This is the climax of a sequence of four problems starting with the more modest 'No perfect powers of 2 and 3 differ by exactly 1, except 2 and 3, 4 and 3, and 8 and 9'.

Several other problems are of an 'elementary number theory' kind (and some not-so-elementary), for example 'The number 1 is not a congruent number, i.e. there is no right-angled triangle with rational sides and area 1', a result due to Fermat; 'Find a large family of integer solutions of $A^4 + B^4 = C^4 + D^4$', a problem with a long history going back to Euler and including Peter Swinnerton-Dyer when he was a scholar at Eton; and 'Let a and b be positive integers such that $q = (a^2 + b^2)/(ab + 1)$ is also an integer. Then q is a square number.' This evidently comes from the 1988 International Mathematical Olympiad, where it did not have so much as a hint.

Some of the problems are decidedly odd (to be included in a problem book, that is) such as the proof that $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$, where if you remember Euler's brilliant misuse of known results then you might have a chance, but a rigorous proof is not for the amateur. Other 'standard result' problems sit more happily, such as the Euler-Feuerbach result about the altitudes of a triangle: 'Let H be the intersection of the altitudes AD, BE, CF of a triangle ABC. Then the midpoints of the sides, the midpoints of the segments AH, BH, CH and the feet of the altitudes all lie on a circle'; or Sylvester's result that every rational number in (0, 1) is the sum of a finite number of reciprocals of distinct natural numbers. Inequalities and identities are scattered through the problems: Tepper's (factorial) identity, Dixon's identity, Hilbert's (square-summable sequence) inequality, Bessel's inequality, inequalities for the central binomial coefficient. Even the Monty Hall problem, the monkey and the coconuts and gambler's ruin find their place.

I have one complaint: I hope that the publisher and not the author is responsible for the sentence in the blurb on the back cover describing the author as 'teaching the very best undergraduates in England'. Brilliance arises everywhere and in any one year who knows where the 'very best undergraduates' are? Cambridge is not the centre of the universe.

So, a very individual problem book, with a huge range of subject matter and difficulty. There is material here for many purposes, even for enrichment material for advanced sixth formers or undergraduates, but you have to dig for it. Many of the problems are very specialised and (to me) not so attractive, but there is certainly something here, if not for everyone, then for a wide range of mathematical readers.

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Once upon a prime by Sarah Hart, pp 304, £14.95 (hard), ISBN 978-1-25085-088-1, Flatiron Books (2023)

Who is the ideal writer of a book for the lay reader that explores what maths and literature have in common? An expert mathematician, of course. Also, someone able to convey mathematical ideas to the general public—the sort of mathematician who might for example make a good Gresham Professor of Geometry, a chair established for that purpose. And a lover of literature, the sort of person who might sit down to read the entirety of the Booker Prize shortlist every year. Step forward Sarah Hart, who ticks all these boxes and more, and has written a delightful book on the 'wondrous connections between mathematics and literature'.

That the book is split into three parts will come as no surprise to anyone who has read chapter 5, in which Hart explains the number 3's 'astonishing hold on Western minds'. Trichotomies are here, there and everywhere. Part I of the book explores the structure of literary texts, Part II the literary manifestations of mathematics, and Part III is concerned with mathematical ideas and characters in fiction. The literature surveyed is pleasingly diverse: old and new, high and low, Western and non-Western. There's something for everyone.

Hart has made a tremendous effort to keep the maths elementary, and to her credit has succeeded. Primes, regular polygons, and other basic mathematical concepts are all defined. This is genuinely a work for the general public, unlike many other half-hearted attempts. Naturally, keeping the mathematics very simple limits the range of topics tackled, but there is still plenty to fill an entertaining and instructive book. The discussion of non-Euclidean geometry and Edwin Abbott's *Flatland*, for example, is a treat. I also particularly enjoyed Hart's mathematical proof of the impossibility of giants, at least giants as they have come down to us in literature. Gulliver during his travels could not really have met Brobdingnagians—giants twelve times our size—as their femurs could not have supported their weight, since the pressure on their scaled-up bones would have been twelve times that on ours. Pressure, you see, is proportional to volume over area, so if you scale lengths by 12 you also multiply pressure by the same amount. King Kong would for the same sort of reason barely manage to slump around, let alone jump up and down skyscrapers.

