DIELECTRONIC RECOMBINATION IN THE CORONA

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RÉSUMÉ. — *Ce n'est que récemment que l'importance de la recombinaison diélectronique dans les plasmas a été reconnue. On discute de ce processus en détails pour le cas des plasmas de faible densité. En employant le principe de correspondance et des arguments basés sur l'équilibre de détail, on obtient des formules pour la vitesse de ce processus. On passe en revue les applications récentes des formules générales simples à l'estimation du coefficient de recombinaison dielectronique et de l'équilibre d'ionization qui sont en bien meilleur accord avec les observations des largeurs de Doppler des raies coronales.*

ABSTRACT. — *The full importance of dielectronic recombination in plasmas has only recently been realized. The process is discussed in detail for the case of low density plasmas, and formulae for the rate of the process are derived using Correspondence Principle and Detailed Balance arguments. Recent work in the derivation of a simple General Formula for estimation of dielectronic recombination coefficients and on the ionization balance for Iron and Calcium ions in the corona is reviewed. The new ionisation balance curves are in much better agreement with observations on the Doppler widths of coronal lines.*

Резюме. — Лишь только недавно была призана важность диэлектронной рекомбинации в плазмах. Этот процесс подробно обсужден для случая плазми малой плотности. Используя принцип соответствия и основанные на детальном равновесии аргументы, получены формулы для скоростей этих процессов. Просмотрены новые применения простых общих формул к оценке коэффициента диэлектронной рекомбинации и ионизационного равновесия ионов железа и кальция в короне. Новые кривые ионизации находятся в намного лучшем согласии с наблюдениями допплеровских широт корональных линий

The possibility of the process of dielectronic recombination in plasmas has of course been realised for some time, and some estimates of its rates for some specific ions have been made (see e. g. MASSEY and BATES [1], BATES and DALGARNO [2], BATES [3]). Also, at the suggestion of UNSÖLD (private communication), SEATON [4] discussed its role in the corona. However, partly due to the fact that the existing calcu ations were all for electron temperatures T_e such that $kT_e \ll E$, where E is the excitation energy of a typical resonance line of the recombining ion (i. e. for plasmas such that the free electrons play an insignificant role in maintaining the ionization of the plasma) — in which case dielectronic recombination is usually negligible — and partly I think due to the influence of the form into which the equations were cast on the way of thinking about the process (see below), the importance of the process was not fully realised until recently [5]. Here, I would like to generalize some of the points in that paper and review further work in hand.

Dielectronic recombination is the result of the two processes

$$(1) \qquad X^{+(z)}(i) + e(E, l') \rightleftarrows X^{+(z-1)}(j, nl),$$

$$(2) \qquad X^{+(z-1)}(j, nl) \rightarrow X^{+(z-1)}(k, nl) + h\nu,$$

where $X^{+(z)}(i)$ is a z-times ionized atom in the state i (we will assume for simplicity that i belongs to the ground configuration) and $X^{+(z-1)}(j, nl)$ is a $(z - 1)$-times ionized atom in a doubly excited state (j, nl). In general i may be a many (say q)-electron state, while (j, nl) will be a $(q + 1)$-electron state based on a q-electron state j plus one electron in an orbital nl. In almost all cases the dominant contributions come from states such that the transition $i \rightarrow j$ is optically allowed, k belongs to the same configuration as i, and $l' = l + 1$. Also, in most cases the important states correspond to quite large values of n and l so that to a good approximation

$$(3) \qquad \frac{E}{I_H} = (z+1)^2 \left(\frac{1}{\nu^2} - \frac{1}{\nu'^2}\right) - \frac{z^2}{n^2}.$$

In order to obtain the rates for the processes in (1) from ordinary excitation cross-section data, we may use Correspondence Principle arguments. We consider a band of incident electron energies lying between E and $E + dE$, and such that $dE \gg 2z^2 n^{-3}$ I_H (i. e. we include several n states within the band). The number of left to right processes in (1) per second is

$$(4) \quad N(X^{+(s)}(i)) \frac{dN_e}{dE} - Q(i, El' \rightarrow j, E'l)\, dE,$$

where

$$(5) \qquad E' = E - (E_j - E_i),$$

v is the incident electron velocity, N_e is the electron density, Q is the cross-section for the process and E_i, E_j are the energies of the states i and j respectively. Classically $Q\, dE$ varies smoothly as E varies from small positive to small negative values, so, by the Correspondence Principle, we may assert that the quantum mechanical $Q\, dE$ extrapolates smoothly from above to just below threshold.

