A DYNAMIC SCALING LAW FOR SOLAR AND STELLAR FLARE LOOPS

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Abstract We discuss an ordinary differential equation which describes how the pressure in a coronal loop may evolve in time under the influence of a uniform, but time varying heating rate.

During the gradual phase of a flare, the most violent motions have ceased. To a good approximation, the pressure P in the corona should be uniform, although changing with time as evaporation or condensation occurs. Our goal is to develop a simple equation to study the evolution of the corona during the gradual phase without having to resort to numerical simulations.

The energy equation can be written

$$\frac{3}{2}\dot{P} = Q - R - \frac{5}{2}P\frac{d\upsilon}{dz} - \frac{dF_c}{dz}\ , \tag{1}$$

where Q is the volumetric heating rate, R is the radiative loss rate, υ is the plasma velocity, $F_c = \kappa_0 T^{5/2} dT/dz$ is the conductive flux, and z is the distance measured from the loop apex toward one of the loop footpoints. The plasma is assumed to be fully ionized, and obeys the equation of state $P = 2nkT$. The radiative loss rate is assumed of the form $R = n^2\Lambda(T)$, where we assume $\Lambda(T) = AT^{\alpha}$, with T being the plasma temperature. The power law index is taken to be $\alpha = -1/2$, consistent with radiative losses at flaring temperatures. Since the pressure P is assumed uniform, its time derivative is as well. Integrating equation (1) from apex to footpoint, the enthalpy and conductive flux terms vanish if the integration is extended into the topmost layer of the chromosphere. The resultant equation is

$$\frac{3}{2}\dot{P} = \langle Q\rangle - \langle R\rangle\ . \tag{2}$$

The quantities $\langle Q\rangle$ and $\langle R\rangle$ are the heating and radiative loss rates averaged over the loop length. If these can be expressed in terms of the loop pressure, then equation (2) provides a way of solving for the coronal pressure as a function of time. In this paper, we assume that Q is uniform in the loop for simplicity. This leaves only the determination of $\langle R\rangle$.

In our first approach to this problem, we treat the temperature structure in the corona as though it were determined to zeroth order by a static loop, and examine the effect of first order perturbations due to evaporation or condensation. This means that to zeroth order, the temperature structure in the corona is determined by

$$Q - R - \frac{dF_c}{dz} = 0 \ . \qquad (3)$$

The solution of equation (3) is straightforward. In its integrated form, it leads to the well known loop scaling law (Craig, McClymont and Underwood, 1978; Rosner, Tucker, and Vaiana, 1978). For our choice of parameters of the loss function, its precise form is

$$LPT_A^{\alpha/2-11/4} = \mathrm{B}\left[\frac{11/4-\alpha/2}{2-\alpha}, \frac{1}{2}\right] \left[2k^2\kappa_0(\alpha+3/2)/A\right]^{1/2} / (2-\alpha) \ , \qquad (4)$$

where L is the apex to footpoint looplength, κ_0 is the Spitzer coefficient, T_A is the apex temperature, and $\mathrm{B}(a,b)$ is the beta function. The position within the loop is related to the temperature by

$$(1 - z/L) = \mathrm{I}_t\left[\frac{11/4-\alpha/2}{2-\alpha}, \frac{1}{2}\right], \qquad (5)$$

where $t = (T/T_A)^{2-\alpha}$, and $\mathrm{I}_x(a,b)$ is the incomplete beta function. Equations (3-5) are sufficient to evaluate $<R>$ if we ignore the enthalpy flux due to evaporation or condensation. We then find

$$<R> = \frac{\mathrm{B}\left[\frac{\alpha/2+3/4}{2-\alpha}, \frac{1}{2}\right]}{\mathrm{B}\left[\frac{11/4-\alpha/2}{2-\alpha}, \frac{1}{2}\right]} R_A \ , \qquad (6)$$

which for $\alpha = -1/2$ gives $<R> = 7/2\, R_A$. R_A is the radiative loss rate at the loop apex. The fact that $<R>$ is 3.5 times larger that R_A means that the bulk of the radiative losses are coming from near the base of the loop where the temperature is cooler than that at the apex. We find it useful to define an effective radiative loss temperature T_e by setting $R(T_e) = <R>$, from which we find

$$T_e = (2/7)^{2/5}\, T_A = 0.606 T_A. \qquad (7)$$

In the static loop model, this occurs at a distance of 89% of L measured from the apex.

Before we examine the effects of enthalpy flux, we digress briefly to discuss the continuity equation

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}\, n\upsilon = 0. \qquad (8)$$

If we divide both sides by n and assume that $\dot{n}/n$ is uniform (which must be true if the loop structure is determined by a sequence of static loop models) then this equation has the solution

$$n\upsilon = -(\dot{n}/n)\, N(z) \ , \qquad (9)$$

where $N(z)$ is the column density $\int_0^z n\, dz'$. From this we find that

https://doi.org/10.1017/S0252921100154521 Published online by Cambridge University Press

$$\frac{\partial \upsilon}{\partial z} = -(\dot{n}/n)\ (1 + 2kN(z)/P\ \frac{dT}{dz}\ ) . \quad (10)$$

We note that the scaling law tells us that for $\alpha = -1/2$, $\dot{n}/n = (2/3)\ \dot{P}/P$.

The enthalpy term $\frac{5}{2} P \frac{d\upsilon}{dz}$ will act to reduce the radiative losses low in the corona during evaporation and increase them during condensation, thereby altering the mean loss rate $<R>$ from its static value in equation (6). To first order, the change in the mean loss rate should be just balanced by the enthalpy term near T_e. We will therefore add this correction to the mean loss rate. All of the quantities in equation (10) needed to evaluate this term can be determined from equations (3-5). At T_e, we find

$$\frac{5}{2} P \frac{d\upsilon}{dz} = \frac{5}{2}\ \frac{2}{3}\ 0.662\ \dot{P} = 1.103\ \dot{P} . \quad (11)$$

This means that to first order in $\dot{P}$, equation (6) can be replaced with

$$<R> = (7/2)\ R_A - 1.103\ \dot{P} . \quad (12)$$

When substituted into equation (2), this results in the loop evolution equation

$$0.40\ \dot{P} = Q - (7/2)\ R_A . \quad (13)$$

Equation (13) can be solved for the time evolution of the coronal pressure. Since the quantity R_A involves just apex densities and temperatures, these can be converted into pressure from the equation of state and the scaling law. Equation (13) is therefore a first order ordinary differential equation for the loop pressure as a function of time given a uniform, time-varying heating function Q. If Q is constant, then the equation is separable and is solvable by quadrature. The solution in this case is monotonic with time. We feel this argues strongly against the "limit cycle" behavior suggested by some authors (*e.g.* Kuin and Martens, 1982).

The approximation of equation (2) by equation (13) assumes that the loop structure is always fairly close to the static scaling law solution. While this should be the case if the heating rate changes on the conductive time scale or longer, more rapid changes in the heating rate will result in evaporating or condensing loops far from static equilibrium. In this case, one must do a more careful job of estimating the average loss rate. We are presently working on such extensions of the model.

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https://doi.org/10.1017/S0252921100154521 Published online by Cambridge University Press