\odot \text{Mpc}^{-3}$
Local halo density	ρ_{halo}	$(2 - 13)10^{-25} \text{ g cm}^{-3} \approx (0.1 - 0.7) \text{ GeV}/c^2 \text{ cm}^{-3}$
Scaled cosmological constant	$\lambda_0 = \Lambda c^2/3H_0^2$	$-1 < \lambda_0 < 2$
Scale factor for cosmological constant	$c^2/3H_0^2$	$2.853 \times 10^{51} h_0^2 \text{ m}^2$