We may now for convenience set $dE = 2I_H z^2 n^{-3}$ and deal with individual n states instead of a band of such states, provided that we finally sum over several n states.

The right to left rate in (1) may be related to the left to right by considering a system in thermal equilibrium and using the SAHA and BOLTZMANN equations and detailed balance arguments, However, we are not completely dependent on these arguments since if the left to right and right to left rates are calculated separately in perturbation theory the same relationship between the two rates results. (These arguments are entirely analogous to those which may be used to deduce the relation between the Einstein A and B coefficients). Also, SEATON (private communication) has shown that the same relationship may be deduced by the Quantum Defect Method. The right to left (Auger) transition probability is given by

$$(6) \quad A_a(j, nl \rightarrow i, El') = \frac{\omega(i)}{\omega(j, nl)} \frac{16\pi m}{h^3} Q(i, El' \rightarrow j, nl)\, E\, dE,$$

where $\omega(s)$ is the statistical weight of the state s.

The rate for process (2) is of course given by the usual Einstein coefficient $A_r(j, nl \rightarrow k, nl)$.

The dielectronic recombination coefficient for a given initial state i and intermediate state (j, nl) is then easily shown to be

$$(7) \quad \alpha_d(i\,;\, j, nl) = \frac{U \dfrac{dN_e}{N_e\, dE} \sum\limits_{l'} Qi, E(l' \rightarrow j, nl)\, dE \sum\limits_k A_r(j, nl \rightarrow k, nl)}{\sum\limits_k \left[A_r(j, nl \rightarrow k, nl) + \dfrac{16\pi m\, \omega(k)}{h^3\, \omega(j, nl)} \sum\limits_{l'} Q(k, El' \rightarrow j, nl)\, E dE \right]}$$

$$(8) \quad \alpha_d(i\,;\, j, nl) = \frac{h^3}{8\pi m^2 v} \frac{dN_e}{N_e\, dE} \frac{\omega(j, nl)}{\omega(i)} \frac{\sum\limits_{l'} A_a(j, nl \rightarrow i, El') \sum\limits_k A_r(j, nl \rightarrow k, nl)}{\sum\limits_k [A_r(j, nl \rightarrow k, nl) + \sum\limits_{l'} A_a(j, nl \rightarrow k, El')]}.$$

The total dielectronic recombination coefficient for a given initial state i is

$$(9) \qquad \alpha_d(i\,;\, \mathrm{tot}) = \sum_{jnl} \alpha_d(i\,;\, j, nl),$$

and the recombination rate is

$$(10) \qquad \sum_i N(X^{+(s)}(i))\, N_e\, \alpha_d(i\,;\, \mathrm{tot}).$$

Earlier treatments [1], [2], [3], [4] made the simplifying assumption that in (8), $k = i$, and

$$(11) \qquad A_a \gg A_r,$$

so that (with a Maxwellian distribution for dN_e/dE)

$$(12) \quad \alpha_d(i\,;\, j, nl) = (2\pi)^{1/2} e^2 h m^{-5/2} c^{-3} k^{-3/2} \frac{\omega(j, nl)}{\omega(j)} (h\nu)^2 T^{-3/2} e^{-E/kT} f(j, i),$$

where $f(j, i)$ is the $i \rightarrow j$ oscillator strength. However, as is discussed in [5], (12) leads to a divergent summation in equation (9). In fact one must keep the denominator in (7) or (8) intact, since, for large l, $A_a \ll A_r$. The summation (9) then converges but can give much larger values of $\alpha_d(i\,;\, \mathrm{tot})$ than previously expected on the basis of earlier explicit calculations (e. g. [3]) which were for temperatures T so low that $e^{-E/kT}$ is very small and decreases rapidly as n increases from small values so that only the first terms in the summation in (9) are effective. Also, the form of equation (12) has, I think, been misleading in that the oscillator strength which appears in it tends to make one think in terms of *upward* transitions [4] rather than downward, and many of the states which are of importance cannot be reached by absorption from the ground configuration of the recombined ion.

The relative sizes of A_a and A_r may be seen from figure 1 which shows the quantity $(2l + 1) \frac{A_a}{A_r + A_a}$ as a function of n and l for the case of recombination of $Fe^{+(15)} + e$ with $i = 3s$ and $j = 3p$. This quantity also indicates the relative contributions to α_d (tot) which come from various values of n and l. We see that quite large values of n and l are important, which indicates that only very small errors should be involved in extrapolating cross-sections as described above.