The book is full of fun asides and interesting anecdotes. We learn for example that Blaise Pascal once fended off a terrible toothache by thinking of cycloids. In an echo of that tactic, the teenage Hart used to bring to mind Pascal's triangle as a distraction from local boys' catcalls. Then there's Tryphiodorus' lipogrammatic (letter-omitting) version of the Odyssey. It cleverly tells the story of Odysseus's return by omitting the first letter of the Greek alphabet (alpha) in its rendering of Book I, the second letter (beta) in Book II, and so on throughout the whole epic, which conveniently has as many Books as there are letters of the Greek alphabet. Alas, it's no longer extant – what a sad loss! With the use of another ancient example, this time from Herodotus, Hart illustrates the concept of steganography, or 'hidden writing', whereby you hide the message rather than put it in code (the latter would be cryptography). One ancient ruler devised a clever way to send a message to another one: he had a trustworthy slave's hair shaved, tattooed the message onto the slave's scalp, waited for the slave's hair to regrow and then sent him off to the intended recipient, who duly shaved the slave's head to read the message.

Is there anything I would have liked to see done differently? Hart has fun pre-emptively asking the reader to forgive her any errors. Borges's Library of Babel contains every possible book, and therefore, Hart has us playfully imagine, it also contains all close approximants of her own book. Perhaps any reader who like me comes across the statement on page 202 that Julius Caesar was a Roman emperor has picked up one of these near-misses rather than the true, flawless version of *Once Upon A Prime*.

I suppose that even near-misses of the book would contain some tenuous connections. Mathematics may be the science of abstract structure, but usually it is the science of drawing consequences from some principles. Bare structural facts do not always count as mathematical. For example, to my mind, that a book omits a certain vowel, or includes only one type of vowel (Perec's La Disparition and Les Revenentes are respective examples), is not really a mathematical property. Part I, fascinating

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though it is, throws up a few of these not quite mathematical properties. In other places, I would have liked to see connections more thoroughly explored. Tolstoy was famous for rejecting the 'great man' theory of history. In an overtly mathematical analogy, he saw history as governed by 'integrating' over all the small individual tendencies—differentials, as it were—rather than the actions of one or two people taken in isolation. Likening historical forces to the calculus is an interesting idea. But does it hold water? Well, sorry to nitpick, but the calculus as Tolstoy would have known it deployed infinitesimals, and only finitely many humans contribute to a historical event. Moreover, it's unclear whether there are any laws of history, and it's even less clear that history is a closed system, determined by humans, somehow splendidly isolated from other natural phenomena. If Tolstoy had lived on another few decades, he might have learned, through chaos theory, that a small disturbance by even a single individual can have an immense impact. History as calculus, then, is an intriguing thought, but if it genuinely forms the intellectual backbone to *War and Peace's* 100+-page Epilogue, a gentle challenge to the idea would not have been amiss.

However, to elaborate on these sorts of concerns would have changed the character of the book, probably for the worse. The book's main aim is to enhance our appreciation of mathematics and literature by exploring their connections and, as far as that goes, we can safely say that it has done its job.

And so to the final little mathematical twist. Throughout the book, especially in Part I, we are told about the various mathematical games literary authors play. They might be hidden occurrences of the number 42 in *Alice Through the Looking-Glass*; or the book might be structured in a mathematically interesting way, a prime example being Eleanor Catton's *The Luminaries*, in which each chapter is half the length of the previous one. That the number of *Once Upon A Prime's* pages is mathematically interesting should come as no surprise, then. It is 256 or 2^8.

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The spirit of mathematics by David Acheson, pp 186, £15.99, ISBN 978-0-19-284508-5, Oxford University Press (2023)

The author's previous books, published by OUP and with a similar format, have all been reviewed in this journal [1, 2, 3]. They share a consistency in approach, in that they all aim to display the joys of the subject in a way which neither baffles nor condescends to the likely readership. The first book covered a wide range of topics from Pythagoras' to number theory, whilst the second and third had, as befits their titles, a more limited focus. There is no doubt that David Acheson is a first-rate communicator and an ambassador for the subject which is dear to *Gazette* readers. However, the obvious question remains: why a fourth book of a similar nature to the first three?

Actually there is not a great deal of overlap. If there is a unifying theme to the narrative, it is algebra, and the author is at pains to show why this area of mathematics, which is clearly unfamiliar to most politicians who are offering 'advice' on how to teach the subject, is important and intriguing. However, I am not sure that the best way to explain to a sceptic why $- \times - = +$ is to derive it formally from the rules of algebra. My own approach would be purely arithmetical and might involve a scenario where students in a communal house are both raiding the fridge for alcoholic refreshment and occasionally topping up the supply. Here 'plus' and