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6th and 8th Order Hermite Integrator Using Snap and Crackle

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Abstract. We present sixth- and eighth-order Hermite integrators for astrophysical N-body simulations, which use the derivatives of accelerations up to second order (*snap*) and third order (*crackle*). These schemes do not require previous values for the corrector, and require only one previous value to construct the predictor. Thus, they are fairly easy to be implemented. The additional cost of the calculation of the higher order derivatives is not very high. Even for the eighth-order scheme, the number of floating-point operations for force calculation is only about two times larger than that for traditional fourth-order Hermite scheme. The sixth order scheme is better than the traditional fourth order scheme for most cases. When the required accuracy is very high, the eighth-order one is the best.

Keywords. stellar dynamics, methods: n-body simulations, methods: numerical

1. Direct calculation of higher order derivatives

The gravitational acceleration from particle j to particle i and its first three time derivatives (we call them *jerk*, *snap* and *crackle*) are expressed as

$$\boldsymbol{A}_{ij} = m_j \boldsymbol{r}_{ij} / r_{ij}^3, \tag{1.1}$$
$$\boldsymbol{J}_{ij} = m_j \boldsymbol{v}_{ij} / r_{ij}^3 - 3\alpha \boldsymbol{A}_{ij}, \tag{1.2}$$
$$\boldsymbol{S}_{ij} = m_j \boldsymbol{a}_{ij} / r_{ij}^3 - 6\alpha \boldsymbol{J}_{ij} - 3\beta \boldsymbol{A}_{ij}, \tag{1.3}$$
$$\boldsymbol{C}_{ij} = m_j \boldsymbol{j}_{ij} / r_{ij}^3 - 9\alpha \boldsymbol{S}_{ij} - 9\beta \boldsymbol{J}_{ij} - 3\gamma \boldsymbol{A}_{ij}, \tag{1.4}$$

with

$$\alpha = (\boldsymbol{r}_{ij} \cdot \boldsymbol{v}_{ij}) / r_{ij}^2, \tag{1.5}$$
$$\beta = (|\boldsymbol{v}_{ij}|^2 + \boldsymbol{r}_{ij} \cdot \boldsymbol{a}_{ij}) / r_{ij}^2 + \alpha^2, \tag{1.6}$$
$$\gamma = (3\boldsymbol{v}_{ij} \cdot \boldsymbol{a}_{ij} + \boldsymbol{r}_{ij} \cdot \boldsymbol{j}_{ij}) / r_{ij}^2 + \alpha(3\beta - 4\alpha^2), \tag{1.7}$$

where $\boldsymbol{r}_i$, $\boldsymbol{v}_i$, $\boldsymbol{a}_i$, $\boldsymbol{j}_i$ and m_i are the position, velocity, total acceleration, total jerk and mass of particle i, and $\boldsymbol{r}_{ij} = \boldsymbol{r}_j - \boldsymbol{r}_i$, $\boldsymbol{v}_{ij} = \boldsymbol{v}_j - \boldsymbol{v}_i$, $\boldsymbol{a}_{ij} = \boldsymbol{a}_j - \boldsymbol{a}_i$ and $\boldsymbol{j}_{ij} = \boldsymbol{j}_j - \boldsymbol{j}_i$ (Aarseth 2003).

The increase of the number of floating point operations required in the high-order schemes is small. In the fourth-order scheme (Makino 1991a; Makino & Aarseth 1992), we need 60 operations for one calculation of acceleration and jerk, if we count 10 operations

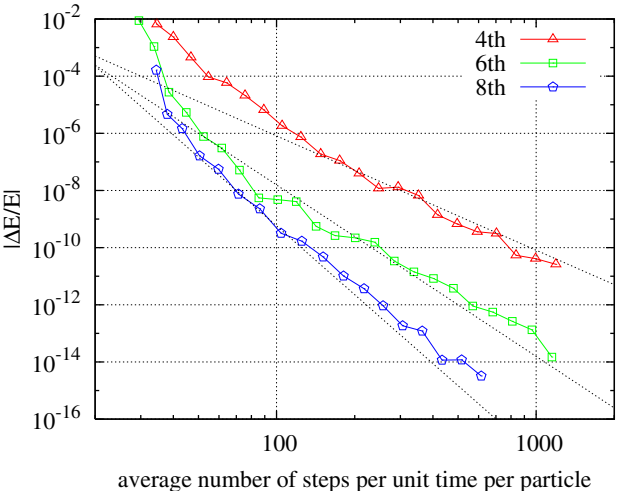

Figure 1. Maximum relative deviation of the total energy during the time integration for 10 time units, as a function of average number of timesteps per particle per unit time. Triangles, squares and pentagons represent the results of 4th-, 6th- and 8th-order schemes. The three dotted lines indicate the expected scaling relations for 4th-, 6th- and 8th-order algorithms.

for each division and square-root (Warren *et al.* 1997; Nitadori *et al.* 2006). In the sixth-order scheme, we need 97 operations for the acceleration, jerk and snap and even in the eighth-order scheme, we need only 144 operation for up to the crackle term (Nitadori & Makino 2007).

Fig. 1 shows the relation between the relative energy error after the integration for 10 time units of a 1024-body Plummer model and the average number of timesteps per particle per unit time. Here, we used the standard N-body unit (Heggie & Mathieu 1986), a softened potential with $\varepsilon = 4/N = 1/256$, and the block timestep algorithm (McMillan 1986; Makino 1991b) where all timesteps are restricted to be powers of two.

We can clearly see that the error of sixth- and eighth-order schemes are proportional to Δt^6 and Δt^8, as expected. For the relative accuracy of 10^{-8}, the sixth-order scheme allows the average timestep which is almost a factor of three larger than that necessary for the fourth-order scheme. For the relative accuracy of 10^{-10}, the eighth-order scheme allows the average timestep which is almost a factor of seven larger than that necessary for the fourth-order scheme. Even for the relatively low accuracy of 10^{-6}, the sixth-order scheme allows about a factor of two larger timestep than the fourth-order scheme does.

References

Aarseth, J. S., 2003, *Gravitational N-Body Simulations* [Cambridge Univ. Pr.]
Heggie, D. C. & Mathieu, R. D. 1986, The Use of Supercomputers in Stellar Dynamics, 267, 233
Makino, J., 1991, *ApJ* 369, 200
Makino, J. 1991b, *PASJ*, 43, 859
Makino, J. & Aarseth, S. 1992, *PASJ*, 44, 141
McMillan, S. L. W. 1986, in The Use of Supercomputer in Stellar Dynamics, ed. P. Hut & S. McMillan (New York: Springer), 156
Nitadori, K., Makino, J., & Hut, P. 2006, *New Astronomy*, 12, 169
Nitadori, K. & Makino, J. 2007, ArXiv e-prints, 708, arXiv:0708.0738
Warren, M. S., Salmon, J. K.,Becker, D. J., Goda, M. P., & Sterling, T. 1997, In *The SC97 Proceedings*, CD–ROM. IEEE, Los Alamitos, CA

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