The values of α_d (tot) obtained may exceed the corresponding radiative recombination coefficients by quite large factors [5] and largely remove the discrepancy [6] between coronal temperatures deduced from ionization balance calculations and from observed Doppler line broadening. Coronal ionization balance calculations including dielectronic recombination have now been carried out for Iron [7] and for Calcium [8]. One important

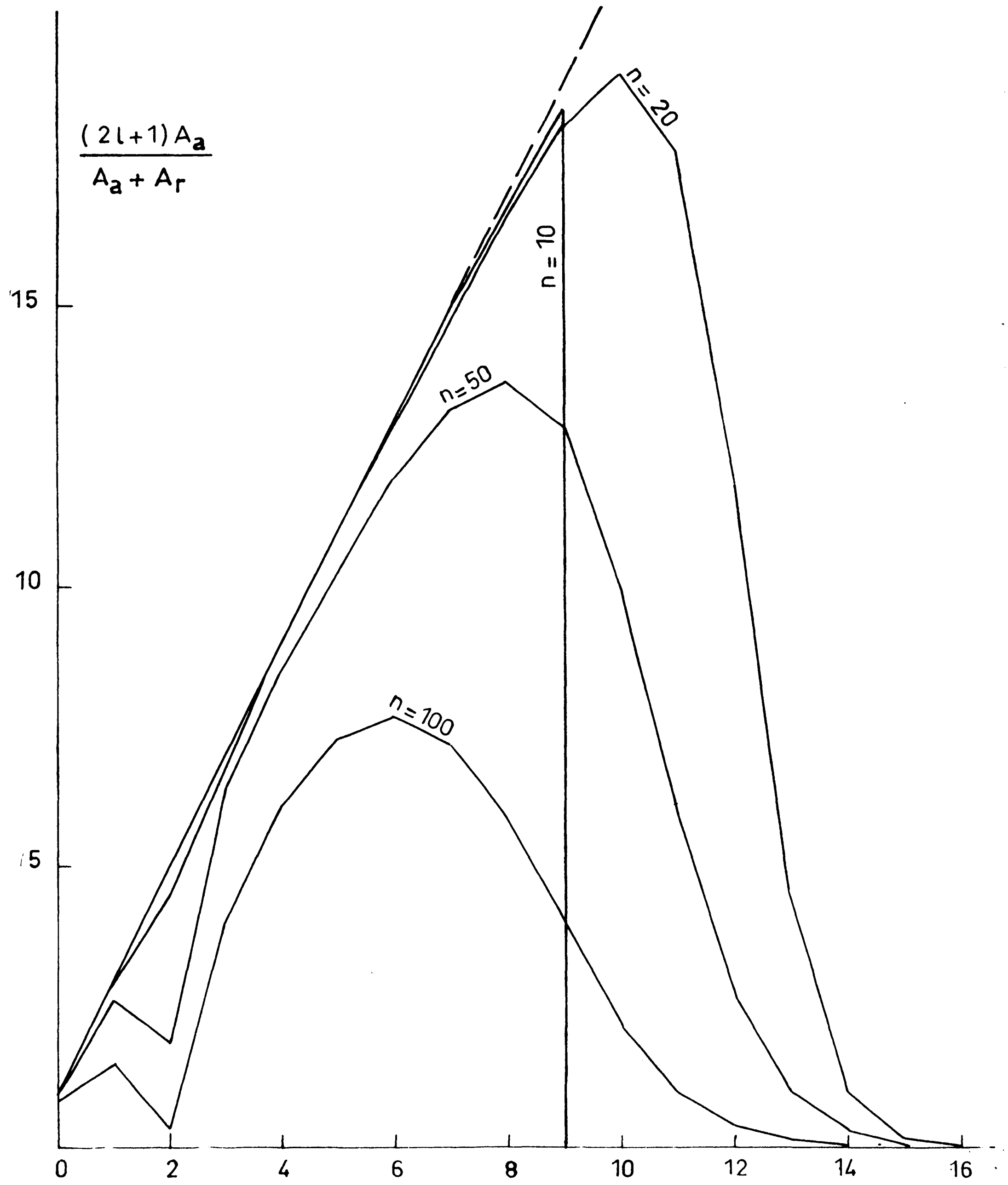

FIG. 1.

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feature of the new ion abundance versus temperature curves is that they have much flatter maxima, which makes it much easier to understand a range of observed line width temperatures for a given stage of ionization of an element, and to understand observations of emissions from several stages of ionization of an element originating from a small region of the corona. The Fe^{+9}/Fe curve reaches its maximum at about 1.2×10^6 °K and falls to 0.1 of its maximum at about 0.6×10^6 °K and 1.9×10^6 °K ; the corresponding temperatures for Fe^{+13}/Fe are 2.3×10^6, 1.5×10^6 and 4.0×10^6 °K, while for Ca^{+14}/Ca they are 5.7×10^6, 3.5×10^6 and 8.3×10^6. Work on other elements is in progress.

