Feedback

The Editor writes: I apologise to Martin Lukarevski for omitting some material from the end of his Note 107.23 on ‘The location of the inarc circle and its point of contact with the circumcircle’, which appeared in the July 2023 issue of the *Gazette*. The following should have been inserted between the end of the published text and the Acknowledgements.

One can consider the related notion of exarc circles and their radii [1,2]. The exarc radii are much larger than the inarc radii and for their sums we have established the inequality

$$8R - 4r \leqslant R_A + R_B + R_C \leqslant \frac{4R^2}{r} - 2R$$

where in the proof we have used Kooi's inequality $s^2 \leqslant \dfrac{R(4R + r)^2}{2(2R - 2)}$ [3, 4, 5]. After much effort to find a linear upper bound, we posed the following conjecture:-

The sum of the exarc radii $R_A + R_B + R_C$ cannot be bounded from above by a linear expression $\lambda R + \mu r$.

1. G. Leversha, The *geometry of the triangle*, UKMT (2013)
2. M. Lukarevski, Exarc radii and the Finsler-Hadwiger inequality, *Math. Gaz*. **106** (March 2022) pp. 138-143.
3. M. Lukarevski, A simple proof of Kooi's inequality, *Math. Mag*. **93**(3) (2022) p. 225.
4. M. Lukarevski, D.S.Marinescu, A refinement of Kooi's inequality, Mittenpunkt and applications, *J. Inequal.Appl*. **13**(3) (2019) pp. 827-832.
5. M. Lukarevski, Wolstenholme's inequality and its relation to the Barrow and Garfunkel-Bankoff inequalities, *Math. Gaz*. **107** (March 2023) pp.70-75.

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On ‘Sums of the first *n* integers’: Paul Stephenson writes: In [1], Chris Sangwin found twenty remarkably different ways to prove that the sum of the first n odd positive integers is n^2. (For ‘number’ read ‘positive integer’ in what follows.) This note discusses three pictorial variants.

In Example 4, the stack of odd numbers is repeated by a fourfold rotation about the centre of a square of side $2n$; it is therefore necessary to divide by 4 to obtain n^2. As an alternative, the stack can be made part of a ‘centred square’ figure, representing the number $(n - 1)^2 + n^2$, $n \geqslant 1$, by adding the light grey piece to achieve symmetry about the midline of the $(2n - 1)$ column (Figure 1). The circles pick out n^2. If the right-hand part is reflected in the dashed line, each circle marks a unit of the stack, as required.