It is easily seen from equations (7) or (8) (also fig. 1) that if $A_a \gg A_r$, α_d $(i\ ;\ j,\ nl)$ is virtually independent of A_a (i. e. of the extrapolated cross-section Q), while if $A_a \ll A_r$,

$$(13)\ \alpha_d\ (i\ ;\ j,\ nl) \simeq v\ \frac{dN_e}{N_e\, dE} \sum_{l'} Q(i, El' \rightarrow i, nl\ )\ dE.$$

Also, the largest contributions to α_d (tot) come from those states for which $A_a \gg A_r$. It is hence obvious that the values of α_d (tot) we obtain should be appreciably more accurate than the cross-section data on which the calculation is based. From this point of view the situation is very satisfactory, especially since a FORTRAN programme is available so as to make the cross-section extrapolations and calculation of α_d virtually automatic. However, if several cases are to be treated, this still involves quite a lot of work, and in many cases adequate cross-section data is not available. For these reasons it was decided to attempt to develop a simpler, if less accurate, general formula for α_d (tot). In spite of the complexity of the problem such a general formula has in fact now been found [9] and estimates of α_d (tot) for plasmas of low electron and radiation density (e. g. the solar corona) may very easily be made to about 20 % accuracy.

At higher densities the problem becomes much more complicated still, due to the effect of collisional transitions between the highly excited states. Some work on this problem has been started.

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Discussion

E. SCHATZMAN. — What is the effect of the density ? The atoms with quantum numbers as large as 100 are very big and can be prevented to exist by their surrounding atoms and electrons.

A. BURGESS. — Yes, I must stress that the present calculations refer to the low density limit. I have checked that at coronal densities, the recombination rates for typical coronal ions are probably not significantly altered by density effects, but, of course, they will be at higher densities and lower stages of ionization. (I think the calculations of BATES, KINGSTON and MCWHIRTER on collisional-radiative recombination suggest that Ne/z^7 is probably the relevant parameter). It will be quite difficult to calculate the density effects properly, in addition to collisional transition of the type $nl \rightarrow n'l'$ and $nl \rightleftarrows kl'$ (with nl, large), collisional transition of the type $nl \rightarrow nl'$ will probably be of importance. Work on the problem is in progress.

A. ARKING. — Your calculations of the dielectric recombination coefficient show that the process depends upon highly excited levels for which n is greater than 10 and important contributions come from levels for which $n \sim 100$. If you take into account the broadening of the levels due to collisions and Doppler effect will not the levels above some critical value of n be wiped out and merge with the continuum ?

A. BURGESS. — With regard to collisional broadening, your question is, I think, the same as that of Schatzman above. I do not think the question of Doppler broadening is really relevant, since this effect just makes the highly excited states unobservable individually, it does not actually destroy them.

A. J. DEUTSCH. — Is it possible yet to estimate whether this process will be important in the ionization equilibrium of the chromosphere and that of interstellar gas ?

A. Burgess. — The process will probably be of importance in all cases where collisional ionization is at all significant.

C. De Jager. — Could you estimate the influence of the new cross-sections on coronal temperatures deduced from the intensities of the forbidden lines in visual spectrum ?

A. Burgess. — The maxima of the ion distribution curves move to higher temperature as stated above. The new curves are in general much less shapely peaked and are reconciliable with Fe XV, Fe XIV and Ca XV Doppler temperatures quoted in Seaton's review article in *Planetary and Space Science*.

L. Biermann. — What would be the effect of your results on the question of how much (mechanical) energy is needed for maintaining the stationary state of the corona ? I understand that more energy is needed than appeared before, but how much more ?

A. Burgess. — I have not estimated the increase in the energy loss due to increased electron temperature.

L. Gratton. — Can these new processes help to reduce the discrepancy between abundances determinations in the solar corona and in the solar photosphere ?

A. Burgess. — I do not think the deduced abundances can be altered much because the line excitation rate will usually be appreciably greater than the ionization rate (and hence the recombination rate) due to the excitation energy being much smaller than the ionization energy in many cases.

L. Goldberg. — (About A. Burgess' reply or L. Gratton.) What you have said refers to the determination of abundances from the forbidden lines but not to those from the far U. V. emission lines.

A. Burgess. — I agree, the lines having excitation energies comparable with the ionization energy will need re-examination.